Light, Camera and Shading

CS 543 / ECE 549 – Saurabh Gupta

Spring 2021, UIUC

http://saurabhg.web.illinois.edu/teaching/ece549/sp2020/

Many slides adapted from S. Seitz, L. Lazebnik, D. Hoiem, D. Forsyth

Overview

- Cameras with lenses
  - Depth of field
  - Field of view
  - Lens aberrations
- Brightness of a pixel
  - Small taste of radiometry
  - In-camera transformation of light
  - Reflectance properties of surfaces
  - Lambertian reflection model
  - Shape from shading

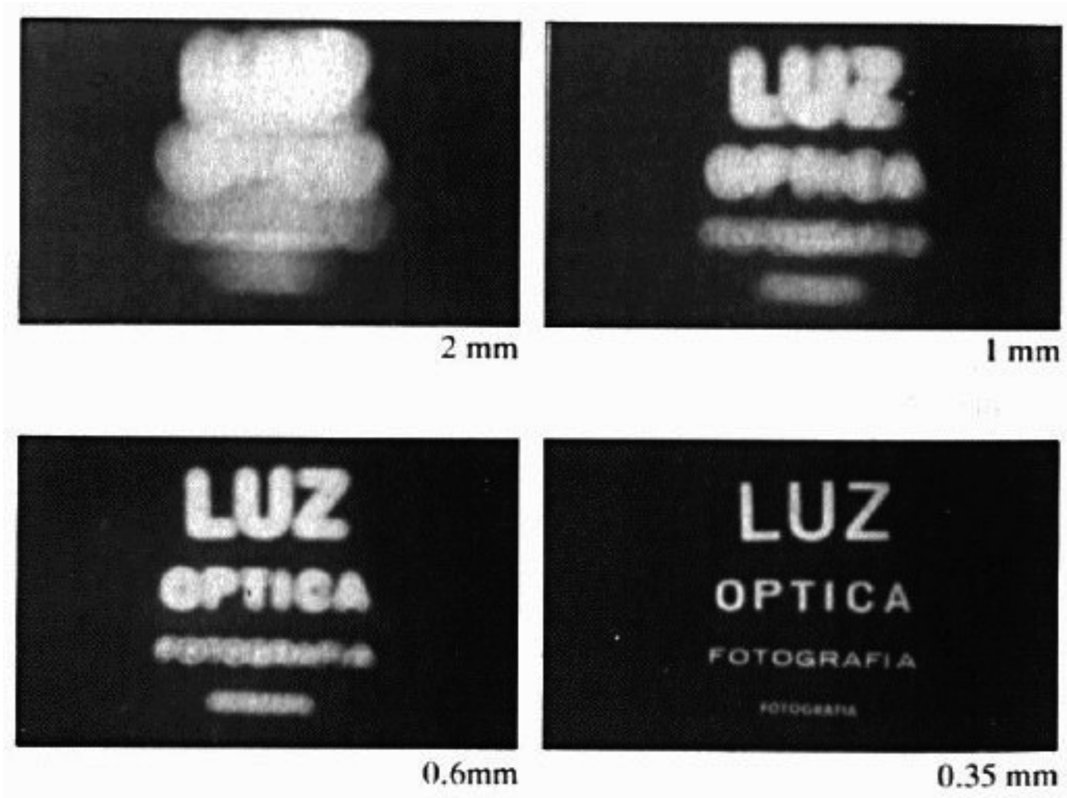
Building a Real Camera

Home-made pinhole camera

http://www.debevec.org/Pinhole/

Slide by A. Efros

Shrinking the aperture

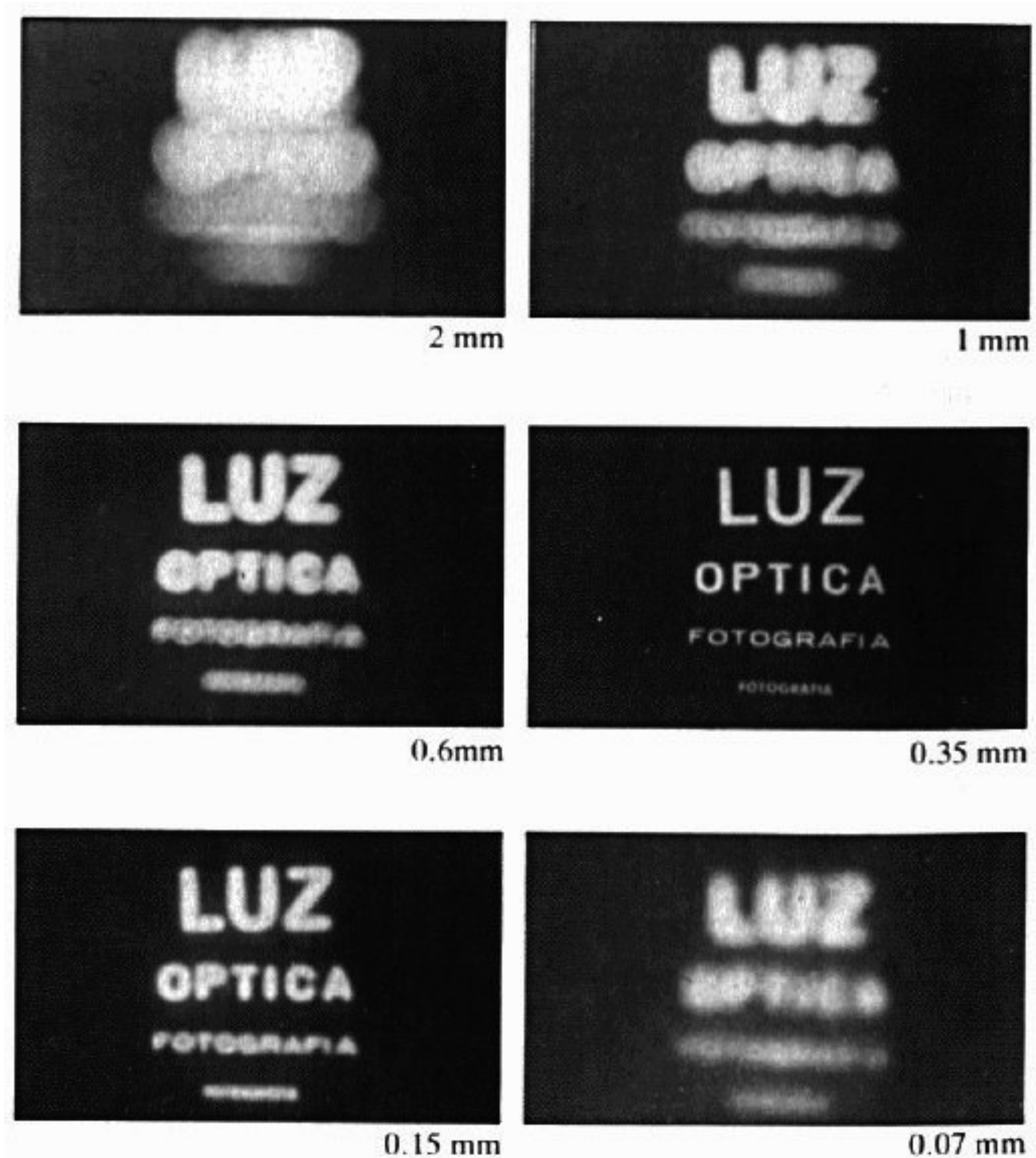


Why not make the aperture as small as possible?

- Less light gets through
- Diffraction effects…

Slide by Steve Seitz

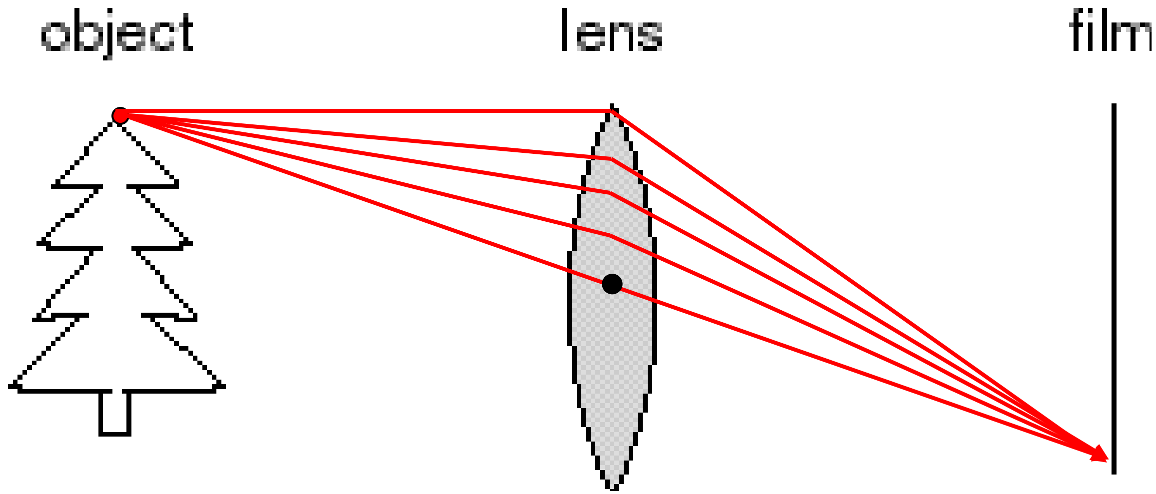
Shrinking the aperture



Slide by Steve Seitz

Adding a lens

Adding a lens

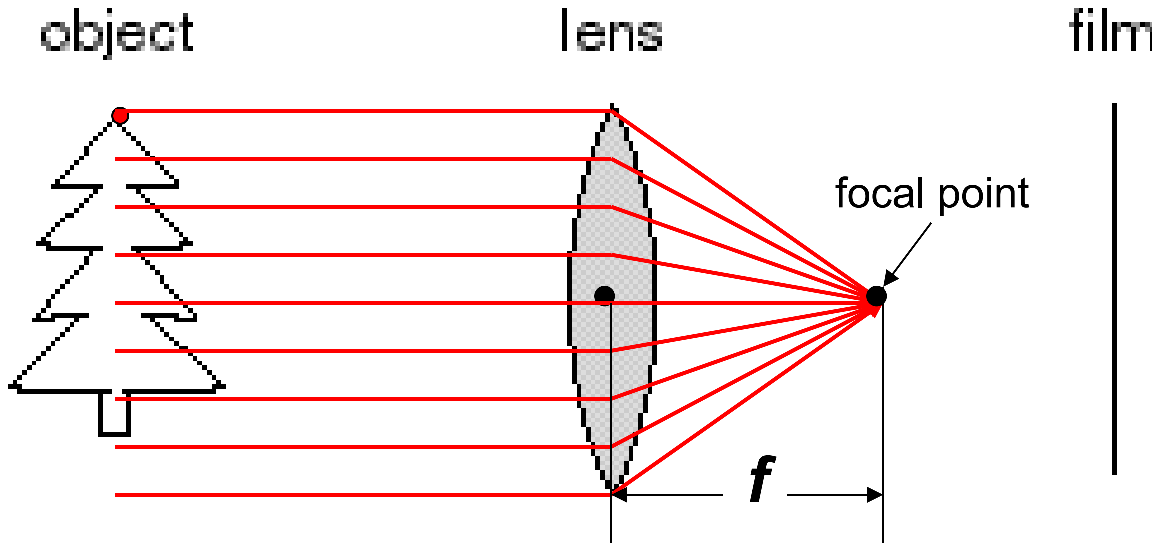


A lens focuses light onto the film

- Thin lens model:
  - Rays passing through the center are not deviated (pinhole projection model still holds)

Slide by Steve Seitz

Adding a lens



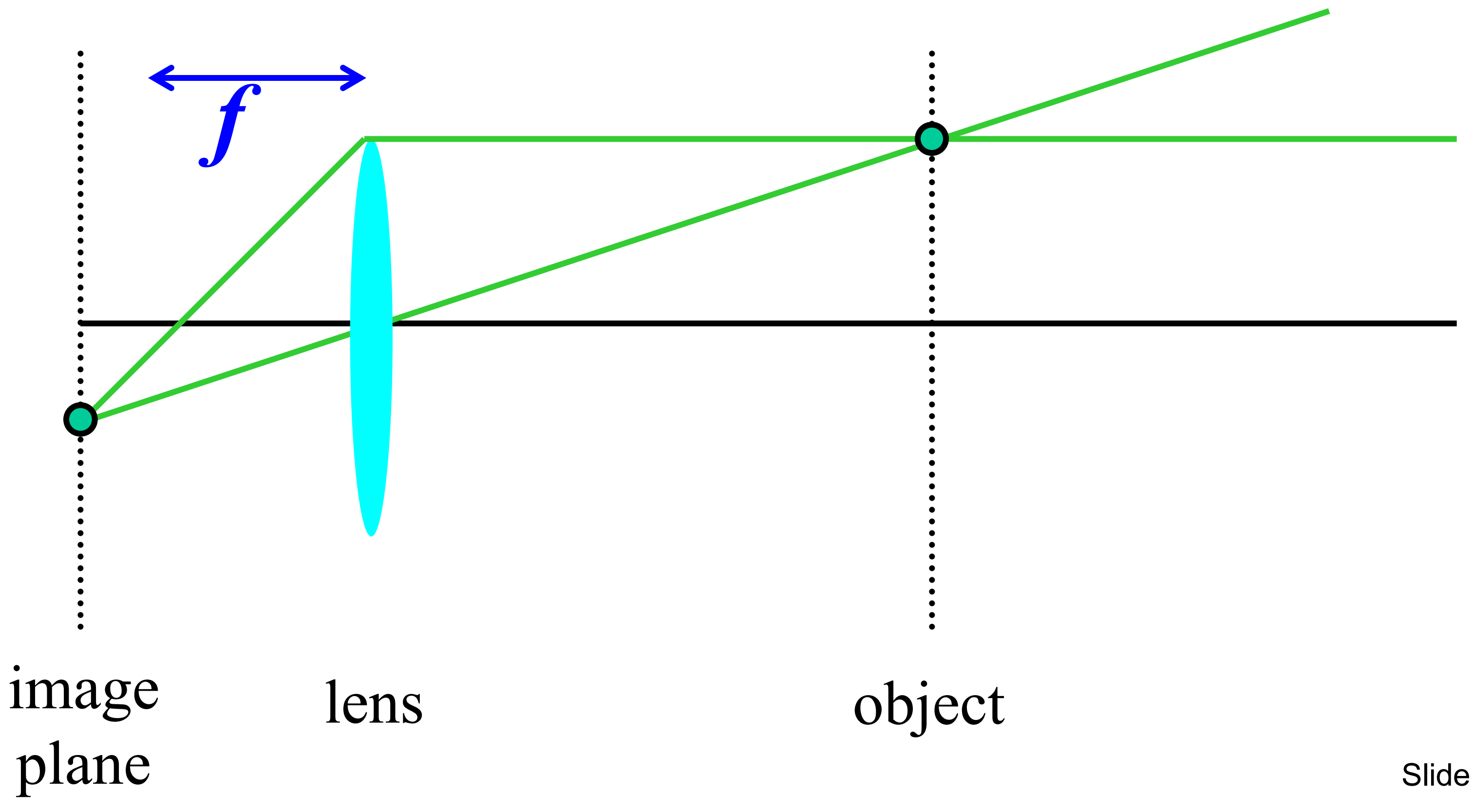
A lens focuses light onto the film

- Thin lens model:
  - Rays passing through the center are not deviated (pinhole projection model still holds)
  - All rays parallel to the optical axis pass through the *focal point*
  - All parallel rays converge to points on the *focal plane*

