# Corner Detection

CS 543 / ECE 549 – Saurabh Gupta

Spring 2020, UIUC

http://saurabhg.web.illinois.edu/teaching/ece549/sp2020/

Slides from S. Lazebnik.

# Why extract keypoints?

- Motivation: panorama stitching
  - We have two images – how do we combine them?

Source: L. Lazebnik

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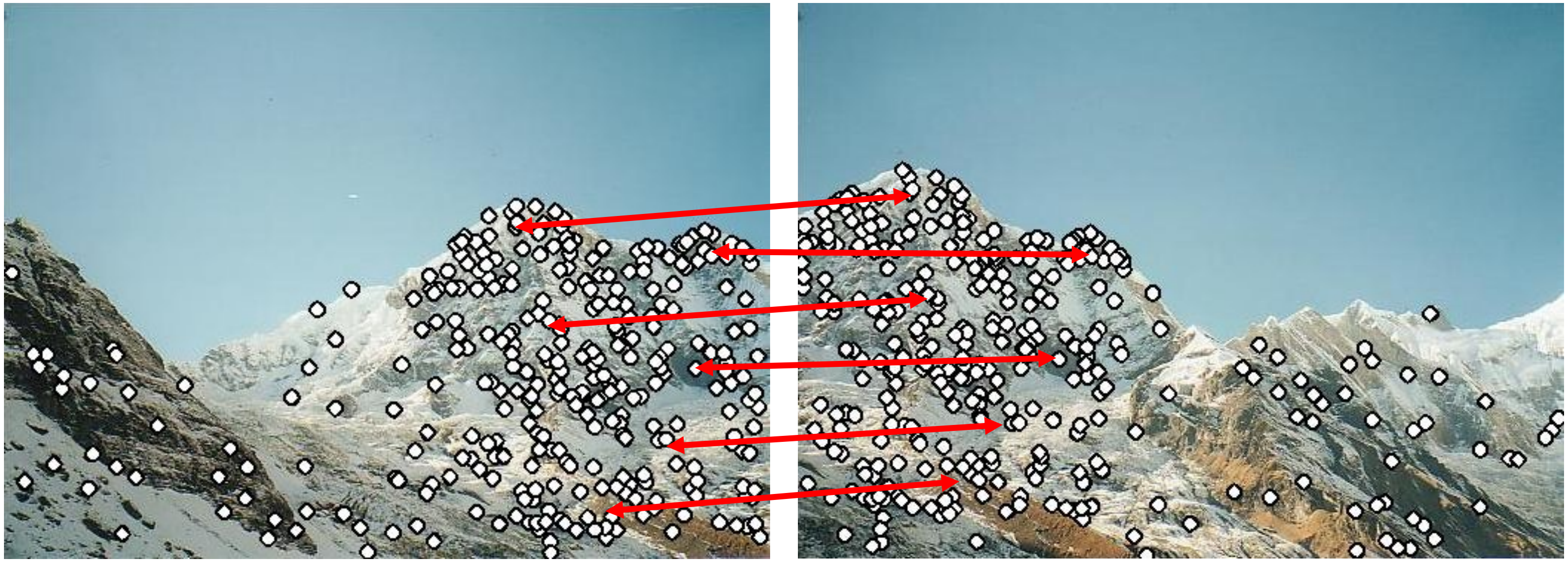


Step 1: extract keypoints

Step 2: match keypoint features

Source: L. Lazebnik

# Why extract keypoints?

- Motivation: panorama stitching
  - We have two images – how do we combine them?

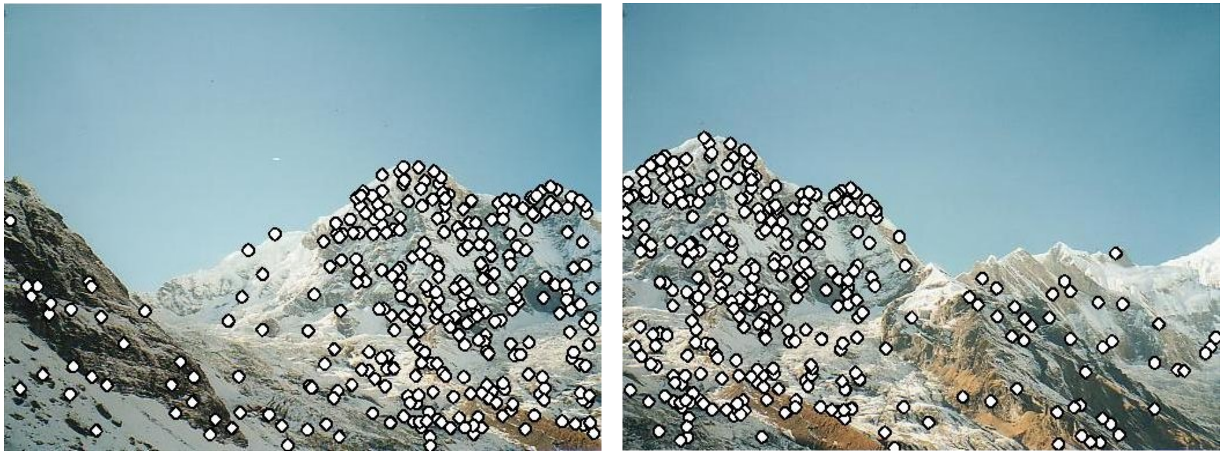
Step 1: extract keypoints

Step 2: match keypoint features

Step 3: align images

Source: L. Lazebnik

# Characteristics of good keypoints



- Compactness and efficiency
  - Many fewer keypoints than image pixels
- Saliency
  - Each keypoint is distinctive
- Locality
  - A keypoint occupies a relatively small area of the image; robust to clutter and occlusion
- Repeatability
  - The same keypoint can be found in several images despite geometric and photometric transformations

Source: L. Lazebnik

# Applications

## Keypoints are used for:

- Image alignment
- 3D reconstruction
- Motion tracking
- Robot navigation
- Database indexing and retrieval
- ~~Object recognition~~

