Corner Detection

CS 543 / ECE 549 – Saurabh Gupta
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Slides from S. Lazebnik.
Why extract keypoints?

- Motivation: panorama stitching
  - We have two images – how do we combine them?

Source: L. Lazebnik
Why extract keypoints?

- Motivation: panorama stitching
  - We have two images – how do we combine them?

Step 1: extract keypoints
Step 2: match keypoint features

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Why extract keypoints?

• **Motivation: panorama stitching**
  • We have two images – how do we combine them?

Step 1: extract keypoints
Step 2: match keypoint features
Step 3: align images

Source: L. Lazebnik
Characteristics of good keypoints

• Compactness and efficiency
  • Many fewer keypoints than image pixels

• Saliency
  • Each keypoint is distinctive

• Locality
  • A keypoint occupies a relatively small area of the image; robust to clutter and occlusion

• Repeatability
  • The same keypoint can be found in several images despite geometric and photometric transformations

Source: L. Lazebnik
Applications

Keypoints are used for:

- Image alignment
- 3D reconstruction
- Motion tracking
- Robot navigation
- Database indexing and retrieval
- Object recognition

Source: L. Lazebnik
Corner detection: Basic idea
Corner detection: Basic idea

- We should easily recognize the point by looking through a small window.
- Shifting a window in *any direction* should give a *large change* in intensity.

- **“flat” region:** no change in all directions
- **“edge”:** no change along the edge direction
- **“corner”:** significant change in all directions

Source: L. Lazebnik
Corner Detection: Derivation

Change in appearance of window $W$ for the shift $[u, v]$:

$$E(u, v) = \sum_{(x,y) \in W} [I(x + u, y + v) - I(x, y)]^2$$

Source: L. Lazebnik
Corner Detection: Derivation

Change in appearance of window $W$ for the shift $[u, v]$:

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Corner Detection: Derivation

Change in appearance of window $W$ for the shift $[u, v]$:

$$E(u, v) = \sum_{(x, y) \in W} [I(x + u, y + v) - I(x, y)]^2$$

We want to find out how this function behaves for small shifts

$E(u, v)$
Corner Detection: Derivation

First-order Taylor approximation for small motions \([u, v]\):

\[
I(x + u, y + v) \approx I(x, y) + I_x u + I_y v
\]

Let’s plug this into \(E(u, v)\):

\[
E(u, v) = \sum_{(x, y) \in W} \left[ I(x + u, y + v) - I(x, y) \right]^2
\]

Source: L. Lazebnik
Corner Detection: Derivation

$E(u, v)$ can be locally approximated by a quadratic surface:

$$E(u, v) \approx u^2 \sum_{x,y} I_x^2 + 2uv \sum_{x,y} I_x I_y + v^2 \sum_{x,y} I_y^2$$

In which directions does this surface have the fastest/slowest change?

Source: L. Lazebnik
Corner Detection: Derivation

$E(u,v)$ can be locally approximated by a quadratic surface:

$$E(u,v) \approx u^2 \sum_{x,y} I_x^2 + 2uv \sum_{x,y} I_x I_y + v^2 \sum_{x,y} I_y^2$$

$$= \begin{bmatrix} u & v \end{bmatrix} \begin{bmatrix} \sum_{x,y} I_x^2 & \sum_{x,y} I_x I_y \\ \sum_{x,y} I_x I_y & \sum_{x,y} I_y^2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

*Second moment matrix $M$*

Source: L. Lazebnik
Interpreting the second moment matrix

A horizontal “slice” of $E(u, v)$ is given by the equation of an ellipse:

$$[u \ v] M \begin{bmatrix} u \\ v \end{bmatrix} = \text{const}$$

Source: L. Lazebnik
Interpreting the second moment matrix

Consider the axis-aligned case (gradients are either horizontal or vertical):

\[
M = \begin{bmatrix}
\sum_{x,y} I_x^2 & \sum_{x,y} I_x I_y \\
\sum_{x,y} I_x I_y & \sum_{x,y} I_y^2 
\end{bmatrix}
\]

\[
[u \ v] \begin{bmatrix}
a & 0 \\
0 & b 
\end{bmatrix} \begin{bmatrix}
u \\
v 
\end{bmatrix} = 1
\]