Slide by Steve Seitz

Thin lens formula

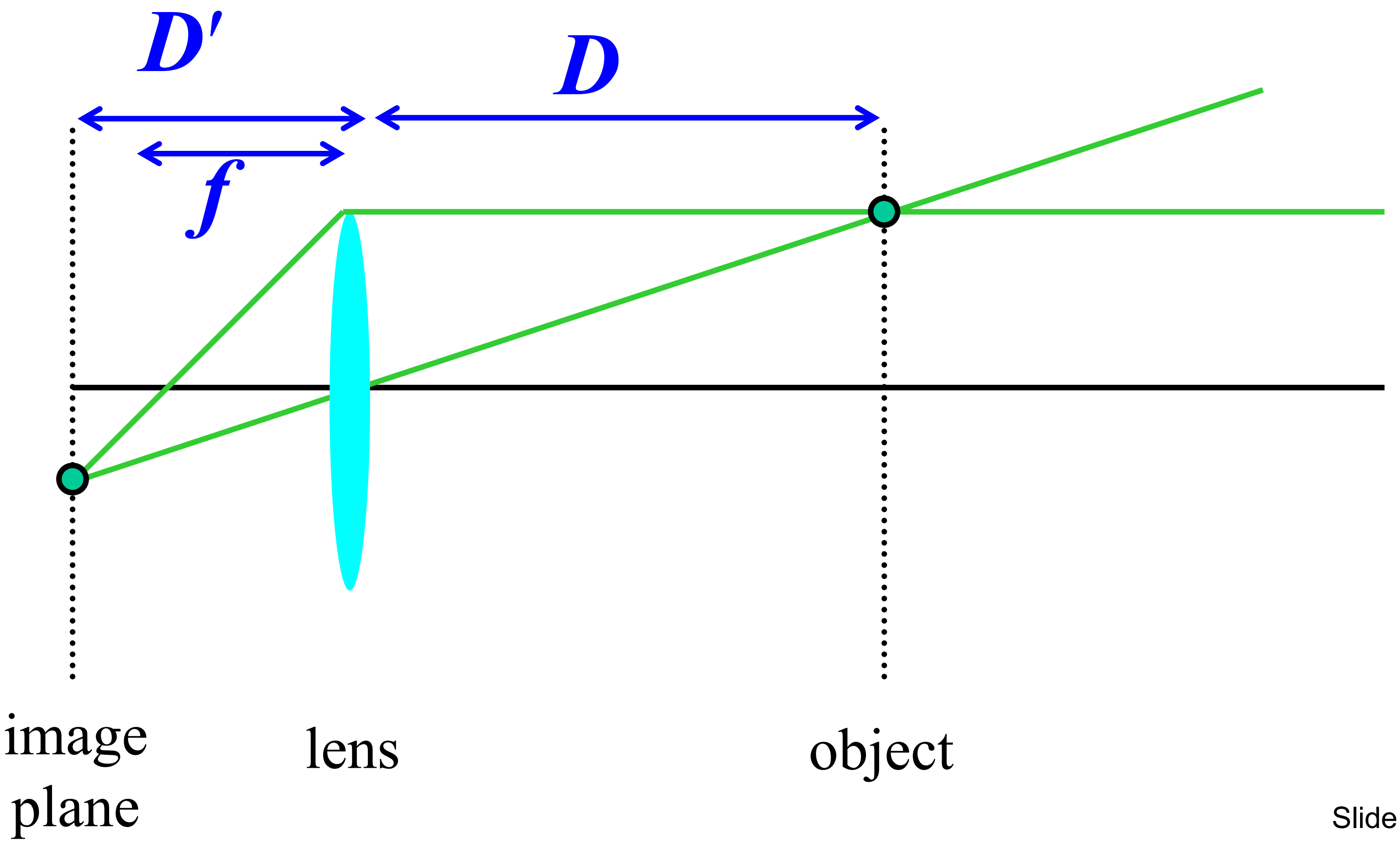
- Where does the lens focus the rays coming from a given point in the scene?



Slide by Frédo Durand

Thin lens formula

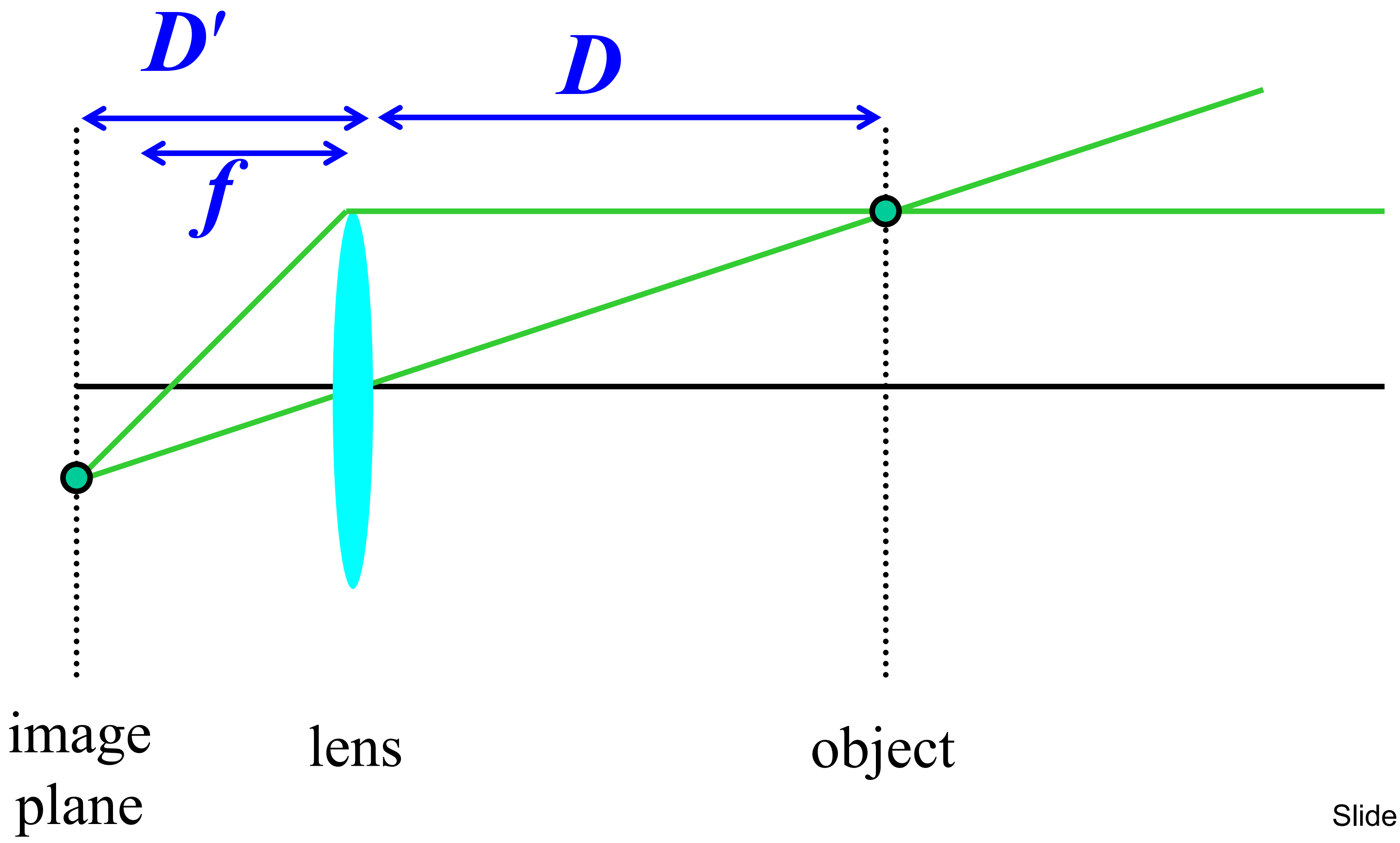
- What is the relation between the focal length (f), the distance of the object from the optical center (D), and the distance at which the object will be in focus (D')?



Slide by Frédo Durand

Thin lens formula

Similar triangles everywhere!

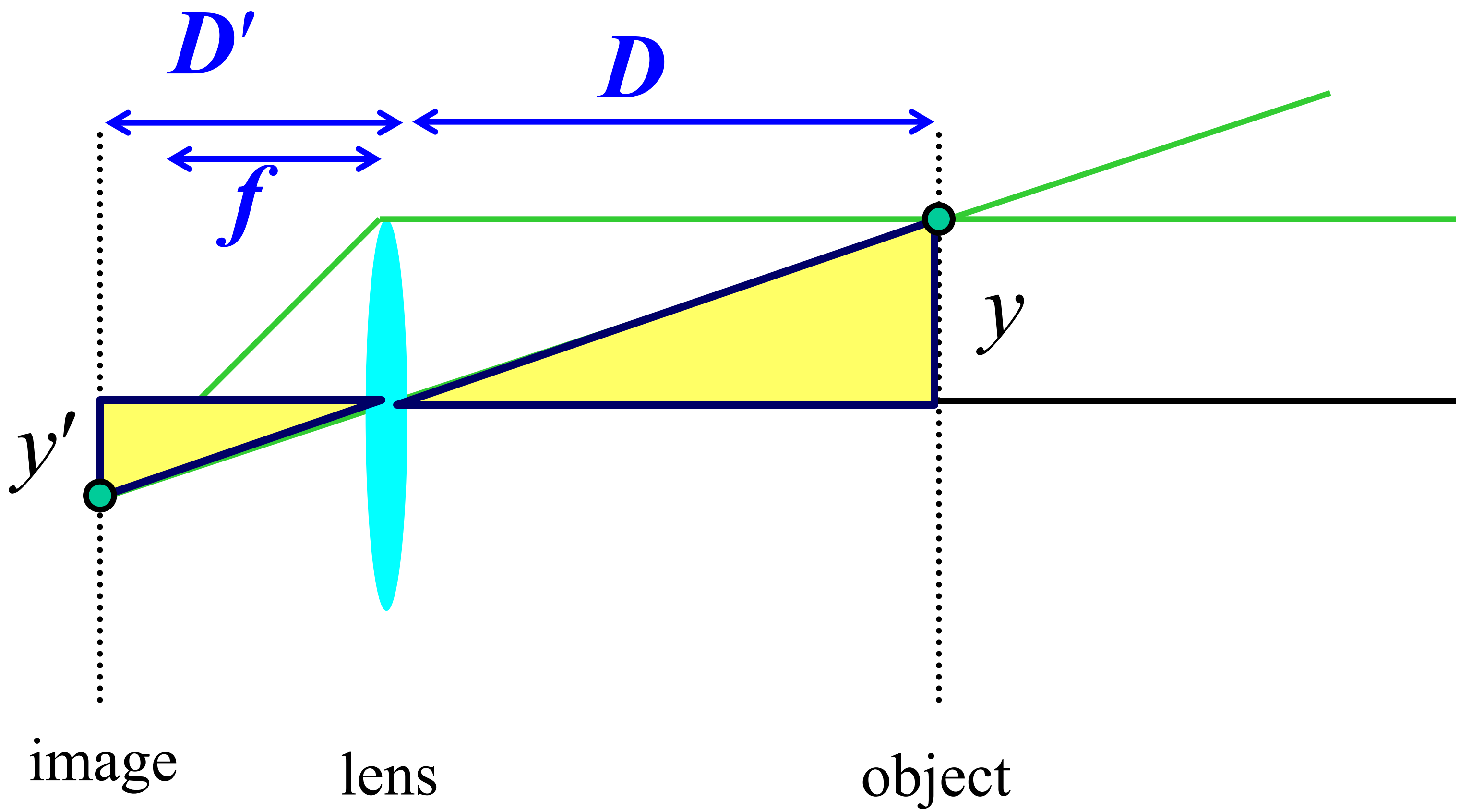


Slide by Frédo Durand

Thin lens formula

Similar triangles everywhere!

$$y'/y = D'/D$$



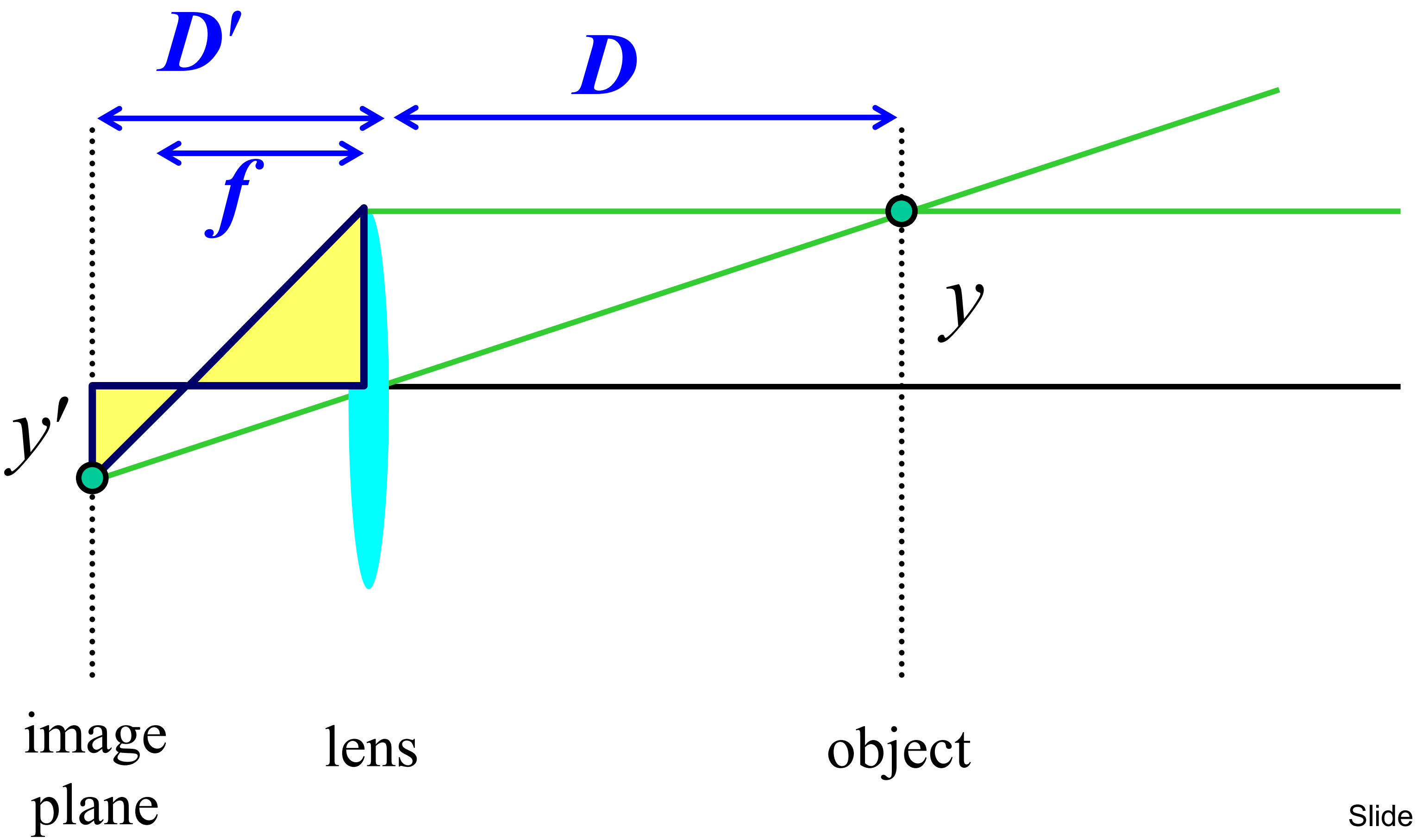
Slide by Frédo Durand

Thin lens formula

Similar triangles everywhere!