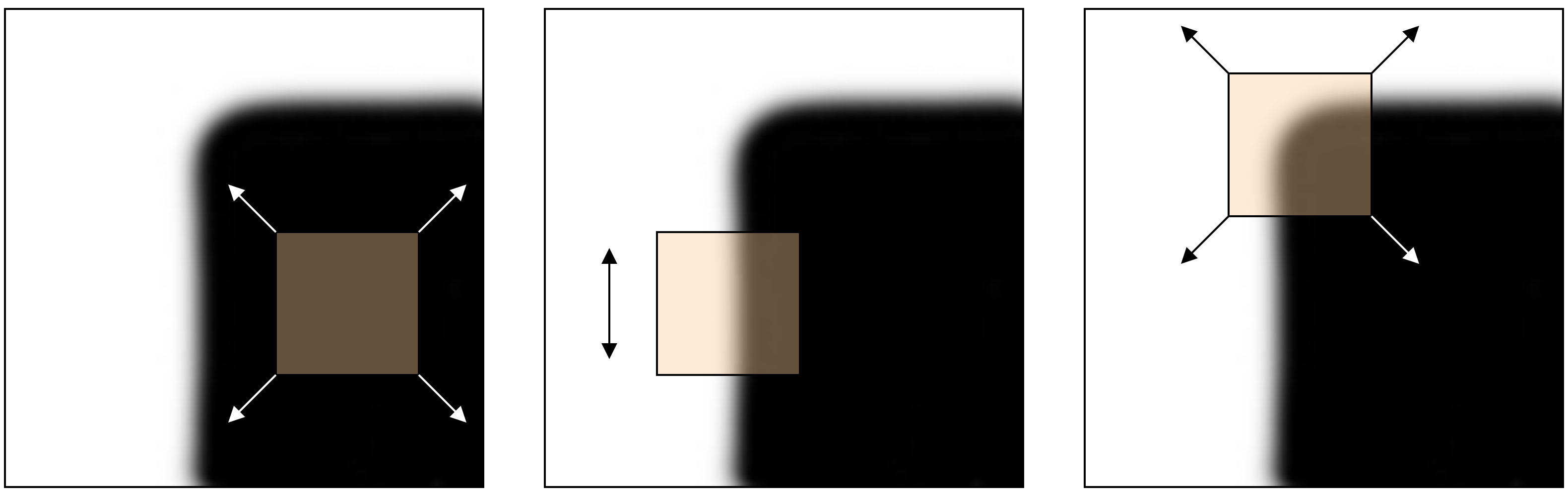
Source: L. Lazebnik

# Corner detection: Basic idea

Source: L. Lazebnik

# Corner detection: Basic idea

- We should easily recognize the point by looking through a small window
- Shifting a window in *any direction* should give *a large change* in intensity



"flat" region: no change in all directions

"edge": no change along the edge direction

"corner": significant change in all directions

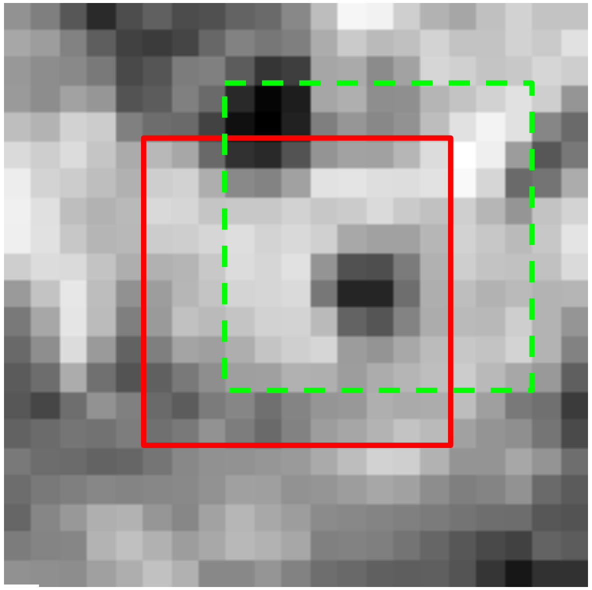
Source: L. Lazebnik

# Corner Detection: Derivation

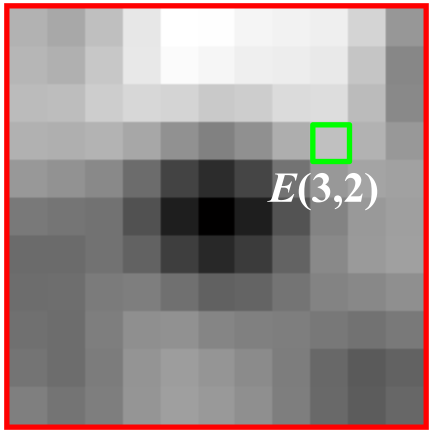
Change in appearance of window *W* for the shift [*u,v*]:

$$E(u,v) = \sum_{(x,y)\in W} [I(x+u, y+v) - I(x,y)]^2$$

$I(x, y)$



$E(u, v)$




Source: L. Lazebnik

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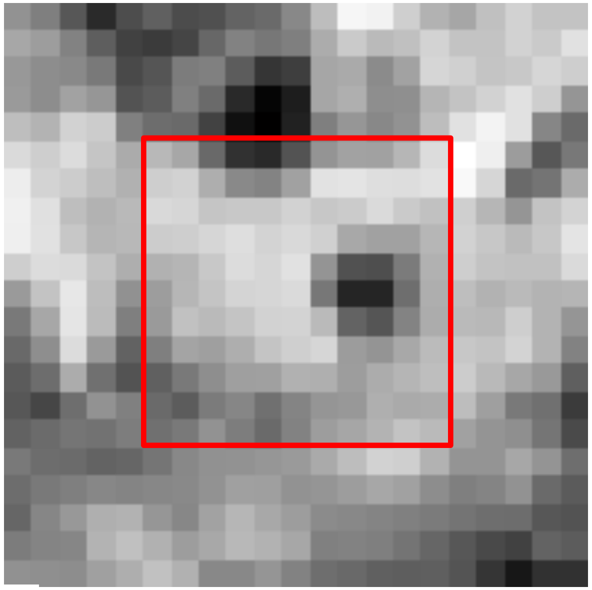
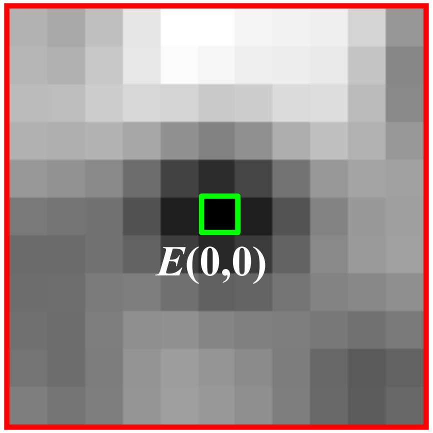


$E(u, v)$




Source: L. Lazebnik

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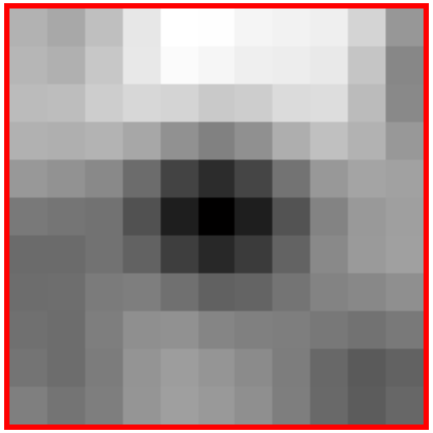
We want to find out how this function behaves for small shifts

$E(u, v)$



Source: L. Lazebnik

# Corner Detection: Derivation

First-order Taylor approximation for small motions [*u*, *v*]:

$$I(x+u, y+v) \approx I(x, y) + I_x u + I_y v$$

Let's plug this into *E*(*u*,*v*):