Source: L. Lazebnik
Interpreting the second moment matrix

Consider the axis-aligned case (gradients are either horizontal or vertical):

\[
M = \begin{bmatrix}
\sum_{x,y} I_x^2 & \sum_{x,y} I_x I_y \\
\sum_{x,y} I_x I_y & \sum_{x,y} I_y^2
\end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}
\]

If either \(a\) or \(b\) is close to 0, then this is not a corner, so we want locations where both are large.

Source: L. Lazebnik
Interpreting the second moment matrix

In the general case, need to *diagonalize* $M$:

$$M = R^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$$

The axis lengths of the ellipse are determined by the eigenvalues and the orientation is determined by $R$:

- Direction of the fastest change: $\lambda_{\text{max}}^{-1/2}$
- Direction of the slowest change: $\lambda_{\text{min}}^{-1/2}$

Source: L. Lazebnik
Visualization of second moment matrices

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Visualization of second moment matrices

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Interpreting the eigenvalues

Classification of image points using eigenvalues of $M$:

- **“Corner”**: $\lambda_1$ and $\lambda_2$ are large, $\lambda_1 \sim \lambda_2$; $E$ increases in all directions.
- **“Edge”**: $\lambda_2 \gg \lambda_1$; $\lambda_1$ and $\lambda_2$ are small; $E$ is almost constant in all directions.
- **“Flat”** region: $\lambda_1 \gg \lambda_2$.

Source: L. Lazebnik
Corner response function

\[ R = \det(M) - \alpha \text{trace}(M)^2 = \lambda_1 \lambda_2 - \alpha(\lambda_1 + \lambda_2)^2 \]

\( \alpha: \) constant (0.04 to 0.06)

Source: L. Lazebnik
The Harris corner detector

1. Compute partial derivatives at each pixel
2. Compute second moment matrix $M$ in a Gaussian window around each pixel:

$$M = \begin{bmatrix}
\sum_{x,y} w(x, y) I_x^2 & \sum_{x,y} w(x, y) I_x I_y \\
\sum_{x,y} w(x, y) I_x I_y & \sum_{x,y} w(x, y) I_y^2
\end{bmatrix}$$


Source: L. Lazebnik
The Harris corner detector

1. Compute partial derivatives at each pixel
2. Compute second moment matrix $M$ in a Gaussian window around each pixel
3. Compute corner response function $R$


Source: L. Lazebnik
Harris Detector: Steps

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Compute corner response $R$

Source: L. Lazebnik
The Harris corner detector

1. Compute partial derivatives at each pixel
2. Compute second moment matrix $M$ in a Gaussian window around each pixel
3. Compute corner response function $R$
4. Threshold $R$
5. Find local maxima of response function (nonmaximum suppression)


Source: L. Lazebnik
Harris Detector: Steps

Find points with large corner response: $R > \text{threshold}$

Source: L. Lazebnik
Harris Detector: Steps

Take only the points of local maxima of $R$

Source: L. Lazebnik
Harris Detector: Steps

Source: L. Lazebnik
Robustness of corner features

• What happens to corner features when the image undergoes geometric or photometric transformations?

Source: L. Lazebnik
Affine intensity change

- Only derivatives are used, so invariant to intensity shift $I \rightarrow I + b$
- Intensity scaling: $I \rightarrow a I$

Partially invariant to affine intensity change

Source: L. Lazebnik
Image translation

- Derivatives and window function are shift-invariant

Corner location is *covariant* w.r.t. translation

Source: L. Lazebnik
Second moment ellipse rotates but its shape (i.e. eigenvalues) remains the same

Corner location is covariant w.r.t. rotation

Source: L. Lazebnik
Corner location is not covariant w.r.t. scaling!

All points will be classified as edges

Source: L. Lazebnik