$$y'/y = D'/D$$

$$y'/y = (D'-f)/f$$



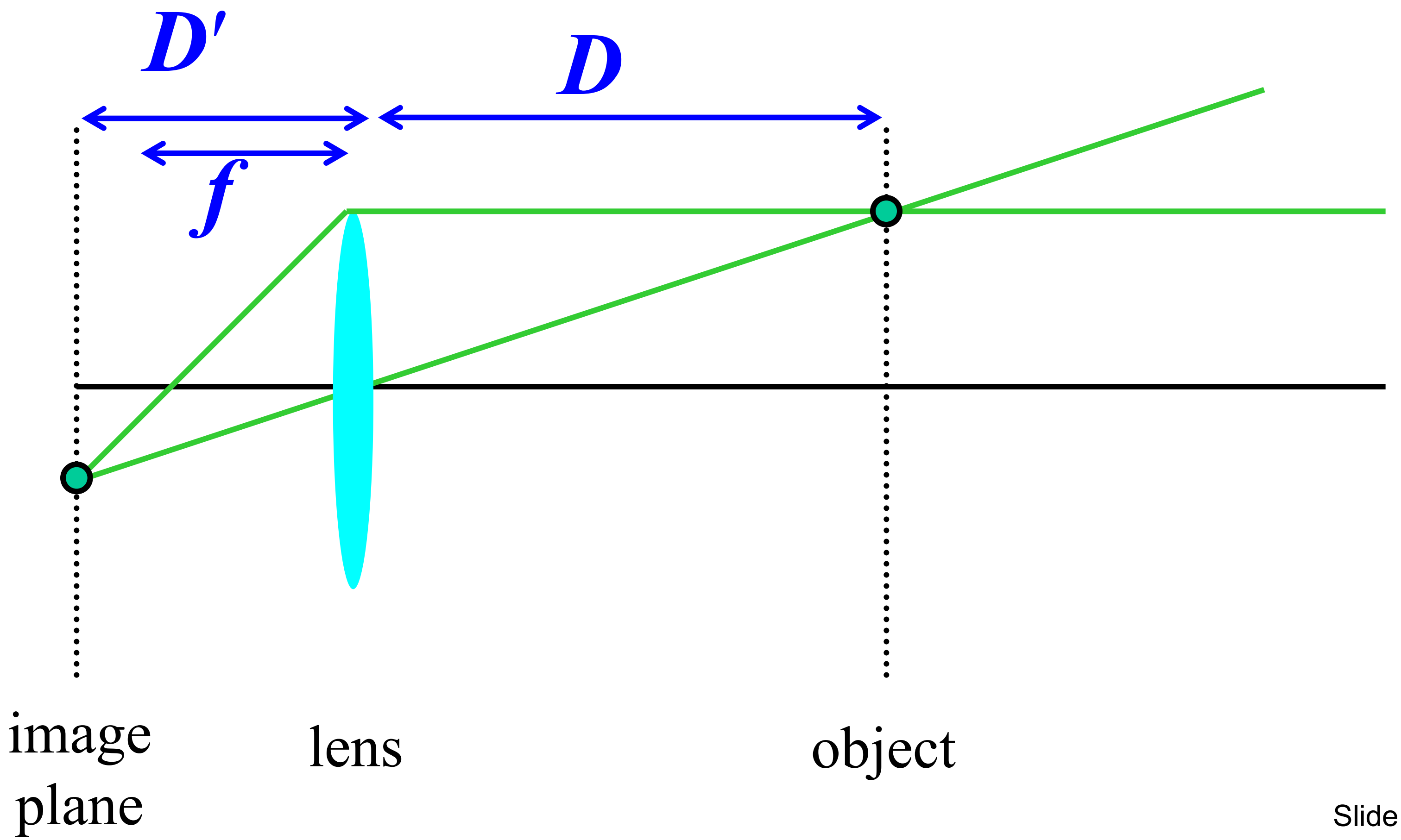
Slide by Frédo Durand

Thin lens formula

$$\frac{1}{D'} + \frac{1}{D} = \frac{1}{f}$$

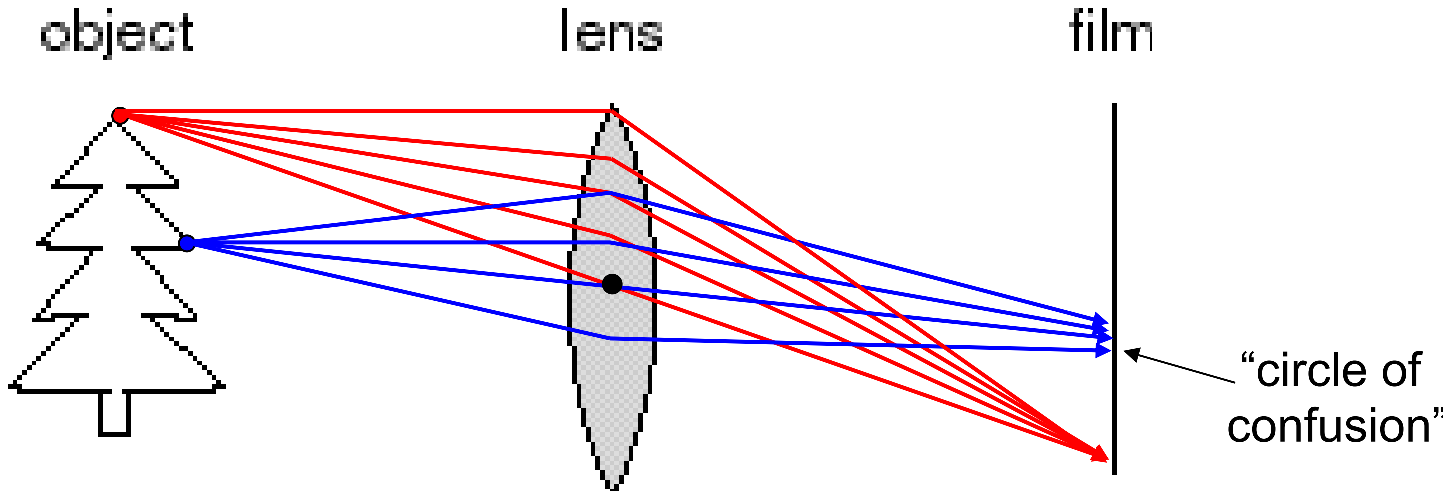
Any point satisfying the thin lens equation is in focus.

What happens when D is very large?



Slide by Frédo Durand

Depth of Field



For a fixed focal length, there is a specific distance at which objects are "in focus"

- Other points project to a "circle of confusion" in the image

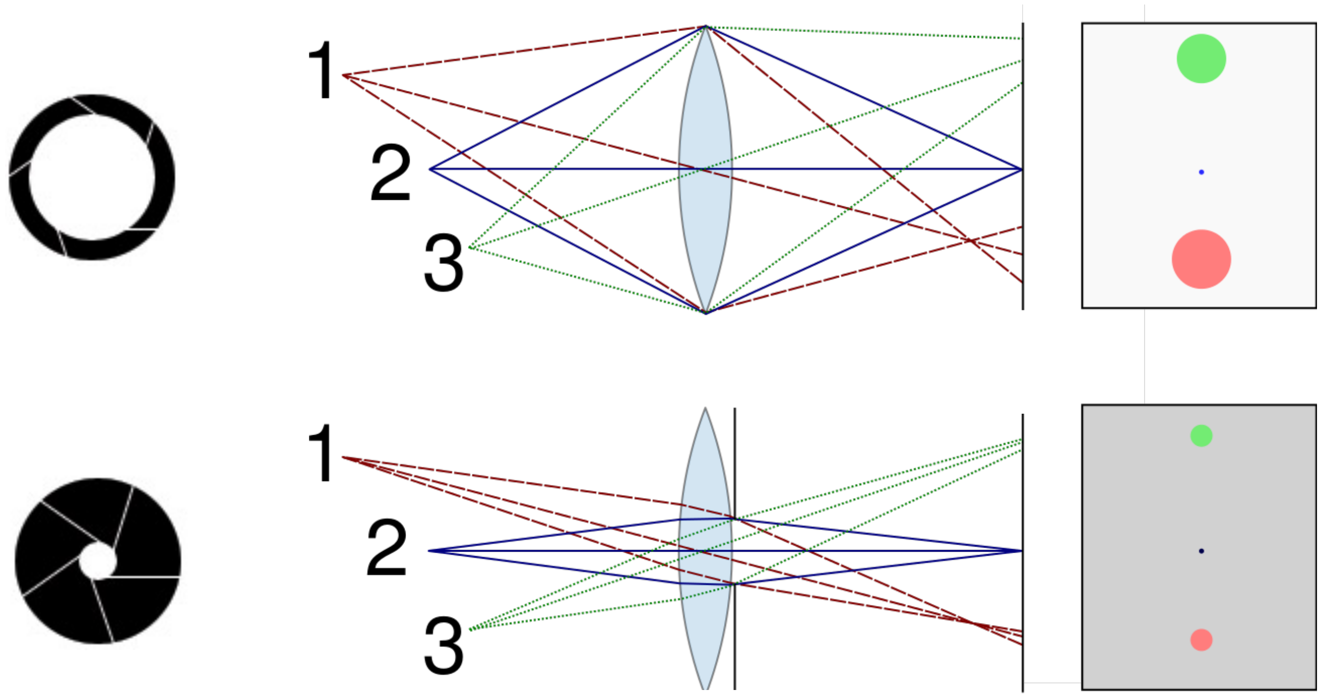
Slide by Steve Seitz

Depth of Field

http://www.cambridgeincolour.com/tutorials/depth-of-field.htm

Slide by A. Efros

Controlling depth of field



Changing the aperture size affects depth of field

- A smaller *aperture* increases the range in which the object is approximately in focus
- But small aperture reduces amount of light – need to increase *exposure*

http://en.wikipedia.org/wiki/File:Depth_of_field_illustration.svg

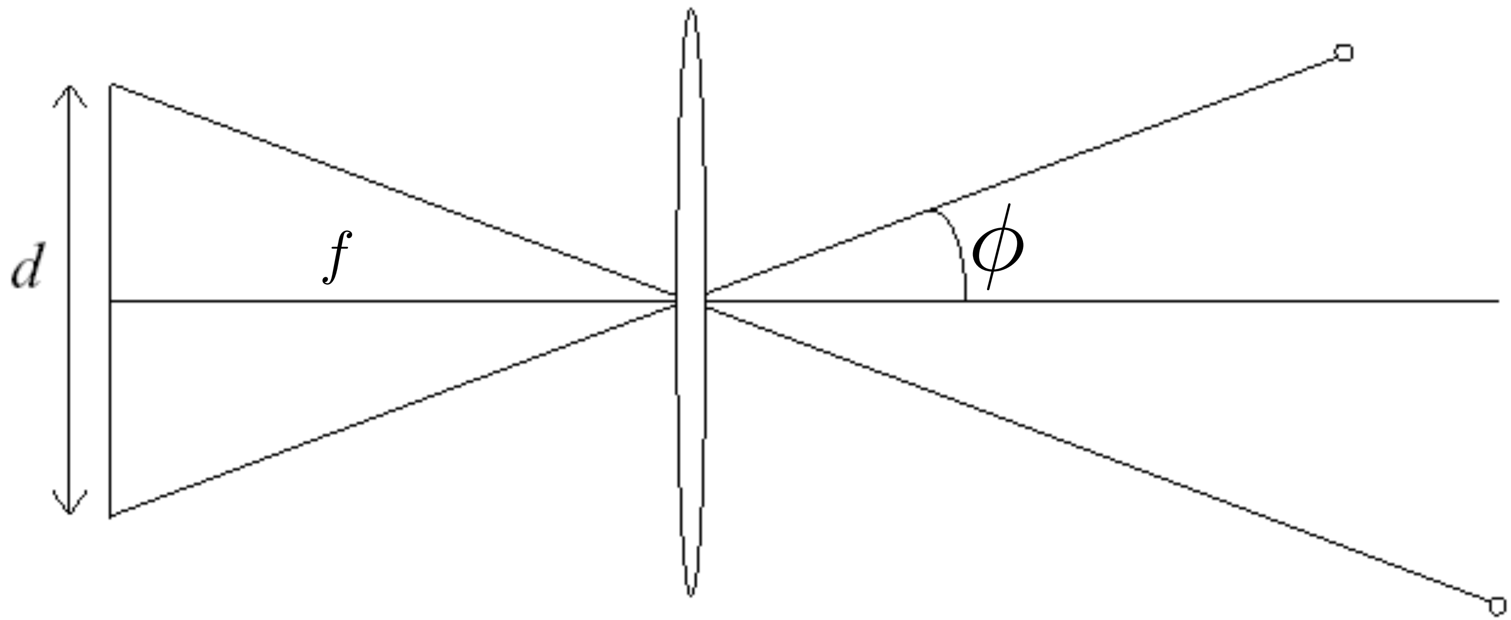
Varying the aperture

Large aperture = small DOF

Small aperture = large DOF

Slide by A. Efros

Field of View



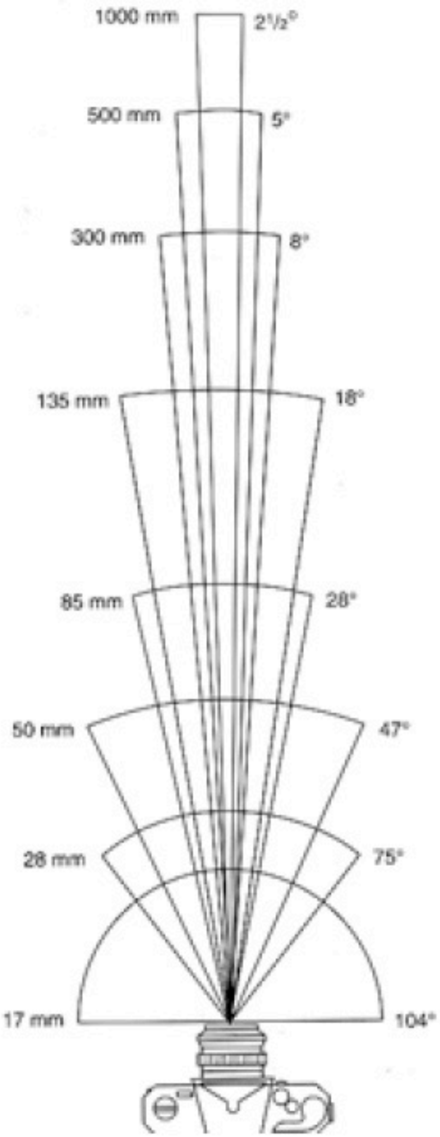
FOV depends on focal length and size of the camera retina