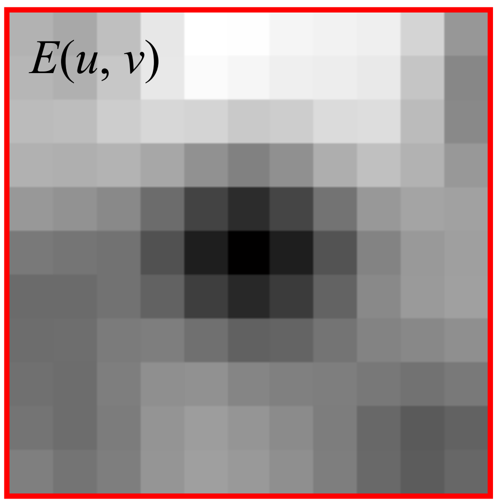
$$E(u,v) = \sum_{(x,y)\in W} [I(x+u, y+v) - I(x,y)]^2$$

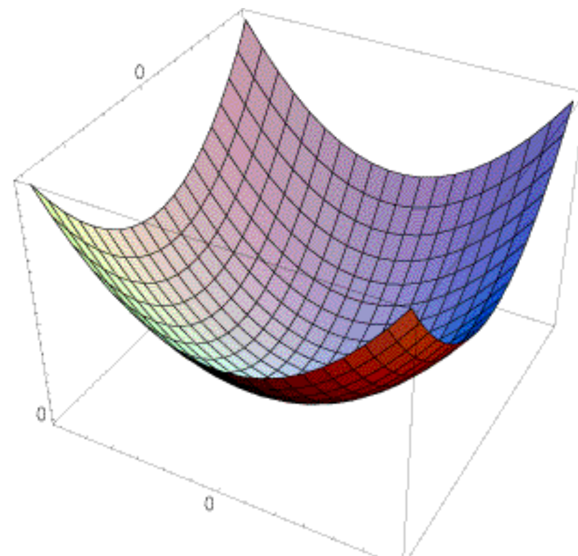
Source: L. Lazebnik

# Corner Detection: Derivation

$E(u,v)$ can be locally approximated by a quadratic surface:

$$E(u,v) \approx u^2 \sum_{x,y} I_x^2 + 2uv \sum_{x,y} I_x I_y + v^2 \sum_{x,y} I_y^2$$



In which directions does this surface have the fastest/slowest change?

Source: L. Lazebnik

# Corner Detection: Derivation

*E*(*u*,*v*) can be locally approximated by a quadratic surface:

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$$= \begin{bmatrix} u & v \end{bmatrix} \begin{bmatrix} \sum_{x,y} I_x^2 & \sum_{x,y} I_x I_y \\ \sum_{x,y} I_x I_y & \sum_{x,y} I_y^2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

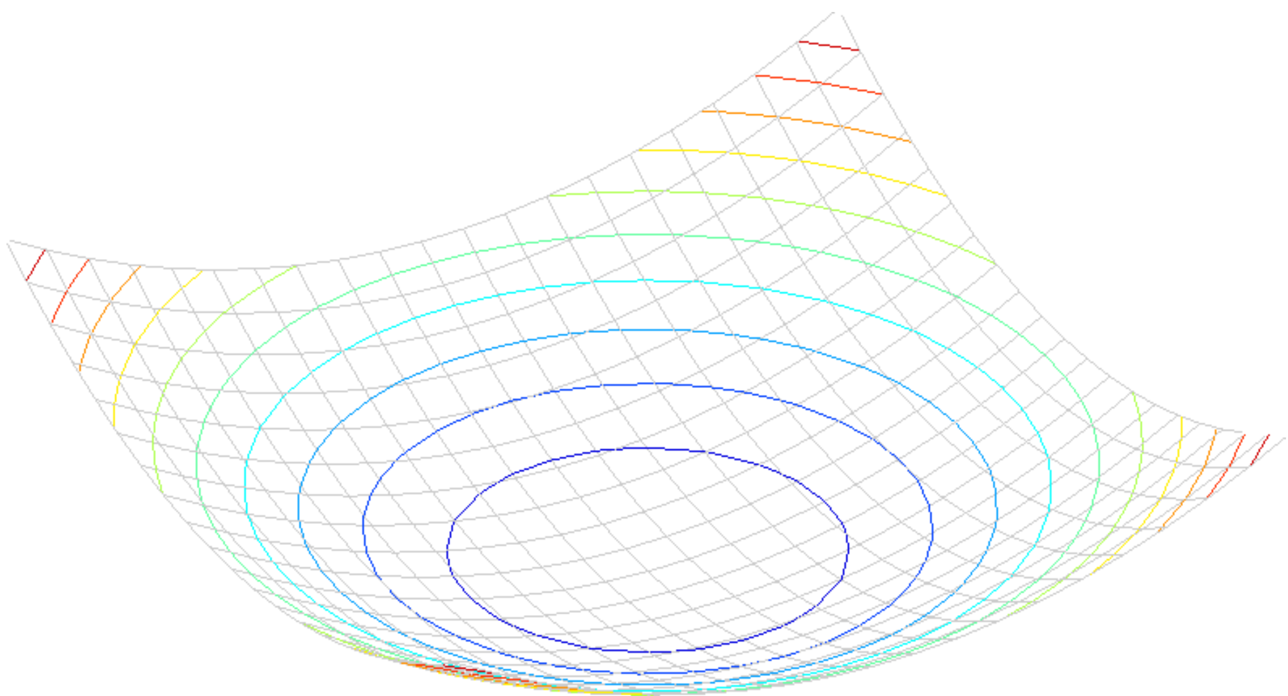
*Second moment matrix M*

Source: L. Lazebnik

# Interpreting the second moment matrix

A horizontal "slice" of $E(u, v)$ is given by the equation of an ellipse:

$$[u \;\; v] \; M \begin{bmatrix} u \\ v \end{bmatrix} = \text{const}$$



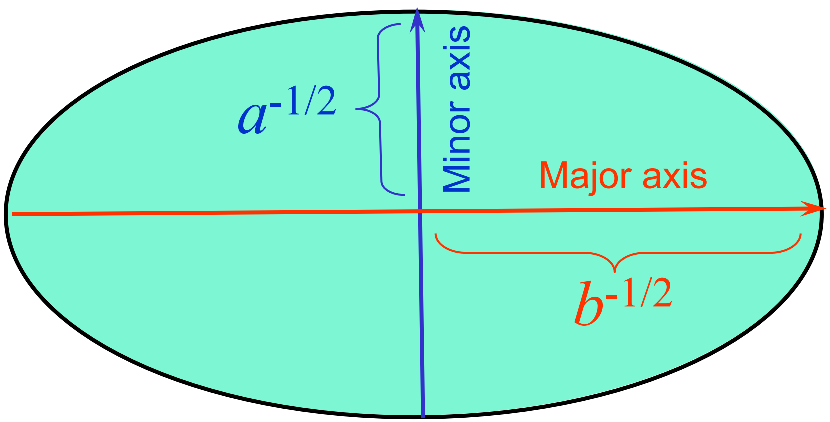
Source: L. Lazebnik

# Interpreting the second moment matrix

Consider the axis-aligned case (gradients are either horizontal or vertical):

$$M = \begin{bmatrix} \sum_{x,y} I_x^2 & \sum_{x,y} I_x I_y \\ \sum_{x,y} I_x I_y & \sum_{x,y} I_y^2 \end{bmatrix}$$

$$[u \;\; v] \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = 1$$

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$$M = \begin{bmatrix} \sum_{x,y} I_x^2 & \sum_{x,y} I_x I_y \\ \sum_{x,y} I_x I_y & \sum_{x,y} I_y^2 \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