$$\phi = \tan^{-1}(\frac{d/2}{f})$$

Larger focal length = smaller FOV

Slide by A. Efros

Field of View




17mm

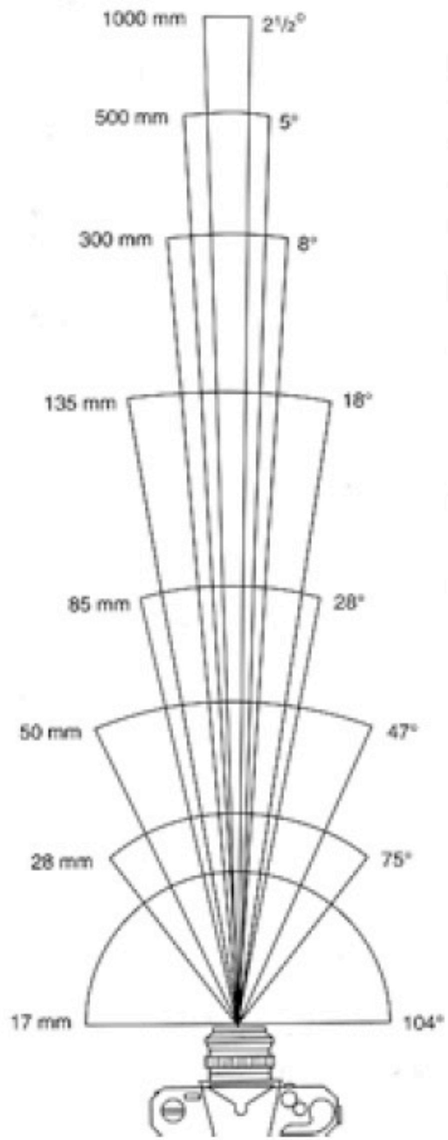

28mm

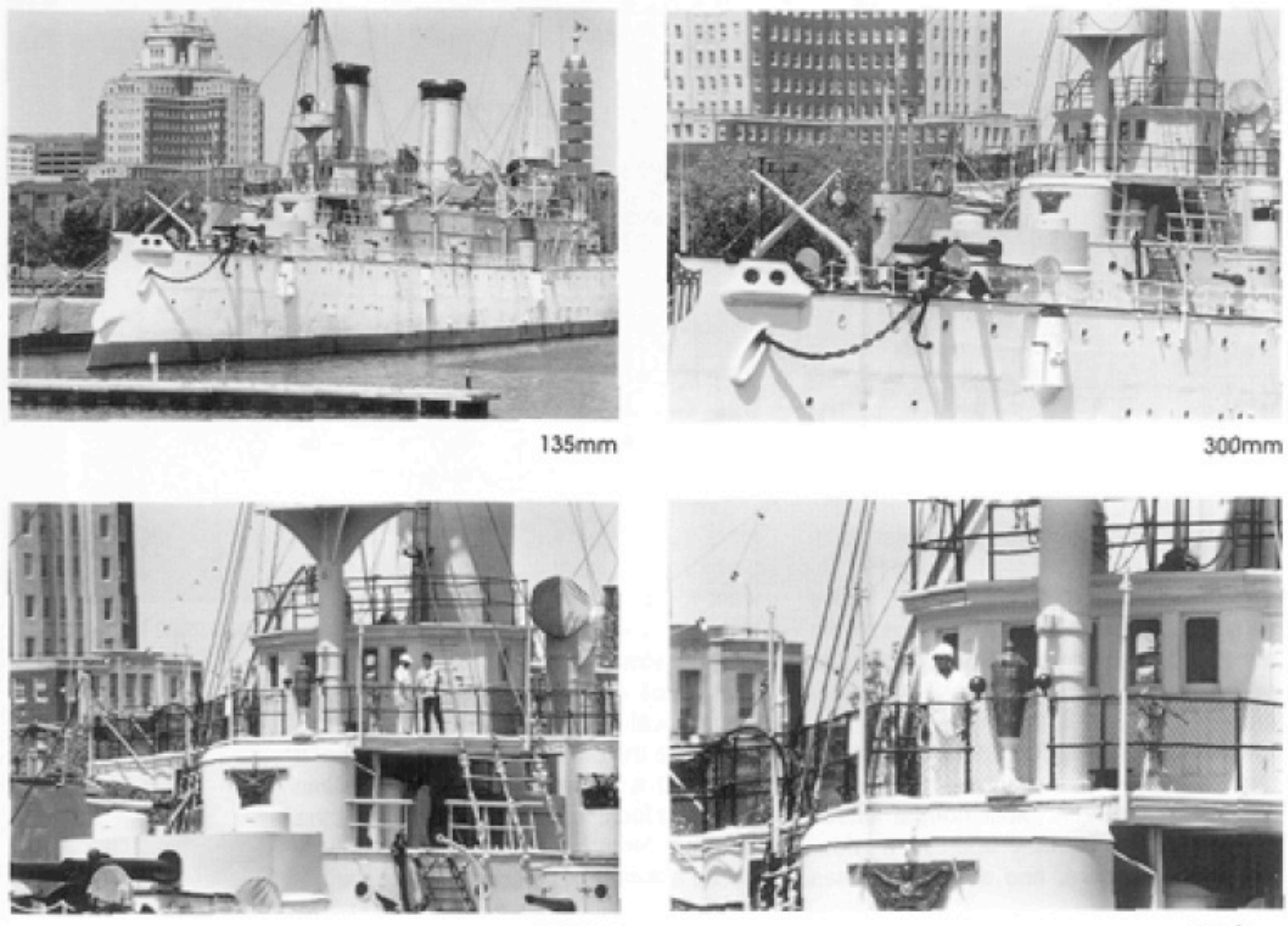

50mm


85mm

Slide by A. Efros

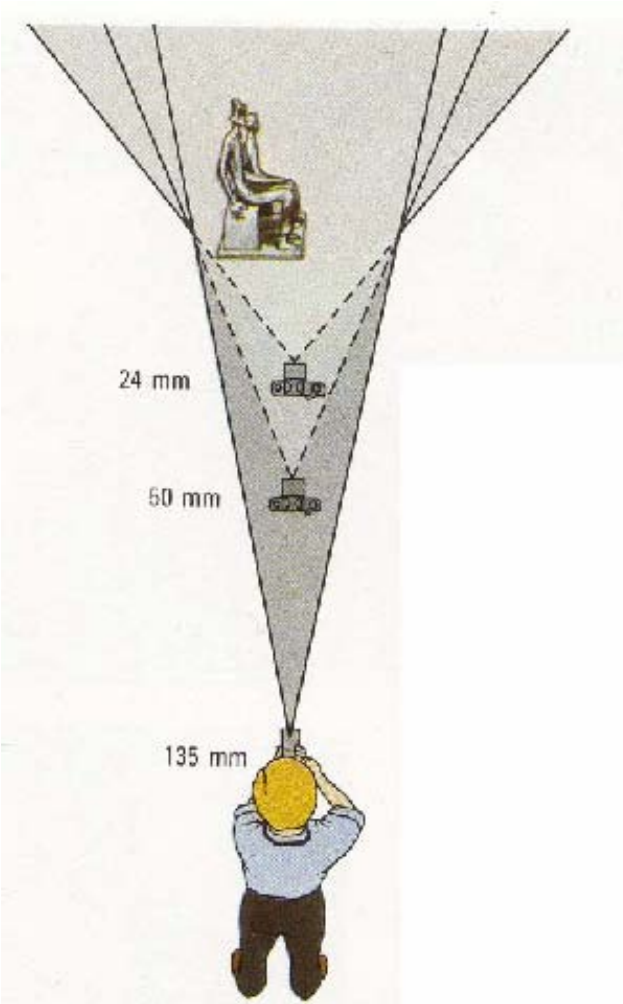
Field of View





Slide by A. Efros

Field of View / Focal Length



Large FOV, small *f*
Camera close to car

Small FOV, large *f*
Camera far from the car

Sources: A. Efros, F. Durand

Same effect for faces

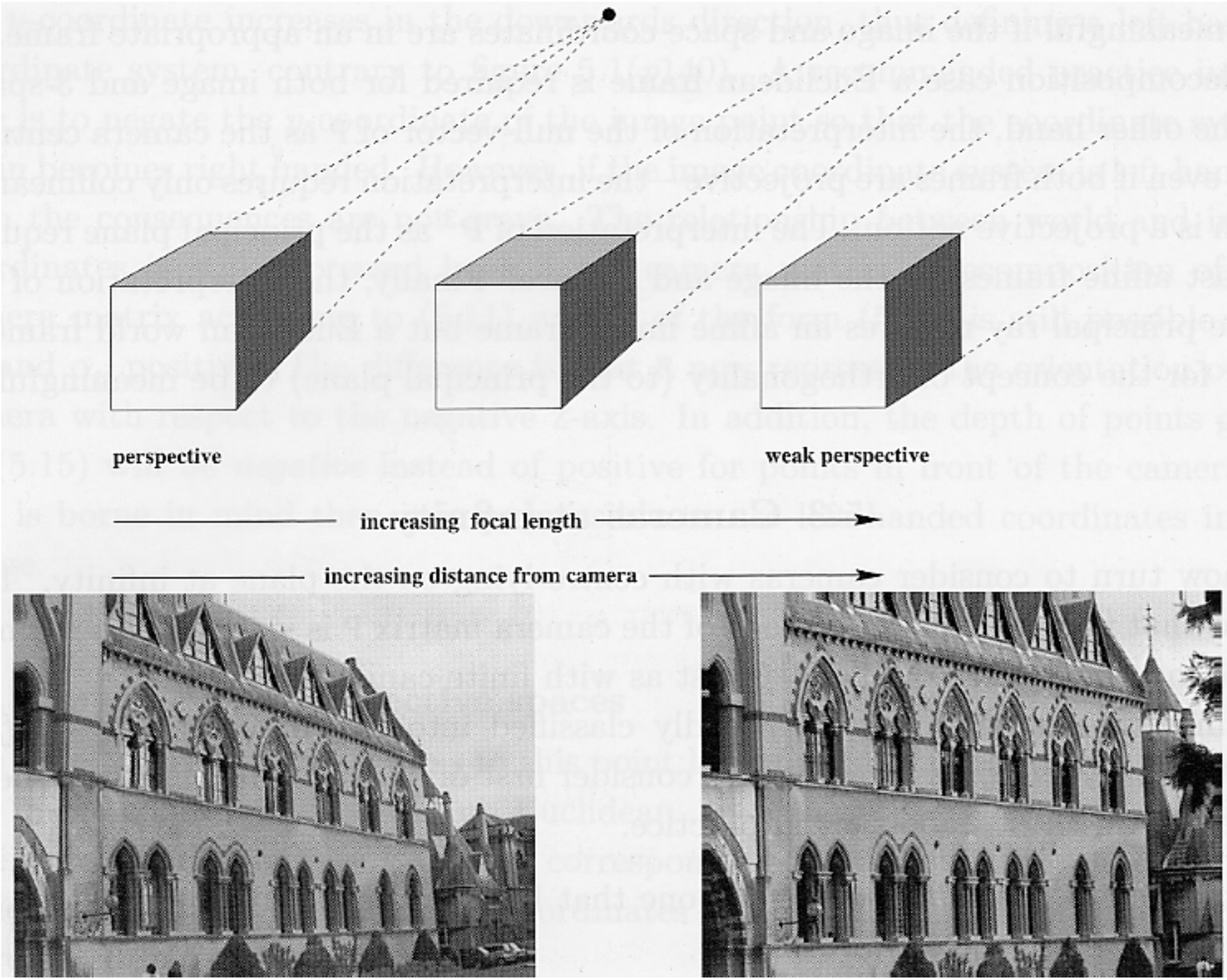
wide-angle

standard

telephoto

Source: F. Durand

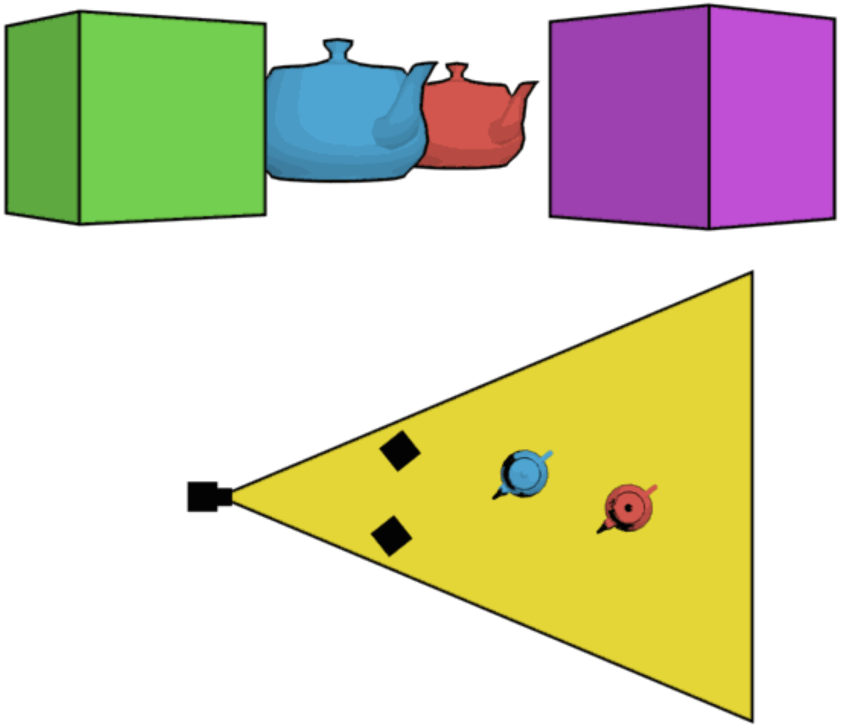
Approximating an orthographic camera



Source: Hartley & Zisserman

The dolly zoom

- Continuously adjusting the focal length while the camera moves away from (or towards) the subject



http://en.wikipedia.org/wiki/Dolly_zoom

The dolly zoom

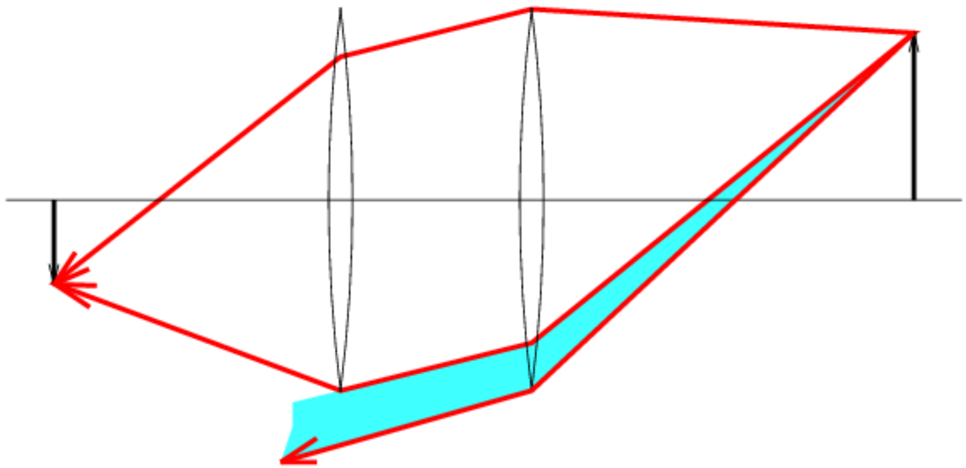
- Continuously adjusting the focal length while the camera moves away from (or towards) the subject
- “The Vertigo shot”

Example of dolly zoom from *Goodfellas* (YouTube)

Example of dolly zoom from *La Haine* (YouTube)

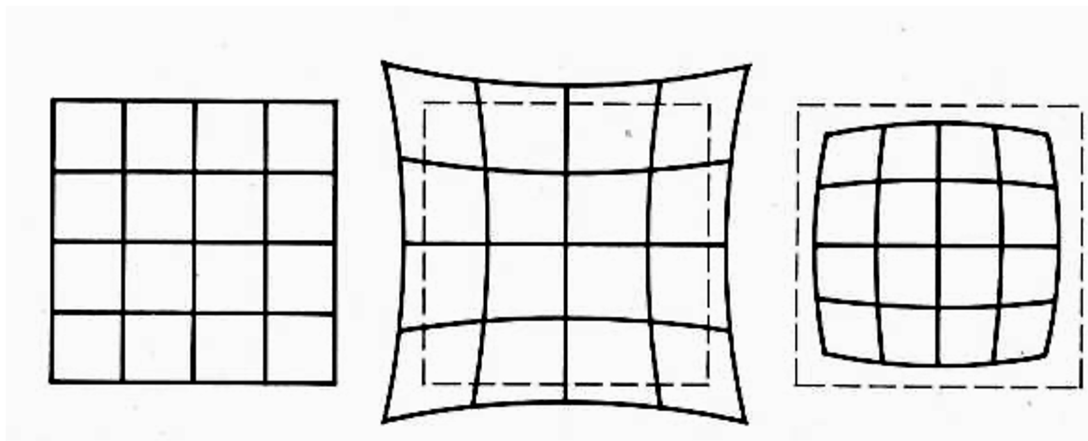
Real lenses

Lens flaws: Vignetting



Radial Distortion

- Caused by imperfect lenses
- Deviations are most noticeable near the edge of the lens

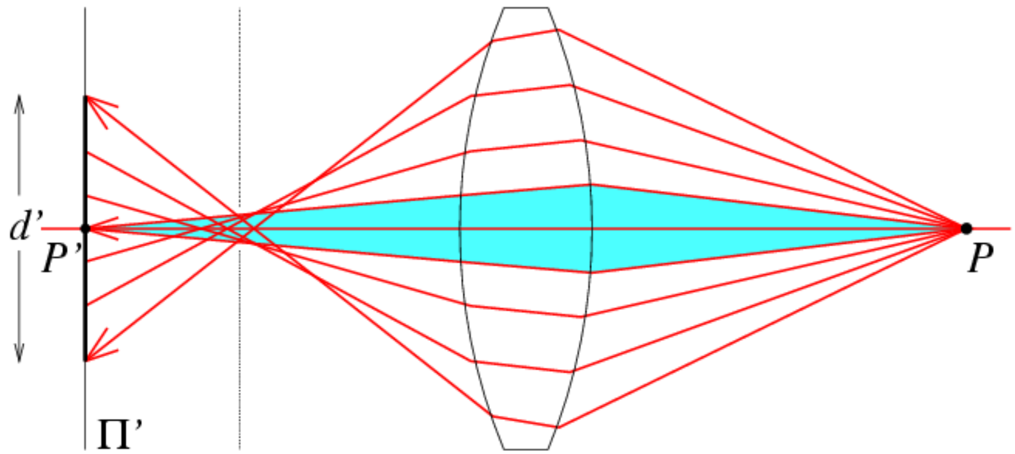


No distortion Pin cushion Barrel

Lens flaws: Spherical aberration

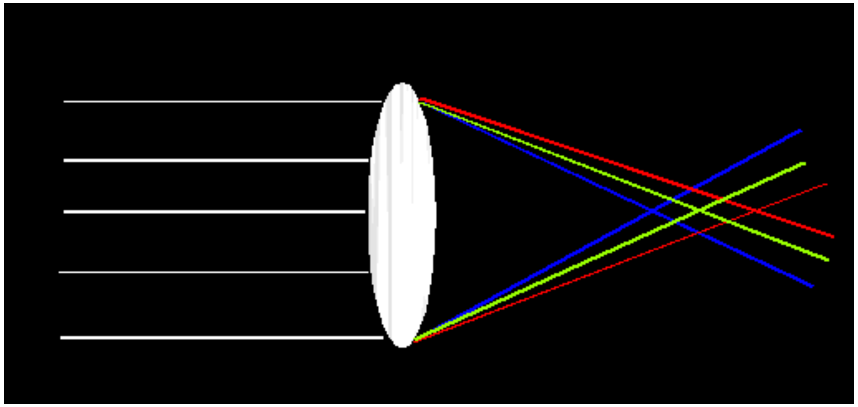
Spherical lenses don't focus light perfectly

- Rays farther from the optical axis focus closer



Lens Flaws: Chromatic Aberration

Lens has different refractive indices for different wavelengths: causes color fringing

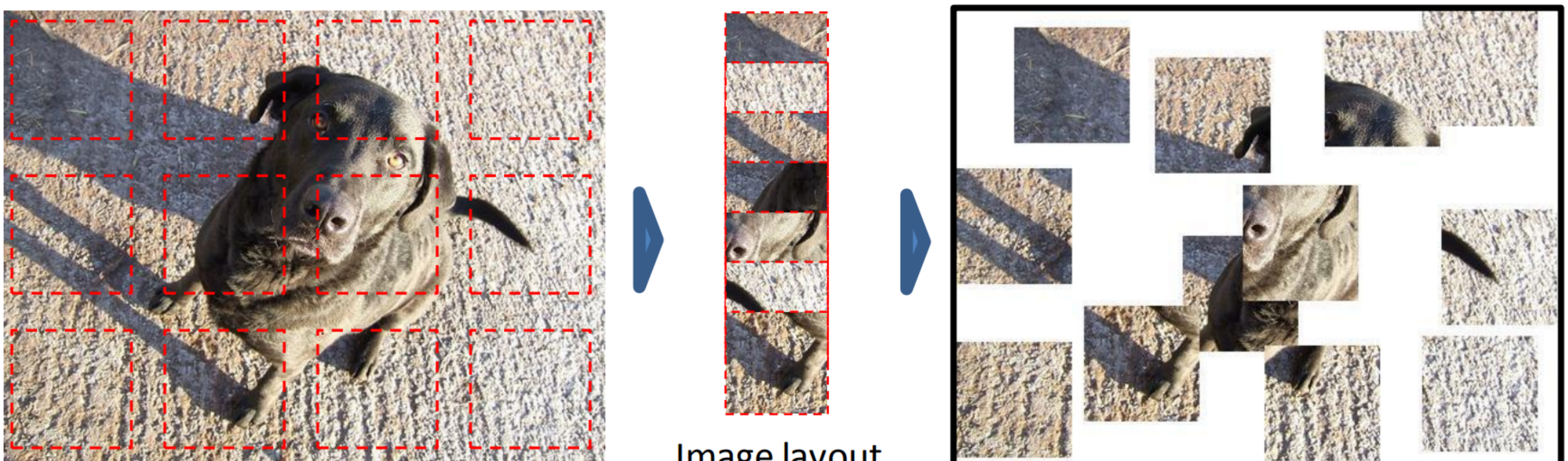


Near Lens Center

Near Lens Outer Edge

Lens Flaws: Chromatic Aberration

Researchers tried teaching a network about objects by forcing it to assemble jigsaws.



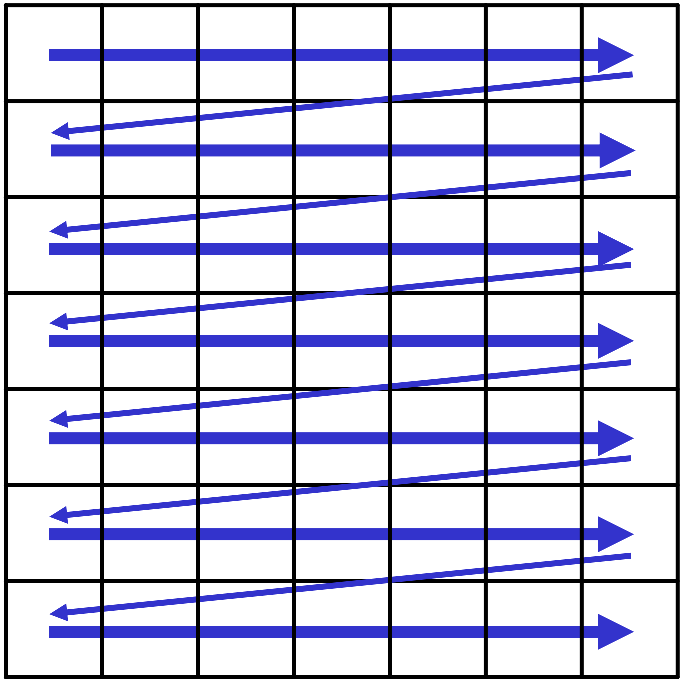
Initial layout, with sampled patches in red | Image layout is discarded | We can recover image layout automatically

Slide Credit: C. Doersch

Rolling Shutter

Rolling Shutter: pixels read in sequence

Can get global reading, but $$$



Overview

- Cameras with lenses
  - Depth of field
  - Field of view
  - Lens aberrations
- Brightness of a pixel
  - Small taste of radiometry
  - In-camera transformation of light
  - Reflectance properties of surfaces
  - Lambertian reflection model
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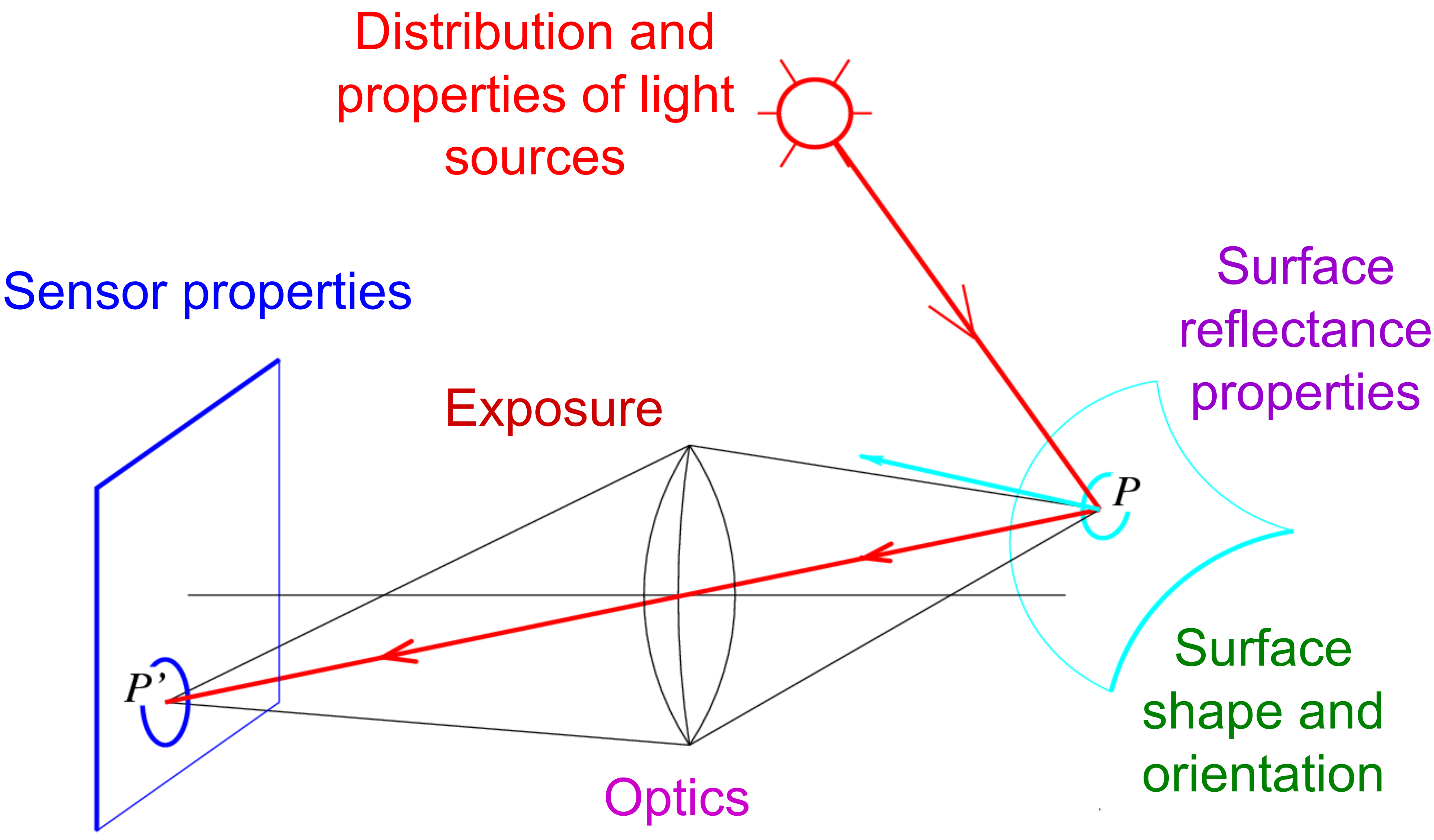
Image formation

What determines the brightness of an image pixel?



Slide by L. Fei-Fei

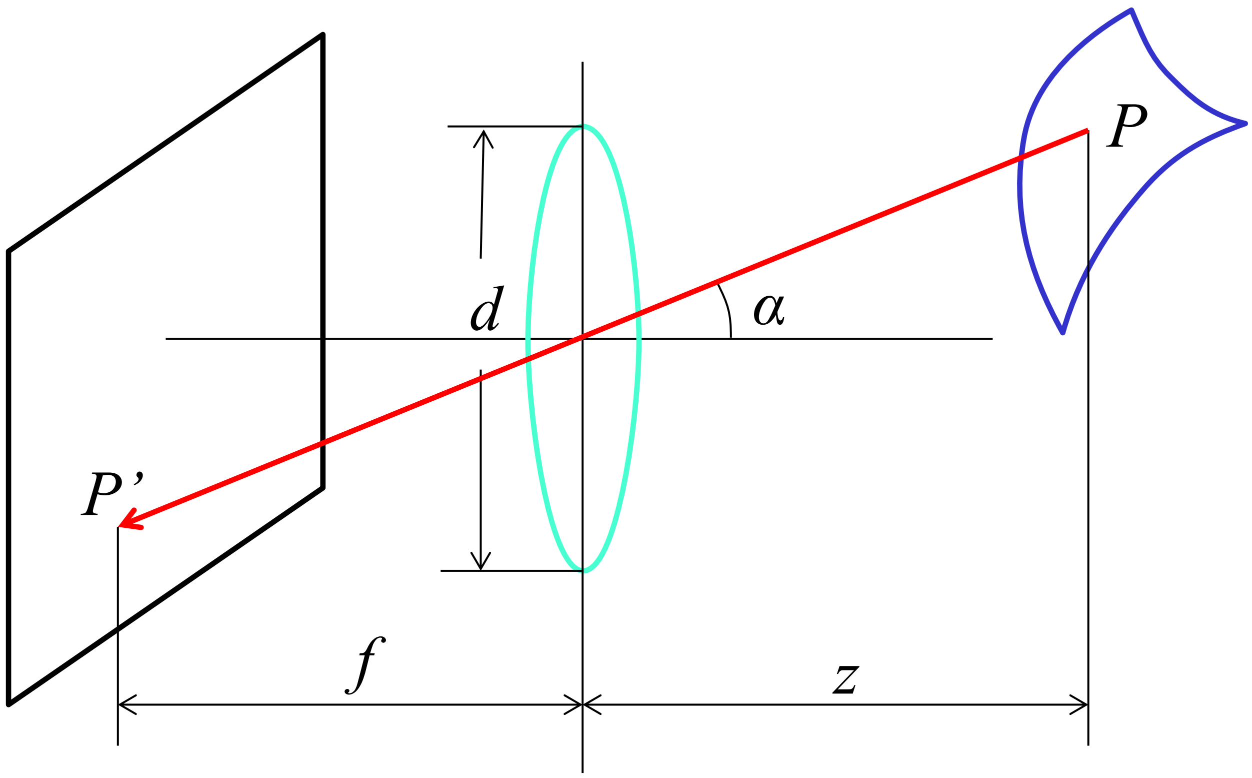
Fundamental radiometric relation

L: *Radiance* emitted from *P* toward *P*'

- Energy carried by a ray (Watts per sq. meter per steradian)

E: *Irradiance* falling on *P*' from the lens

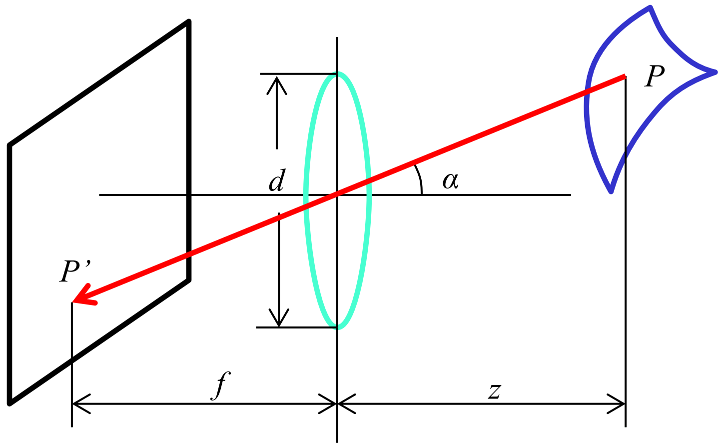
- Energy arriving at a surface (Watts per sq. meter)



What is the relationship between *E* and *L*?

Szeliski 2.2.3

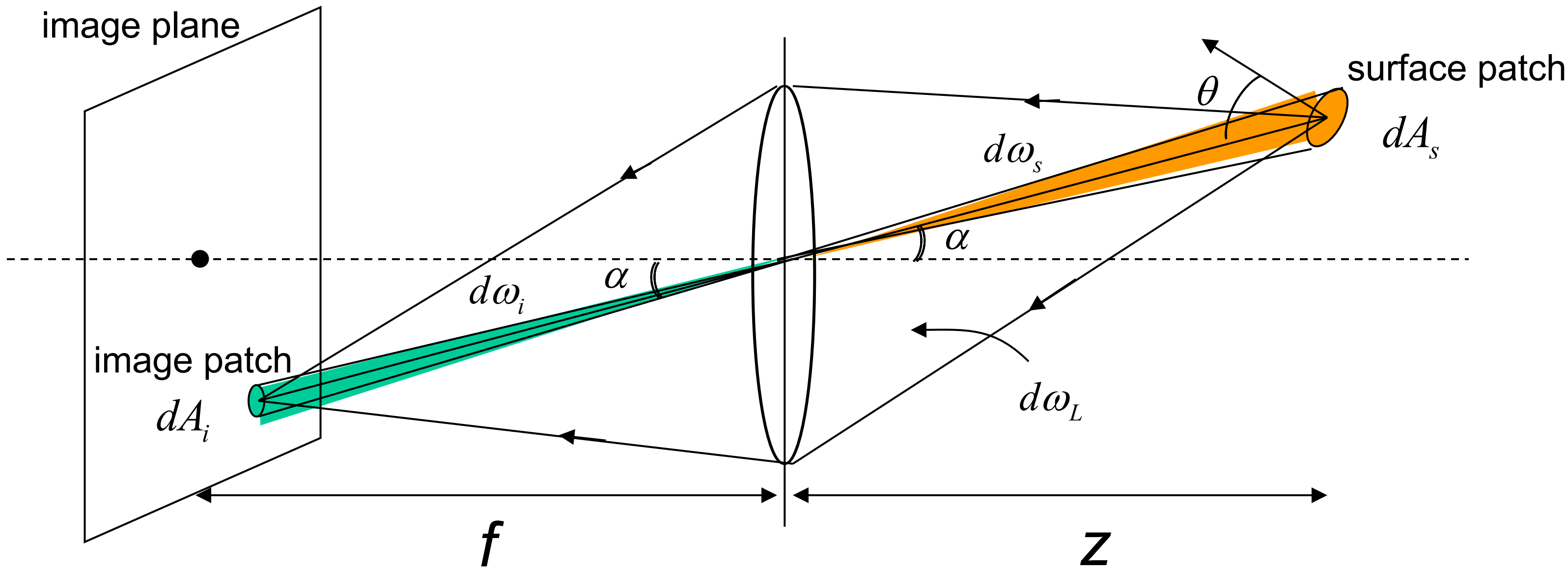
Fundamental radiometric relation



$$E = \left[\frac{\pi}{4} \left(\frac{d}{f} \right)^2 \cos^4 \alpha \right] L$$

- Image irradiance is linearly related to scene radiance
- Irradiance is proportional to the area of the lens and inversely proportional to the squared distance between the lens and the image plane
- The irradiance falls off as the angle between the viewing ray and the optical axis increases