If either *a* or *b* is close to 0, then this is **not** a corner, so we want locations where both are large

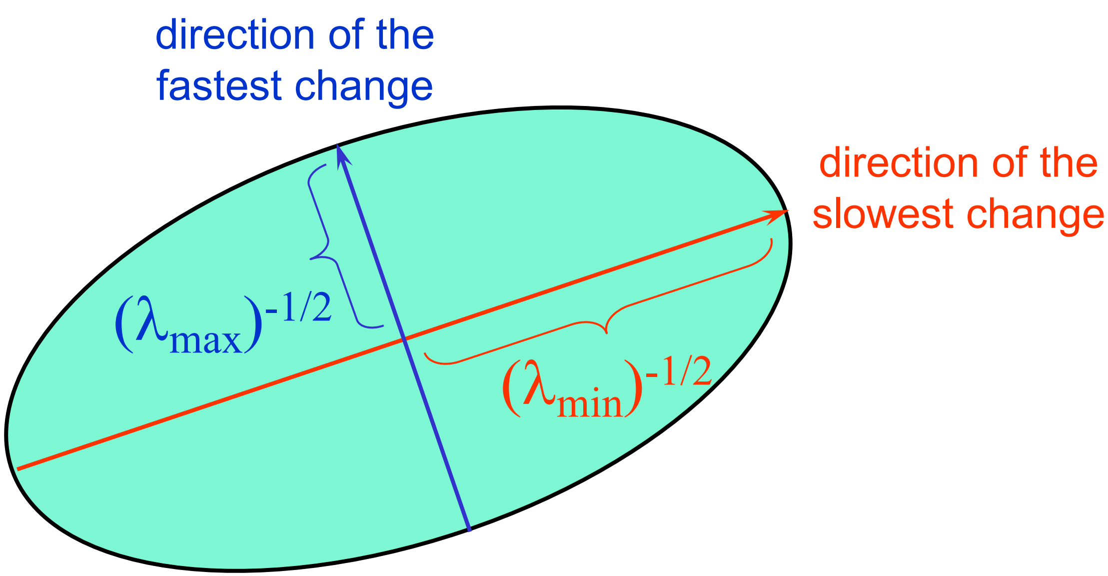
Source: L. Lazebnik

# Interpreting the second moment matrix

In the general case, need to *diagonalize* M:

$$M = R^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$$

The axis lengths of the ellipse are determined by the eigenvalues and the orientation is determined by *R*:




Source: L. Lazebnik

# Visualization of second moment matrices

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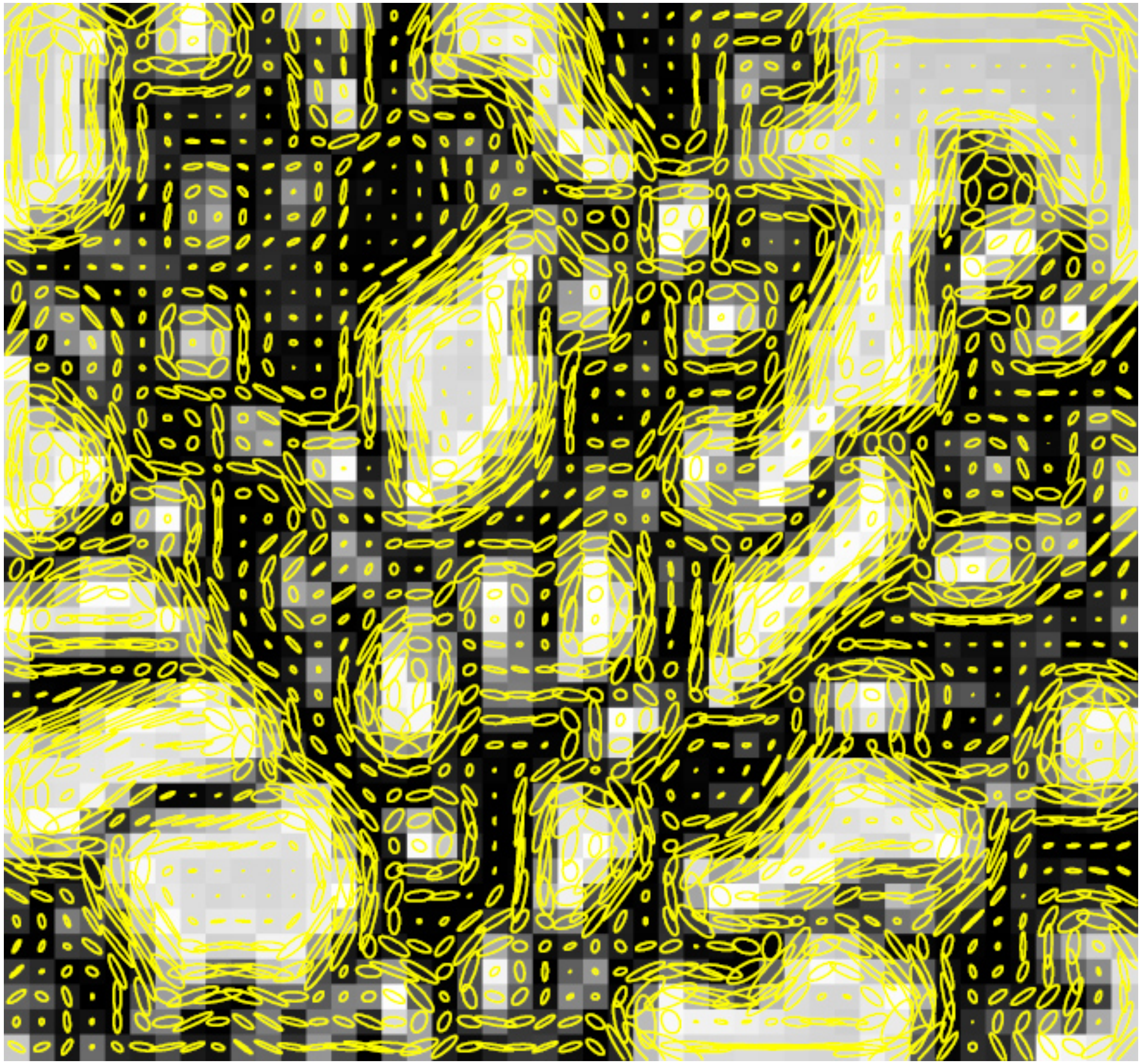
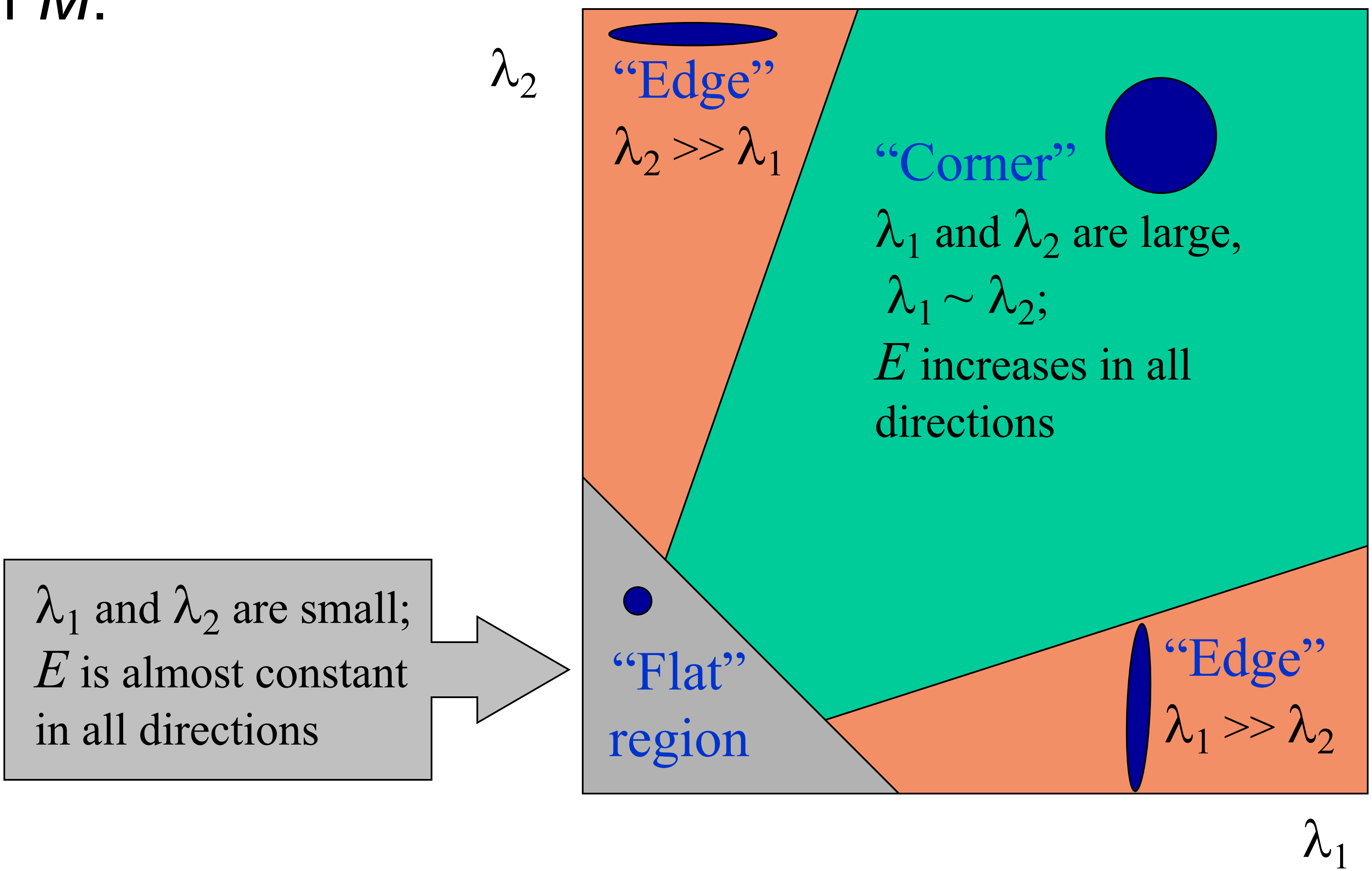


Source: L. Lazebnik

# Interpreting the eigenvalues

Classification of image points using eigenvalues of *M*:

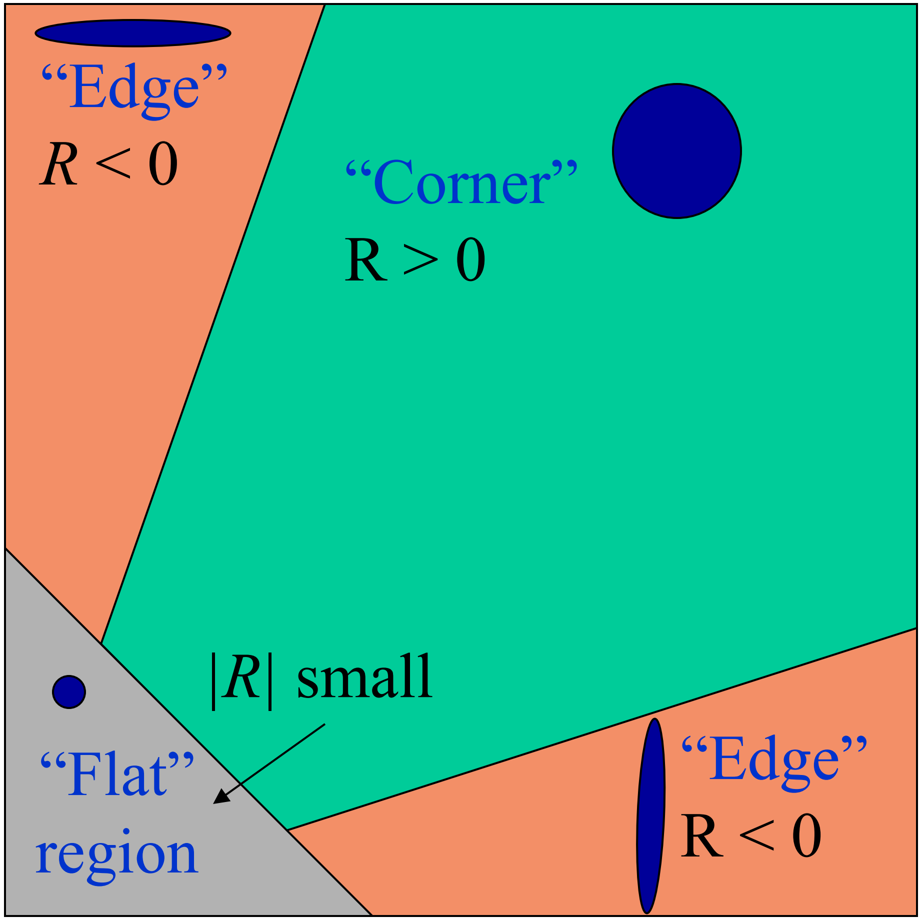



Source: L. Lazebnik

# Corner response function

$$R = \det(M) - \alpha\,\mathrm{trace}(M)^2 = \lambda_1\lambda_2 - \alpha(\lambda_1 + \lambda_2)^2$$

$\alpha$: constant (0.04 to 0.06)




Source: L. Lazebnik

# The Harris corner detector

1. Compute partial derivatives at each pixel
2. Compute second moment matrix *M* in a Gaussian window around each pixel:

$$M = \begin{bmatrix} \sum_{x,y} w(x,y) I_x^2 & \sum_{x,y} w(x,y) I_x I_y \\ \sum_{x,y} w(x,y) I_x I_y & \sum_{x,y} w(x,y) I_y^2 \end{bmatrix}$$

C.Harris and M.Stephens, A Combined Corner and Edge Detector, *Proceedings of the 4th Alvey Vision Conference*: pages 147—151, 1988.