Relation between Image Irradiance E and Scene Radiance L



- Solid angles of the double cone (orange and green):

$$d\omega_i = d\omega_s \qquad \frac{dA_i \cos\alpha}{(f/\cos\alpha)^2} = \frac{dA_s \cos\theta}{(z/\cos\alpha)^2}$$

$$\frac{dA_s}{dA_i} = \frac{\cos\alpha}{\cos\theta}\left(\frac{z}{f}\right)^2$$

(1)

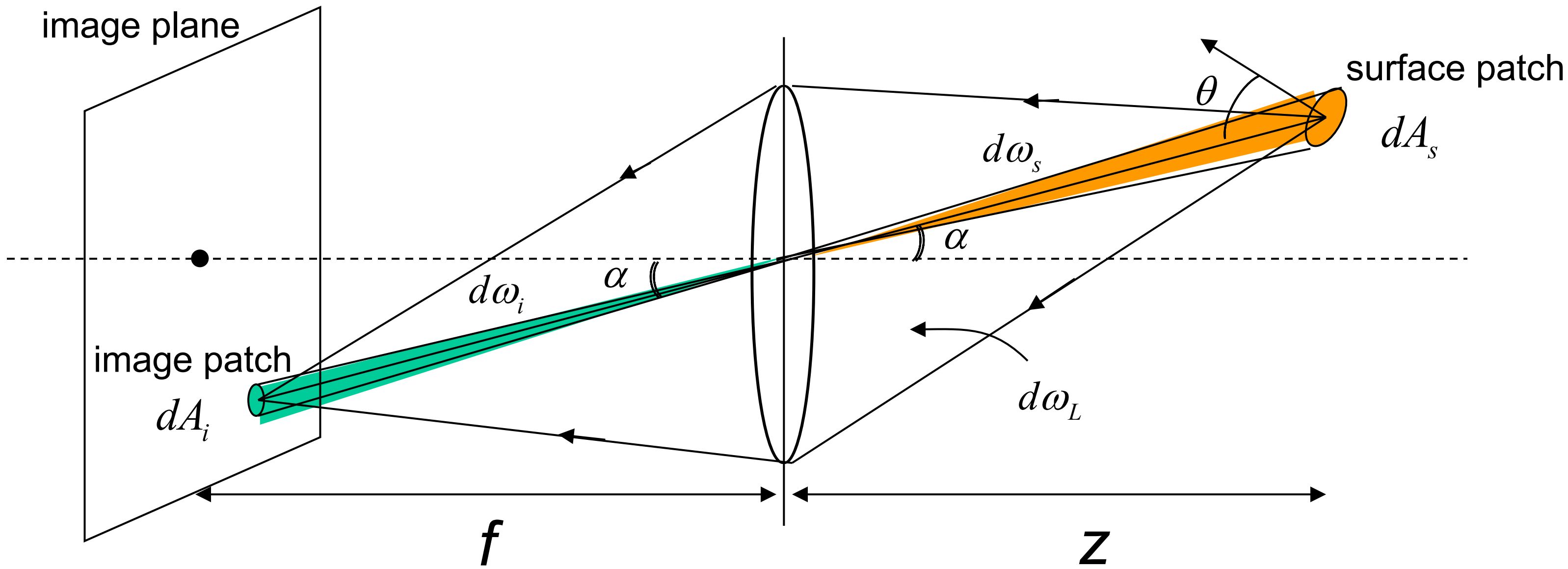
- Solid angle subtended by lens:

$$d\omega_L = \frac{\pi d^2}{4} \frac{\cos\alpha}{(z/\cos\alpha)^2}$$

(2)

Slide from S Narasimhan.

Relation between Image Irradiance E and Scene Radiance L



- Flux received by lens from dA_s = Flux projected onto image dA_i

$$L\,(dA_s \cos\theta)\,d\omega_L = E\,dA_i \quad (3)$$

- From (1), (2), and (3):

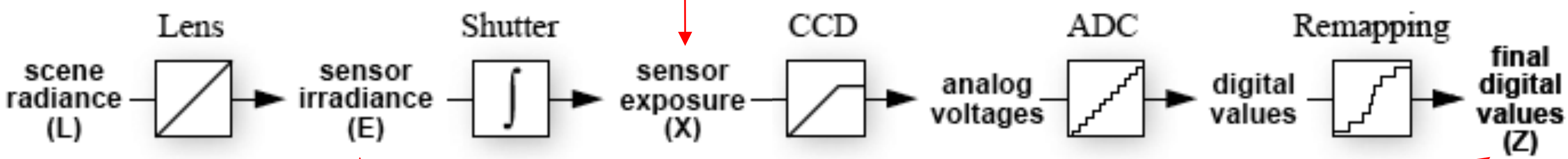
$$E = L\,\frac{\pi}{4}\left(\frac{d}{f}\right)^2 \cos\alpha^4$$

Slide from S Narasimhan.

- Image irradiance is proportional to Scene Radiance!
- Small field of view → Effects of 4th power of cosine are small.

From light rays to pixel values

$$X = E \cdot \Delta t$$



$$E = \left[\frac{\pi}{4} \left(\frac{d}{f} \right)^2 \cos^4 \alpha \right] L$$

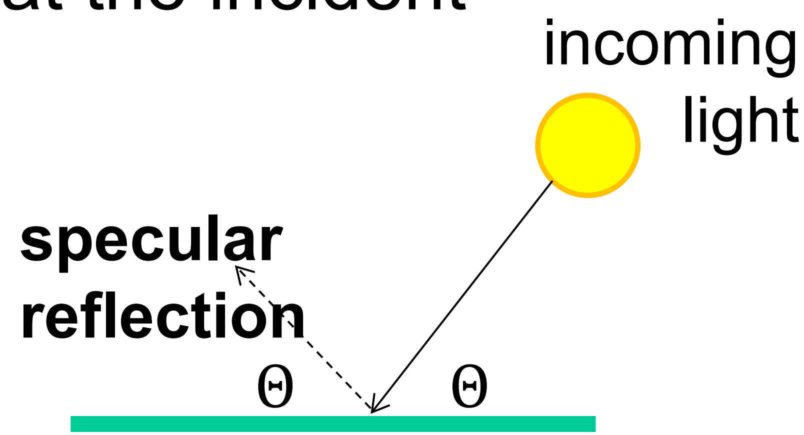
$$Z = f\left(E \cdot \Delta t\right)$$

- Camera response function: the mapping f from irradiance to pixel values
  - Useful if we want to estimate material properties
  - Enables us to create *high dynamic range (HDR) images*
  - Classic reference: P. E. Debevec and J. Malik, *Recovering High Dynamic Range Radiance Maps from Photographs*, *SIGGRAPH 97*

Basic models of reflection

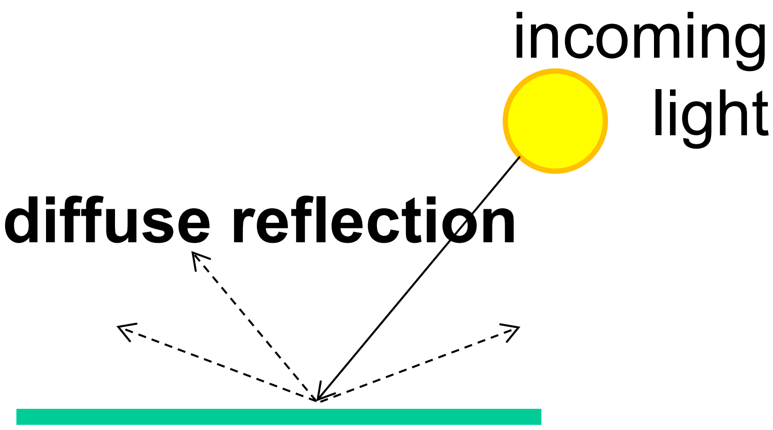
Specular: light bounces off at the incident angle

- E.g., mirror



Diffuse: light scatters in all directions

- E.g., brick, cloth, rough wood

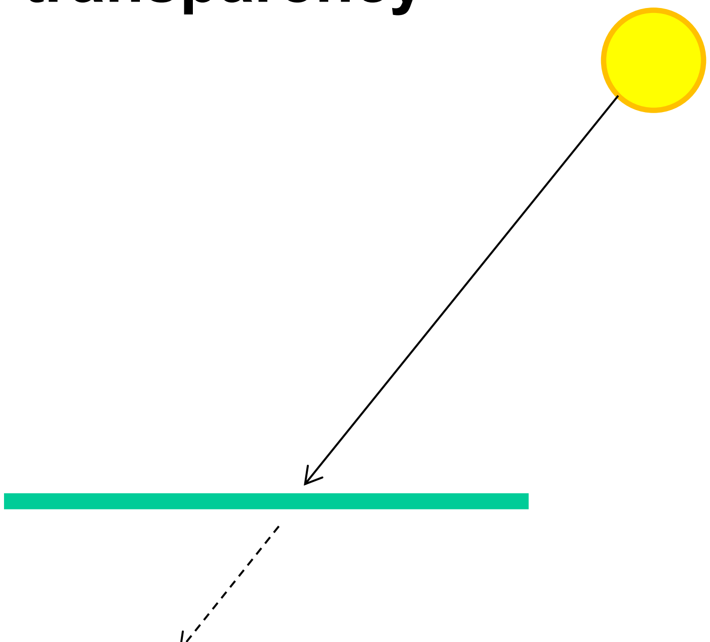


Slide from D. Hoiem

Other possible effects

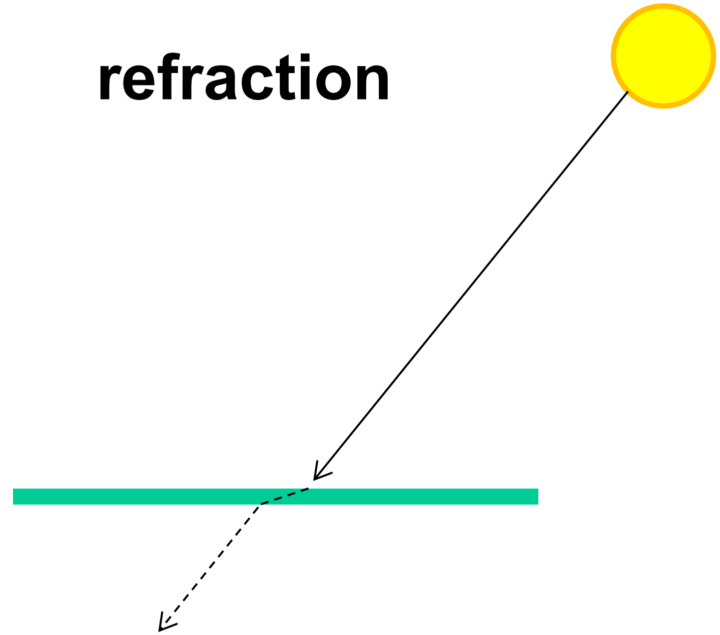
transparency

light source



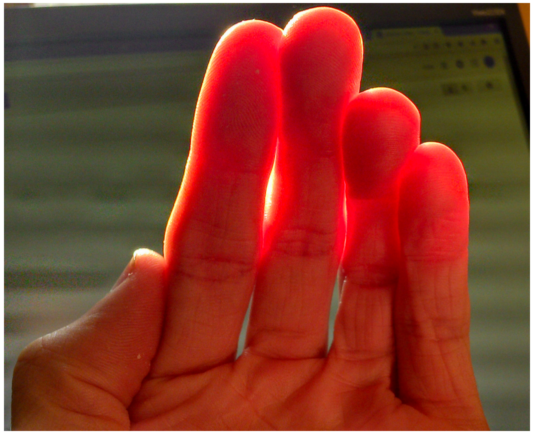
refraction

light source

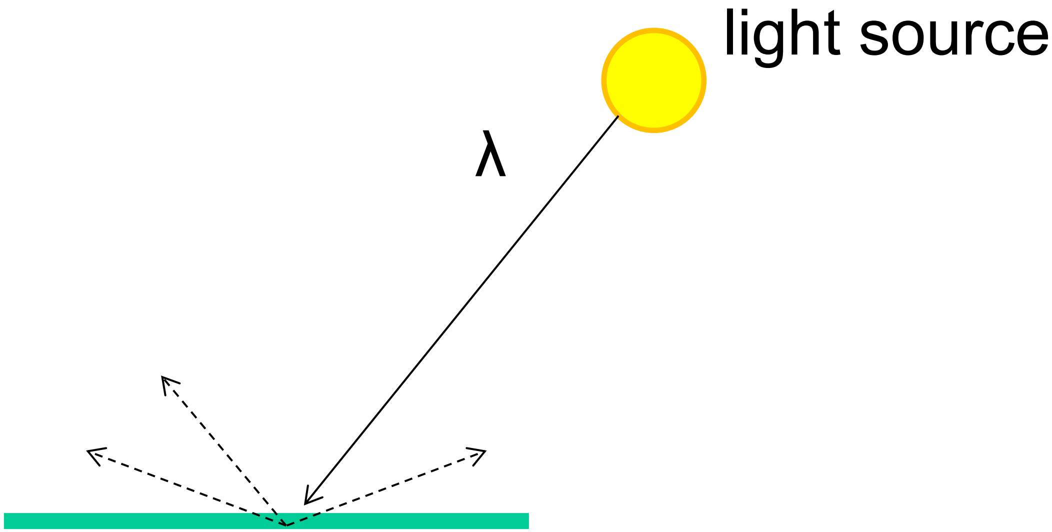


Slide from D. Hoiem

Other possible effects



subsurface scattering

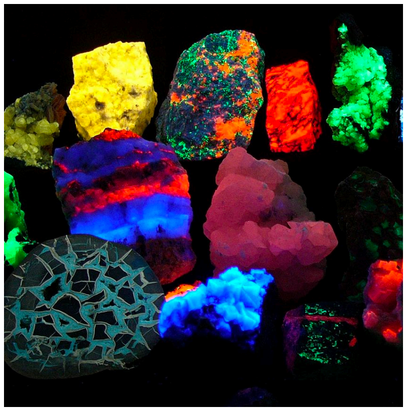


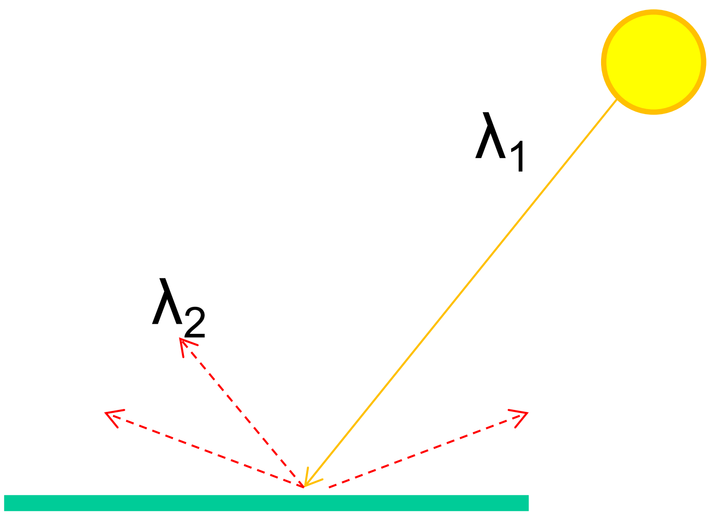
Slide from D. Hoiem

https://en.wikipedia.org/wiki/Subsurface_scattering#/media/File:Skin_Subsurface_Scattering.jpg

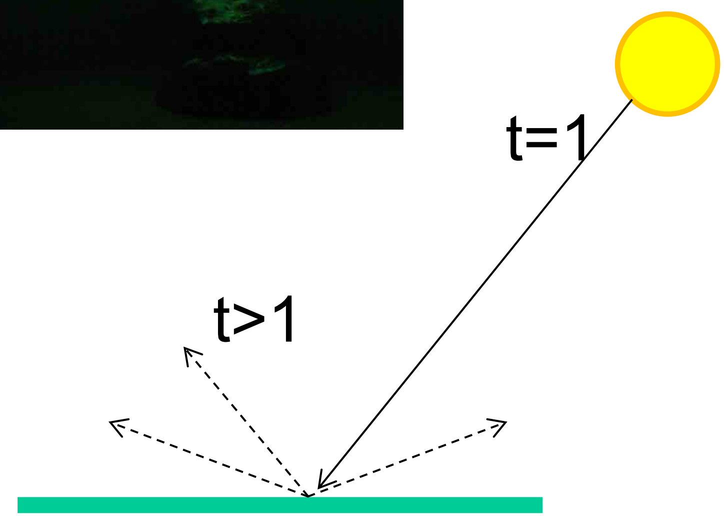
Other possible effects

fluorescence





phosphorescence



https://en.wikipedia.org/wiki/Fluorescence#/media/File:Fluorescent_minerals_hg.jpg

Slide from D. Hoiem

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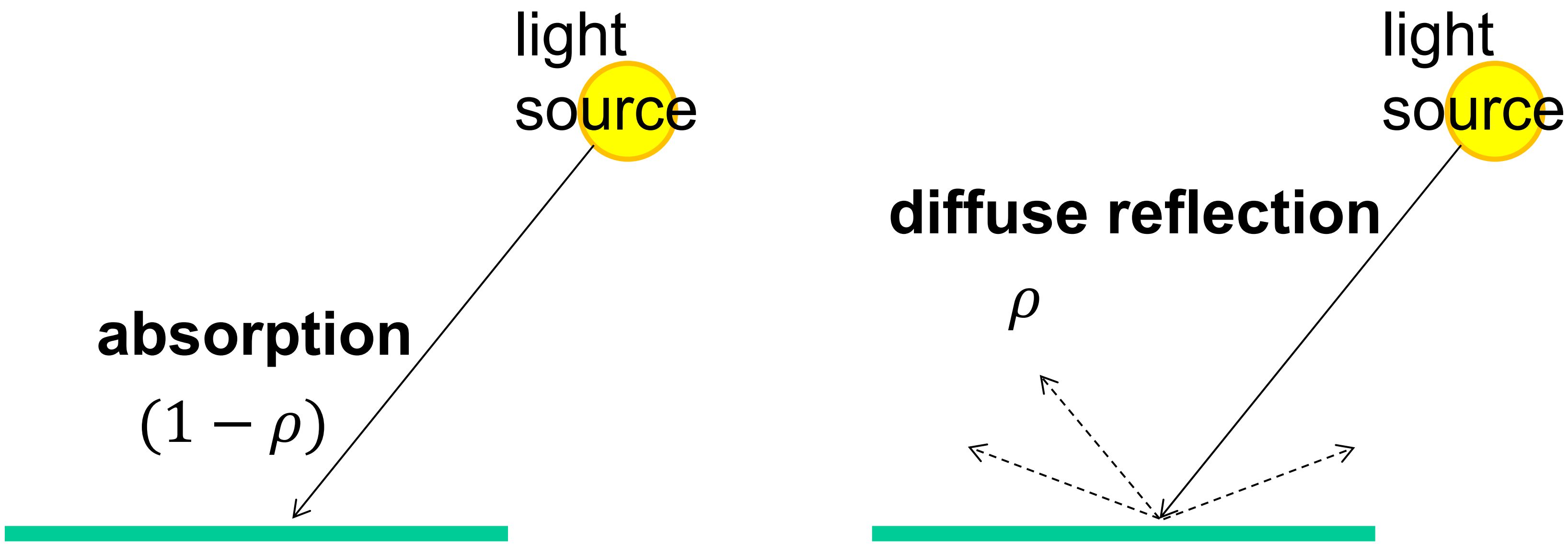
Lambertian reflectance model

Some light is absorbed (function of albedo ρ)

Remaining light is scattered (diffuse reflection)

Examples: soft cloth, concrete, matte paints



Slide from D. Hoiem

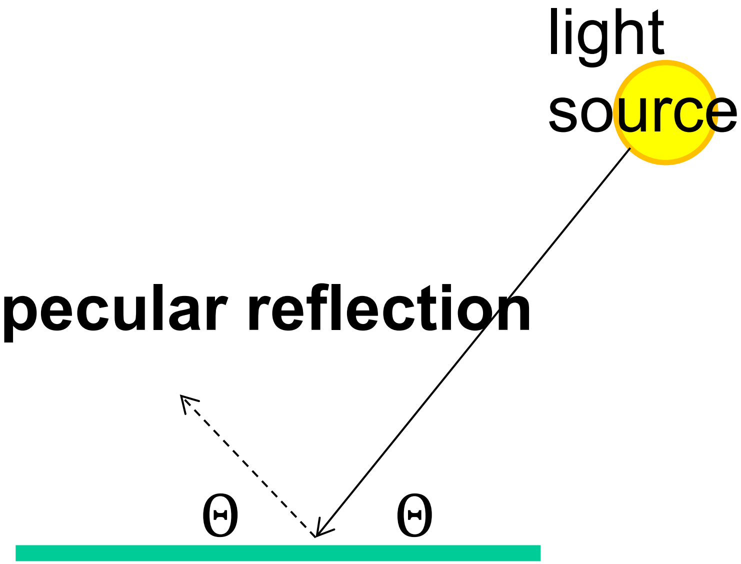
Specular Reflection

Reflected direction depends on light orientation and surface normal

- E.g., mirrors are fully specular
- Most surfaces can be modeled with a mixture of diffuse and specular components

Flickr, by suzysputnik

specular reflection



Flickr, by piratejohnny

Slide from D. Hoiem

Most surfaces have both specular and diffuse components

Specularity = spot where specular reflection dominates (typically reflects light source)

Photo: northcountryhardwoodfloors.com

Typically, specular component is small

Slide from D. Hoiem

Intensity and Surface Orientation

Intensity depends on illumination angle because less light comes in at oblique angles.

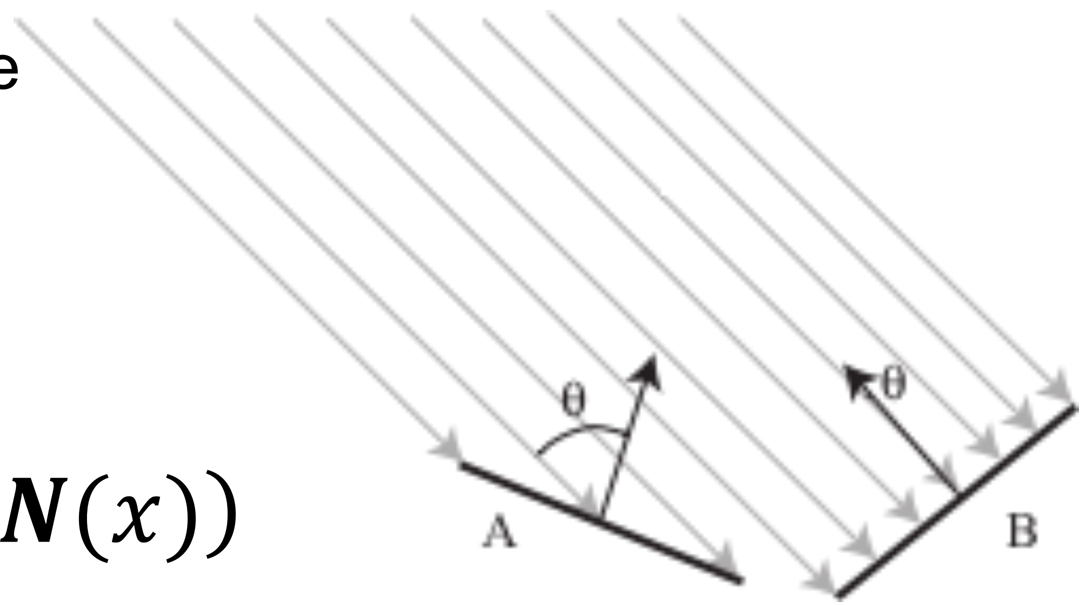
$\rho =$ albedo

$\boldsymbol{S} =$ directional source

$\boldsymbol{N} =$ surface normal

$\mathrm{I} =$ reflected intensity

$$I(x) = \rho(x)(\boldsymbol{S} \cdot \boldsymbol{N}(x))$$



Slide: Forsyth

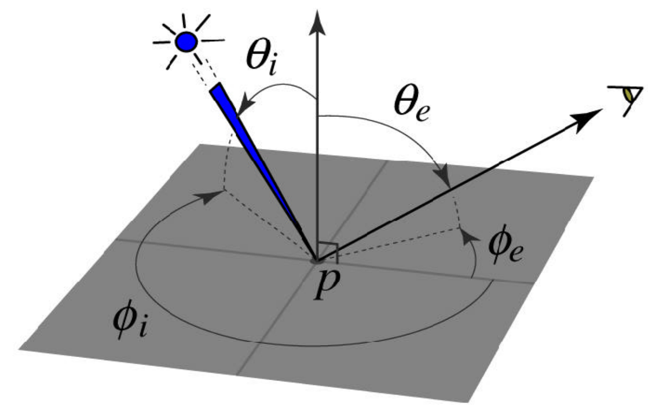
The interaction of light and surfaces

What happens when a light ray hits a point on an object?

- Some of the light gets **absorbed**
  - converted to other forms of energy (e.g., heat)
- Some gets **transmitted** through the object
  - possibly bent, through refraction
  - or scattered inside the object (subsurface scattering)
- Some gets **reflected**
  - possibly in multiple directions at once
- Really complicated things can happen
  - fluorescence

Bidirectional reflectance distribution function (BRDF)

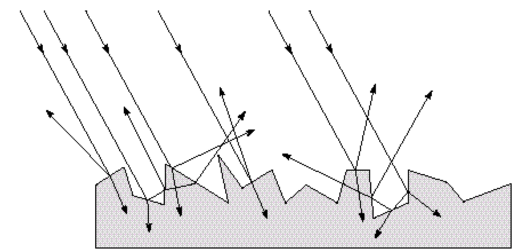
- How bright a surface appears when viewed from one direction when light falls on it from another
- Definition: ratio of the radiance in the emitted direction to irradiance in the incident direction



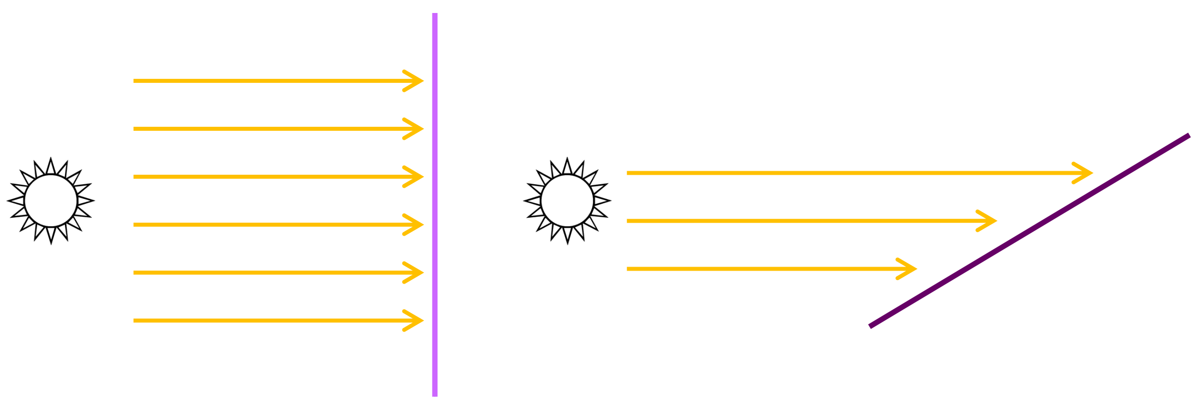
Source: Steve Seitz

Diffuse reflection

- Light is reflected equally in all directions
  - Dull, matte surfaces like chalk or latex paint
  - Microfacets scatter incoming light randomly

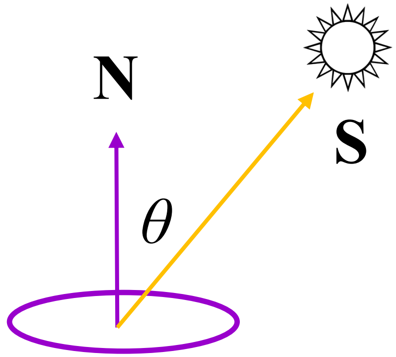


- Brightness of the surface depends on the incidence of illumination



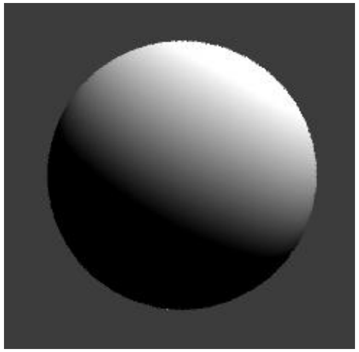
brighter darker

Diffuse reflection: Lambert's law



$$B = \rho\, \mathbf{N} \cdot \mathbf{S}$$

$$= \rho \left\| \mathbf{S} \right\| \cos\theta$$

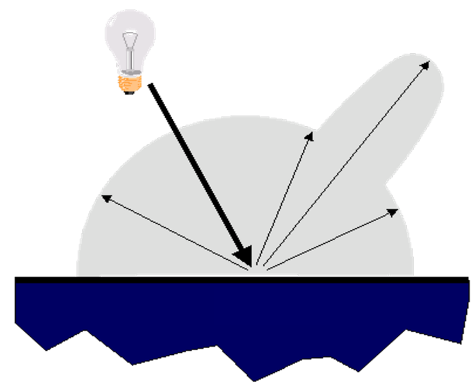


B: radiosity (total power leaving the surface per unit area)
ρ: albedo (fraction of incident irradiance reflected by the surface)
N: unit normal
S: source vector (magnitude proportional to intensity of the source)

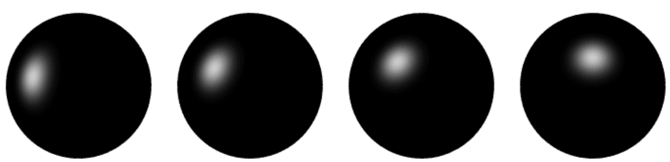
Slide from L. Lazebnik

Specular reflection

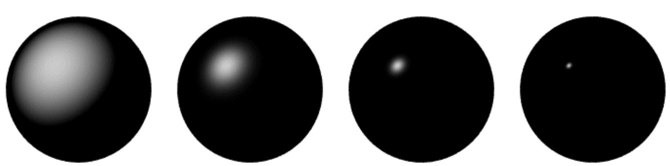
- Radiation arriving along a source direction leaves along the **specular direction** (source direction reflected about normal)
- On real surfaces, energy usually goes into a lobe of directions
- **Phong model**: reflected energy falls of with $\cos^n(\delta\theta)$