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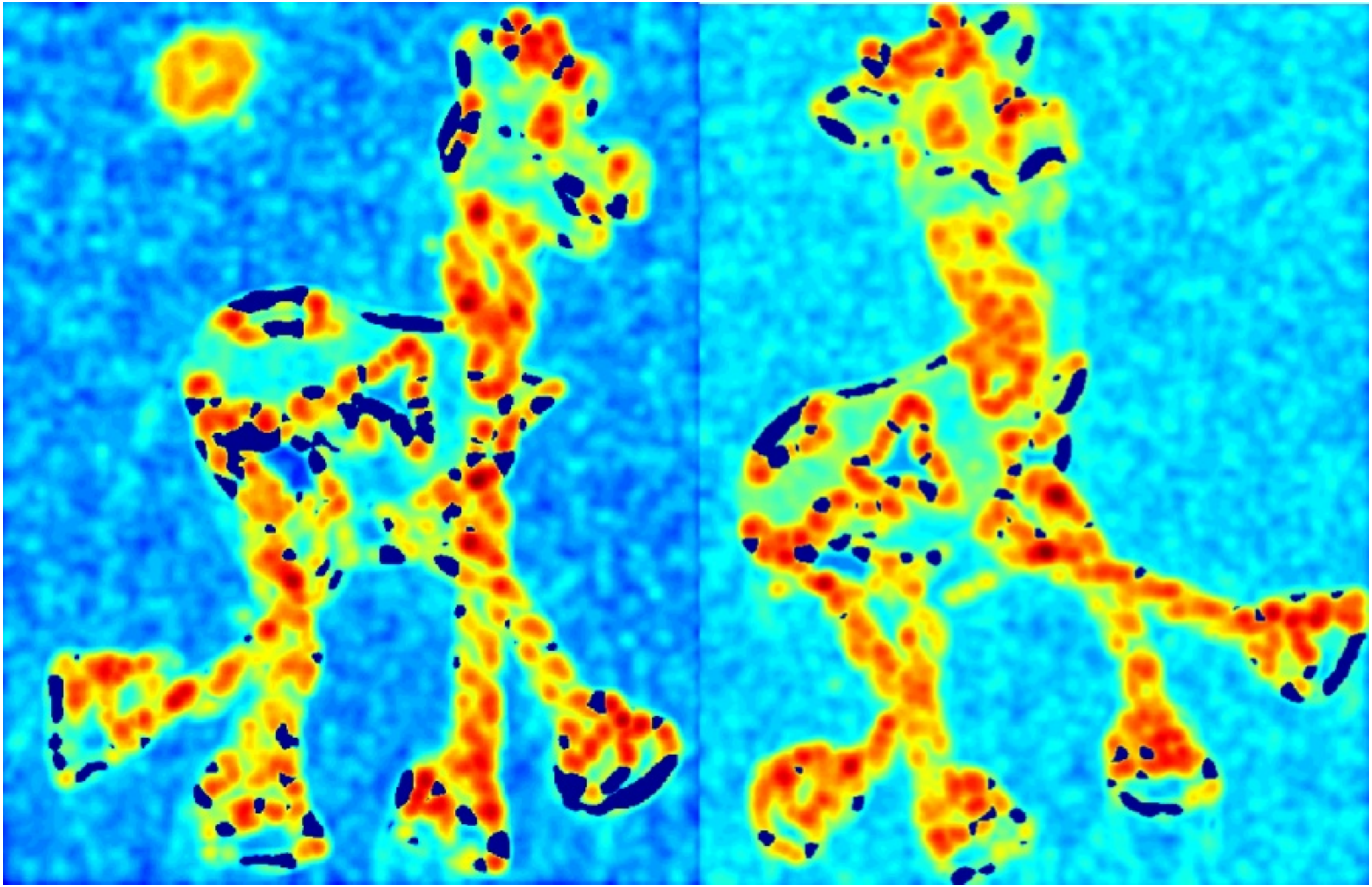
Source: L. Lazebnik

# Harris Detector: Steps

Source: L. Lazebnik

# Harris Detector: Steps

Compute corner response $R$



Source: L. Lazebnik

# The Harris corner detector

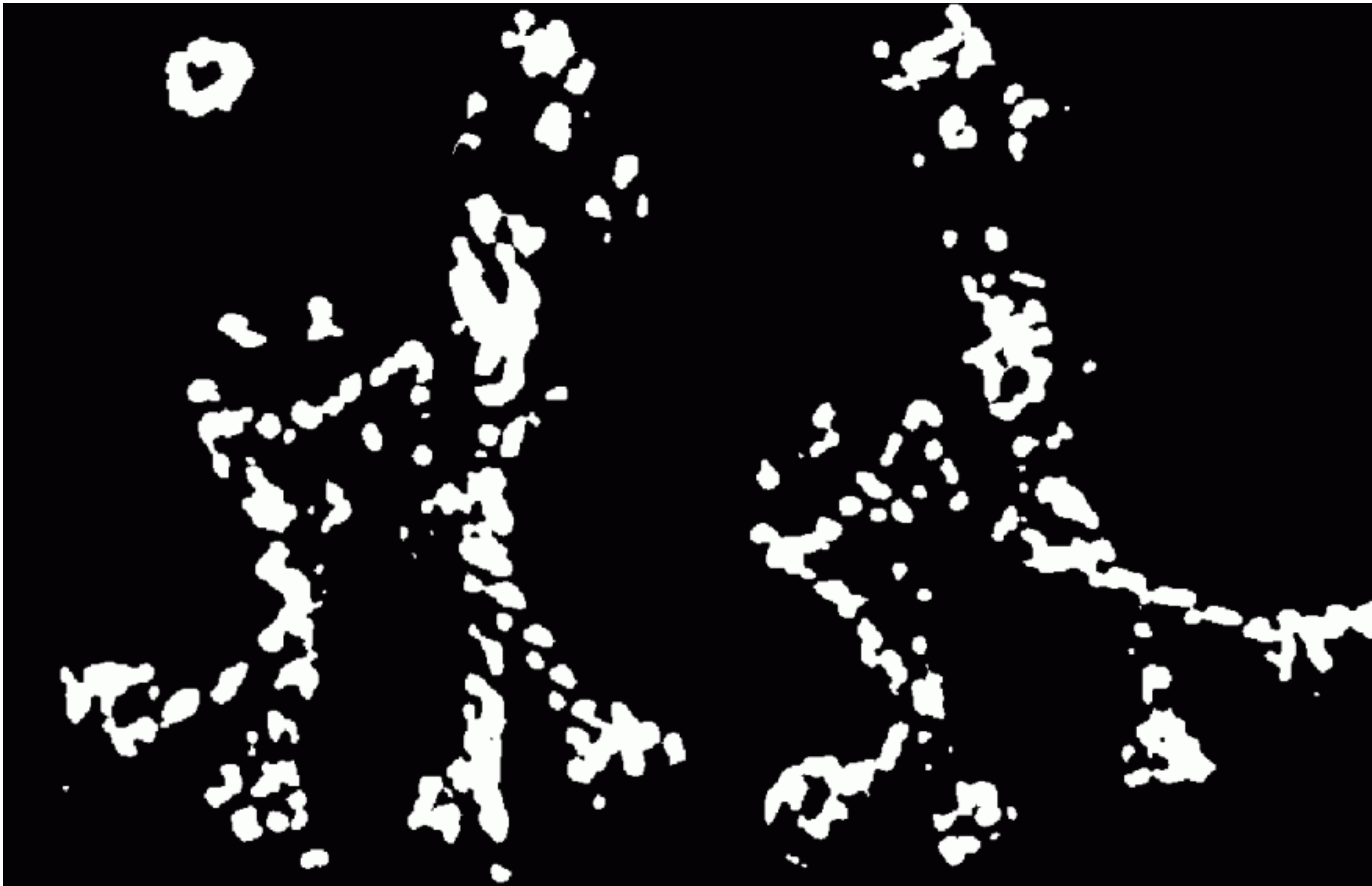
1. Compute partial derivatives at each pixel
2. Compute second moment matrix *M* in a Gaussian window around each pixel
3. Compute corner response function *R*
4. Threshold *R*
5. Find local maxima of response function (nonmaximum suppression)

C.Harris and M.Stephens, A Combined Corner and Edge Detector, *Proceedings of the 4th Alvey Vision Conference*: pages 147—151, 1988.

Source: L. Lazebnik

# Harris Detector: Steps

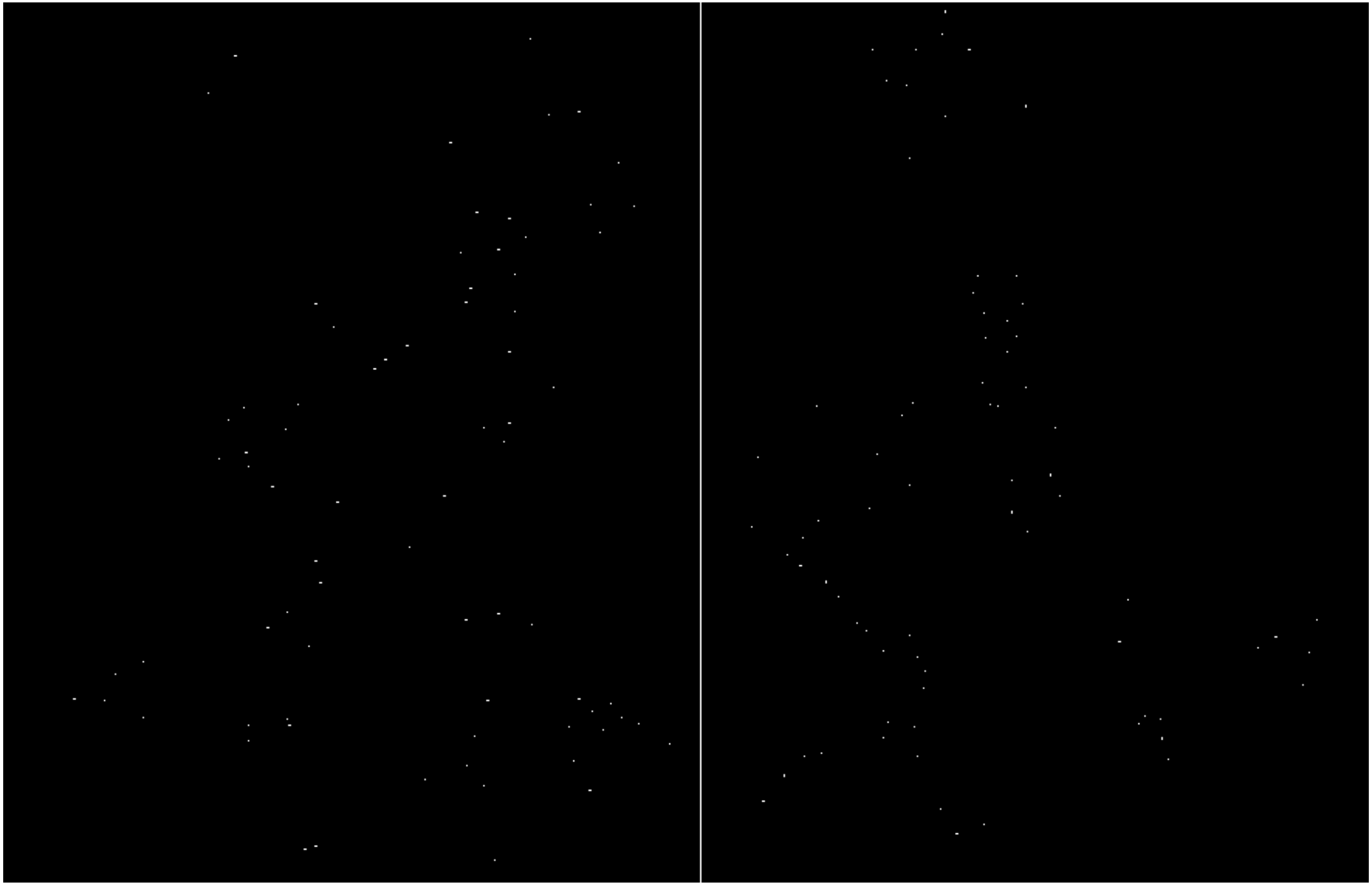
Find points with large corner response: $R$ > threshold



Source: L. Lazebnik

# Harris Detector: Steps

Take only the points of local maxima of $R$



Source: L. Lazebnik

# Harris Detector: Steps

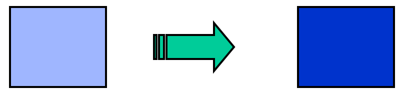
Source: L. Lazebnik

# Robustness of corner features

- What happens to corner features when the image undergoes geometric or photometric transformations?

Source: L. Lazebnik

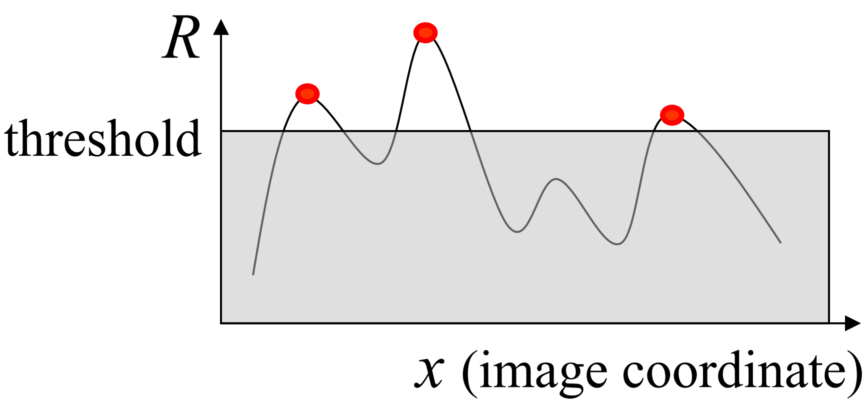
# Affine intensity change

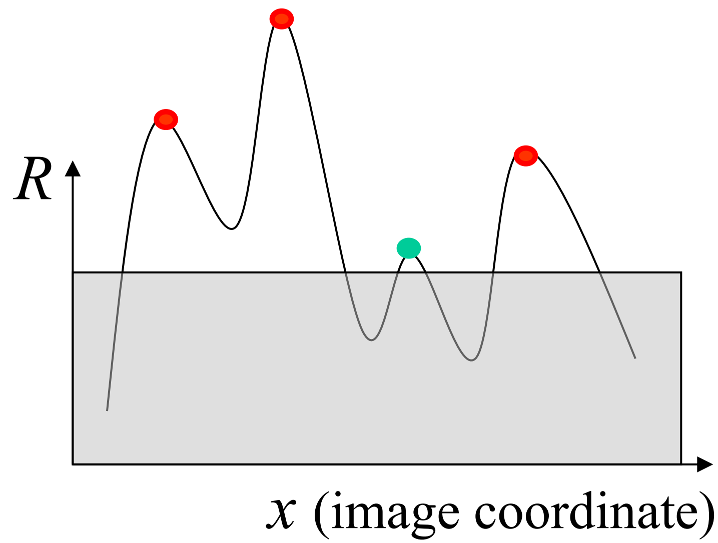


$$I \rightarrow a\,I + b$$

- Only derivatives are used, so invariant to intensity shift $I \rightarrow I + b$

- Intensity scaling: $I \rightarrow a\,I$

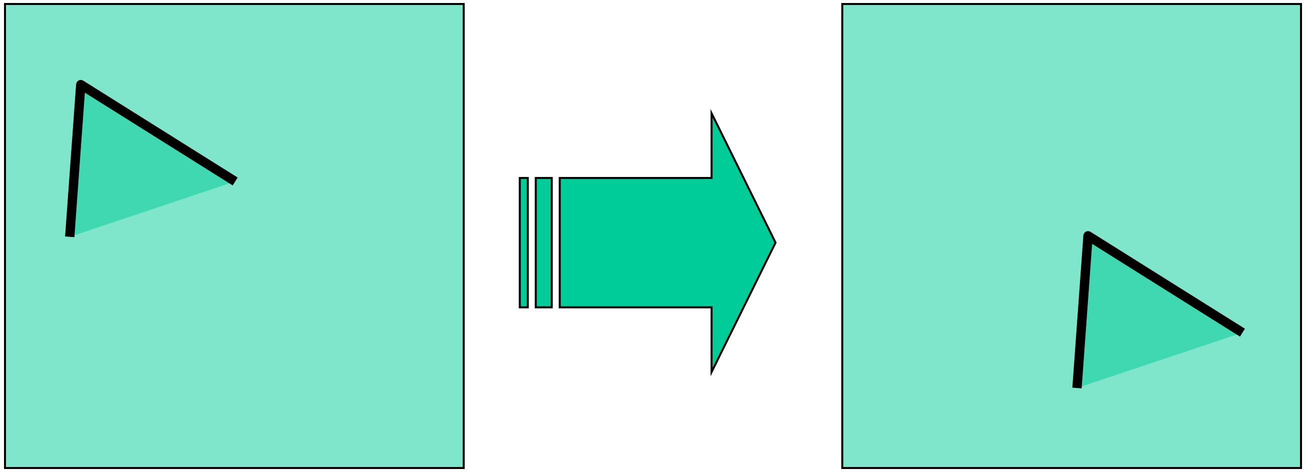



*Partially invariant* to affine intensity change

Source: L. Lazebnik

# Image translation

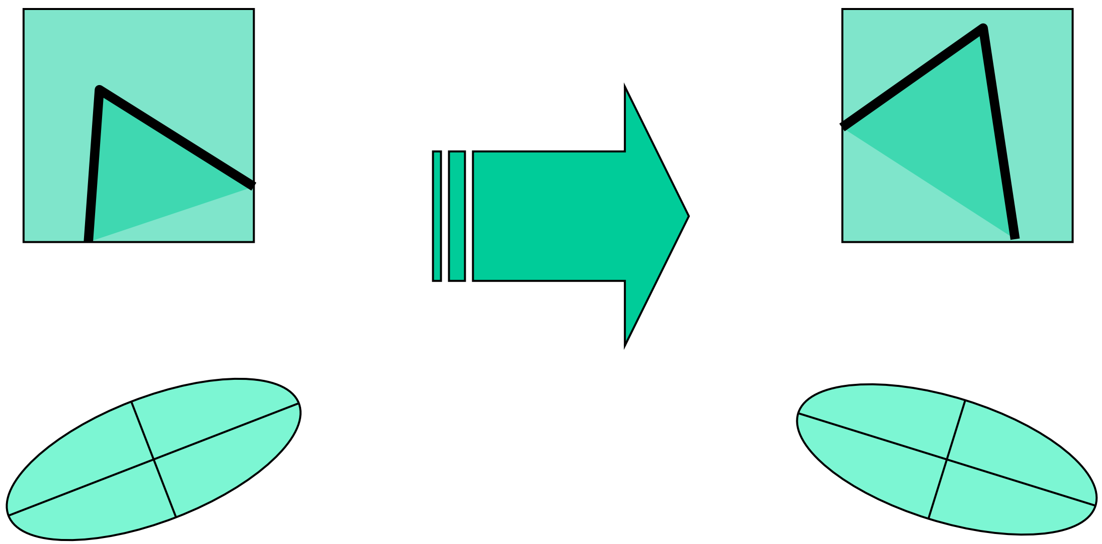


- Derivatives and window function are shift-invariant

Corner location is *covariant* w.r.t. translation

Source: L. Lazebnik

# Image rotation

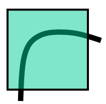


Second moment ellipse rotates but its shape (i.e. eigenvalues) remains the same

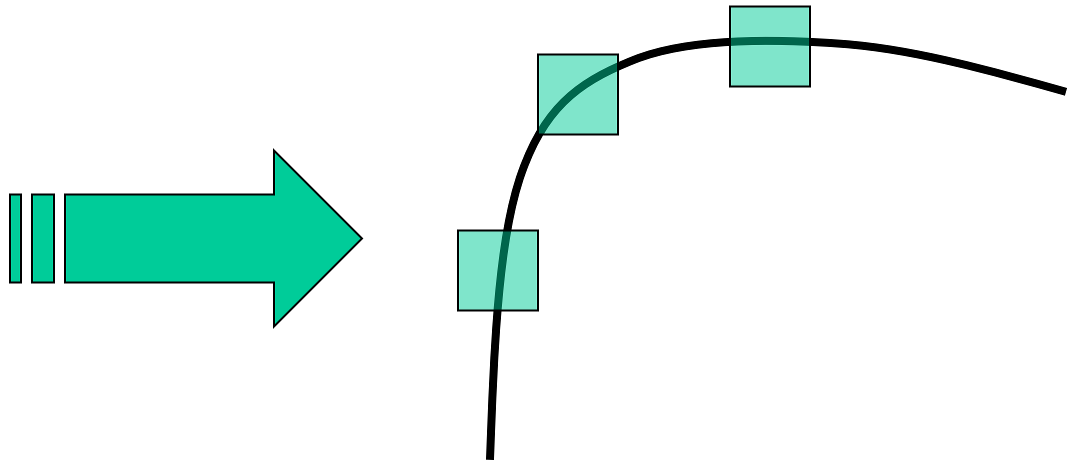
Corner location is covariant w.r.t. rotation

Source: L. Lazebnik

# Scaling



Corner



All points will be classified as edges

Corner location is not covariant w.r.t. scaling!

Source: L. Lazebnik