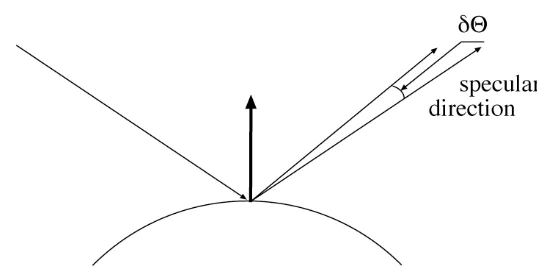


Moving the light source



Changing the exponent





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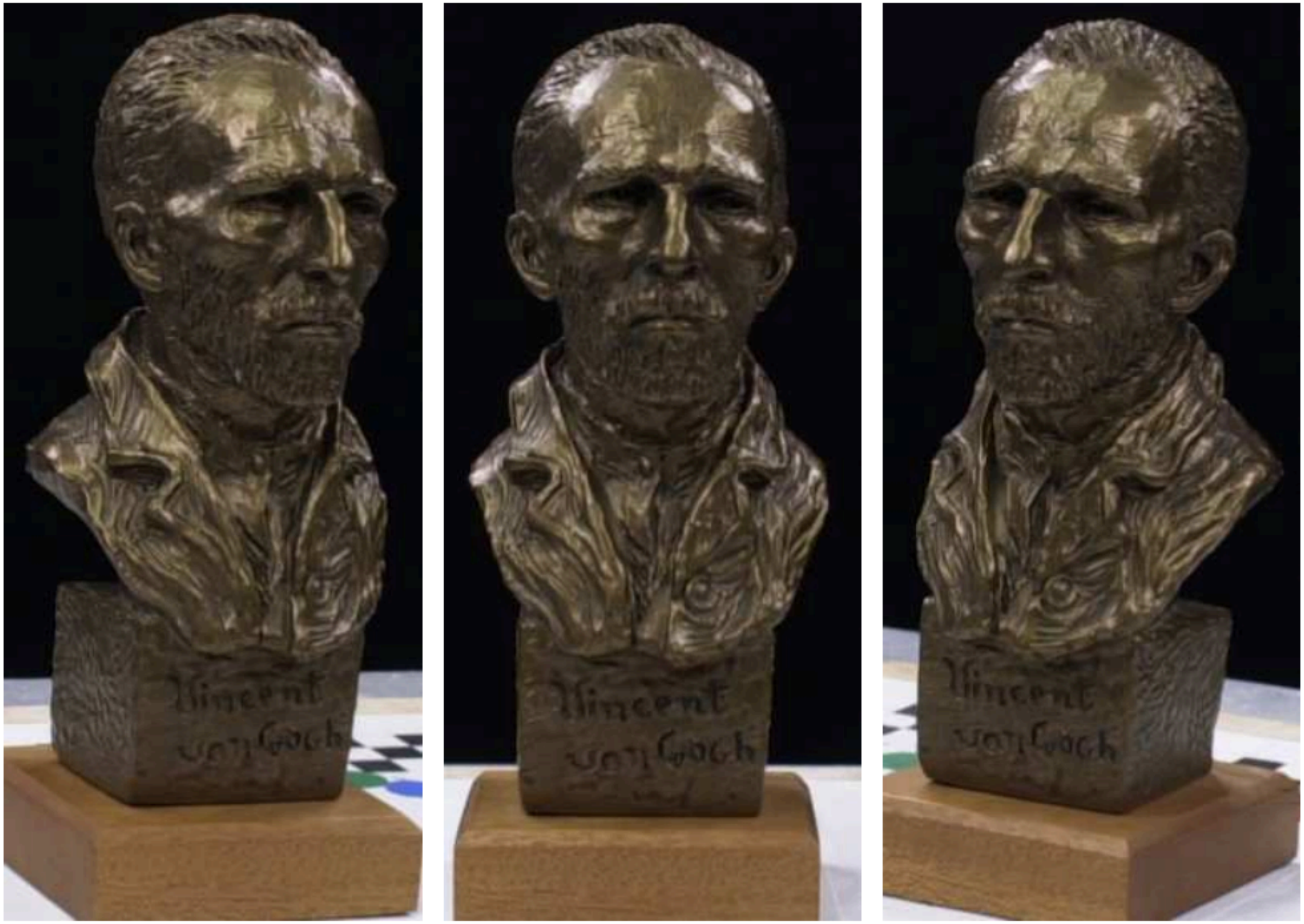
Specular reflection



Picture source

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Photometric stereo (shape from shading)

- Can we reconstruct the shape of an object based on shading cues?

Luca della Robbia,
Cantoria, 1438

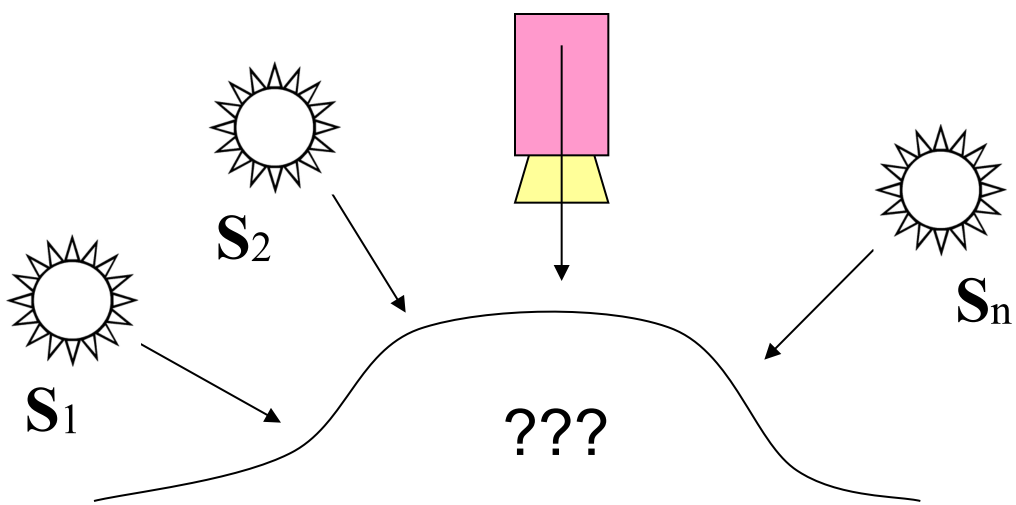
Slide from L. Lazebnik

Photometric stereo

Assume:

- A Lambertian object
- A *local shading model* (each point on a surface receives light only from sources visible at that point)
- A set of *known* light source directions
- A set of pictures of an object, obtained in exactly the same camera/object configuration but using different sources
- Orthographic projection

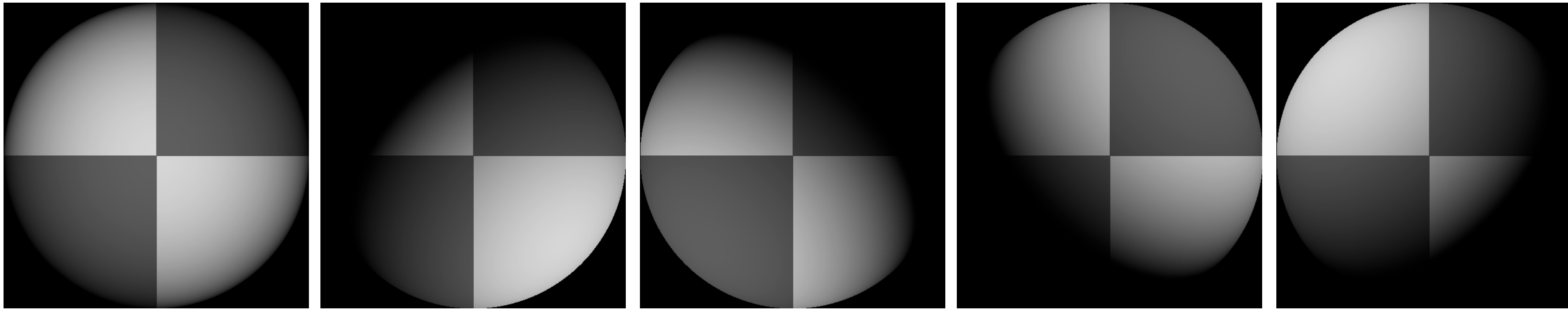
Goal: reconstruct object shape and albedo



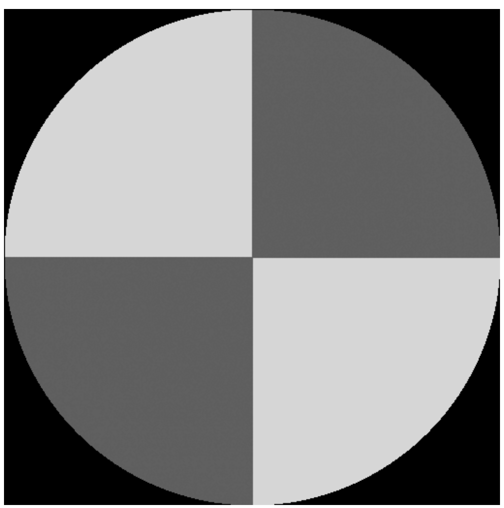
Slide from L. Lazebnik

F&P 2nd ed., sec. 2.2.4

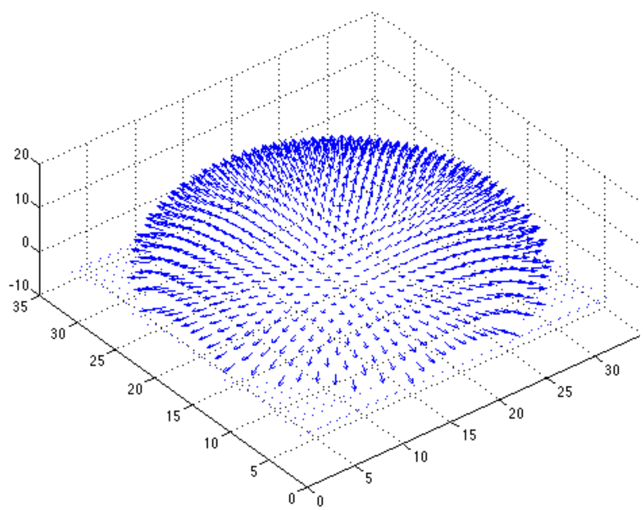
Example 1



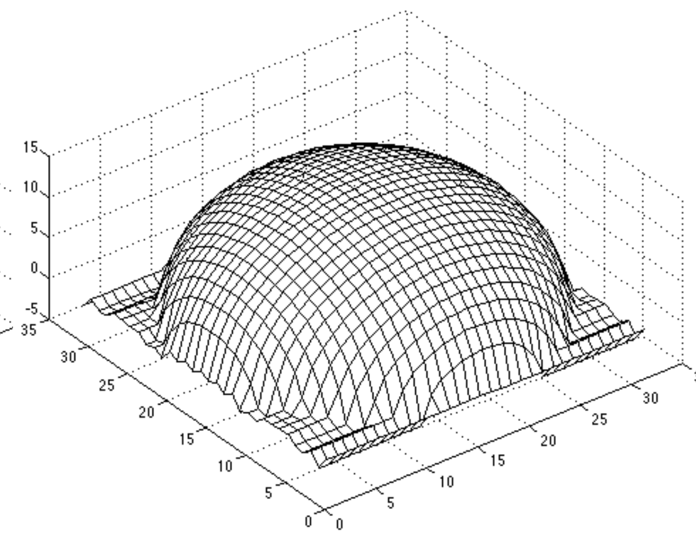
Recovered albedo



Recovered normal field



Recovered surface model



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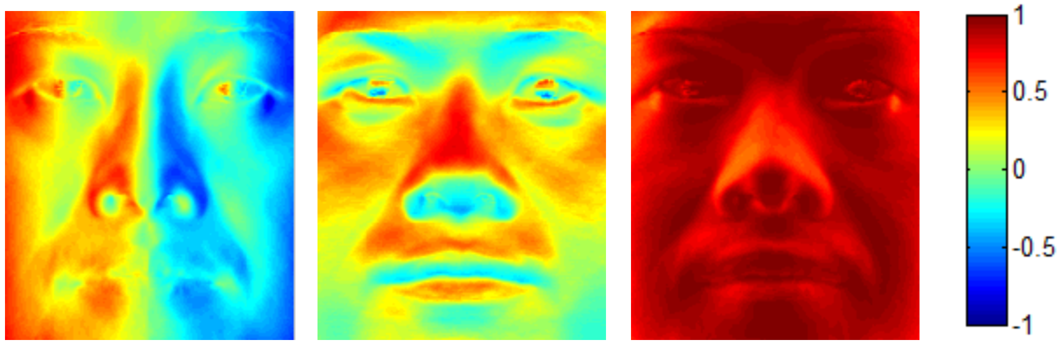
Example 2

Input

Recovered albedo

Recovered normal field



x y z

Recovered surface model

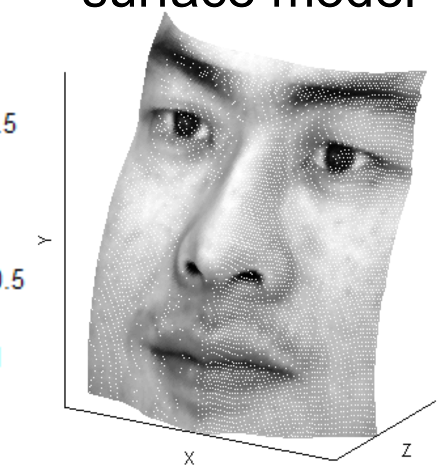


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Image model

- **Known:** source vectors $\mathbf{S}_j$ and pixel values $I_j(x,y)$
- **Unknown:** surface normal $\mathbf{N}(x,y)$ and albedo $\rho(x,y)$

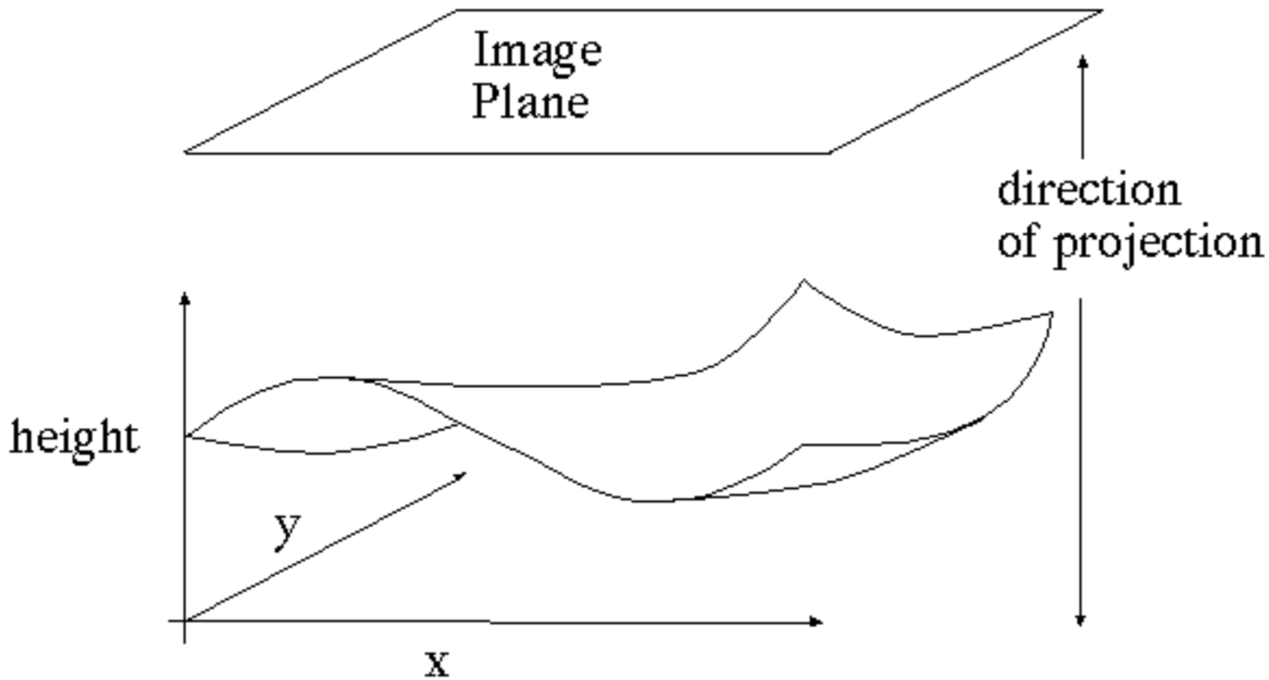

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Image model

- **Known:** source vectors $\mathbf{S}_j$ and pixel values $I_j(x,y)$
- **Unknown:** surface normal $\mathbf{N}(x,y)$ and albedo $\rho(x,y)$
- Assume that the response function of the camera is a linear scaling by a factor of k
- Lambert's law:

$$\begin{aligned} I_j(x,y) &= k\,\rho(x,y)\big(\mathbf{N}(x,y)\cdot\mathbf{S}_j\big) \\ &= \big(\rho(x,y)\mathbf{N}(x,y)\big)\cdot(k\mathbf{S}_j) \\ &= \mathbf{g}(x,y)\cdot\mathbf{V}_j \end{aligned}$$

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Least squares problem

- For each pixel, set up a linear system:

$$\begin{bmatrix} I_1(x,y) \\ I_2(x,y) \\ \vdots \\ I_n(x,y) \end{bmatrix} = \begin{bmatrix} \mathbf{V}_1^T \\ \mathbf{V}_2^T \\ \vdots \\ \mathbf{V}_n^T \end{bmatrix} \mathbf{g}(x,y)$$

(n × 1) known — (n × 3) known — (3 × 1) unknown

- Obtain least-squares solution for $\mathbf{g}(x,y)$ (which we defined as $\mathbf{N}(x,y)\ \rho(x,y)$)
- Since $\mathbf{N}(x,y)$ is the unit normal, $\rho(x,y)$ is given by the magnitude of $\mathbf{g}(x,y)$
- Finally, $\mathbf{N}(x,y) = \mathbf{g}(x,y)\ /\ \rho(x,y)$

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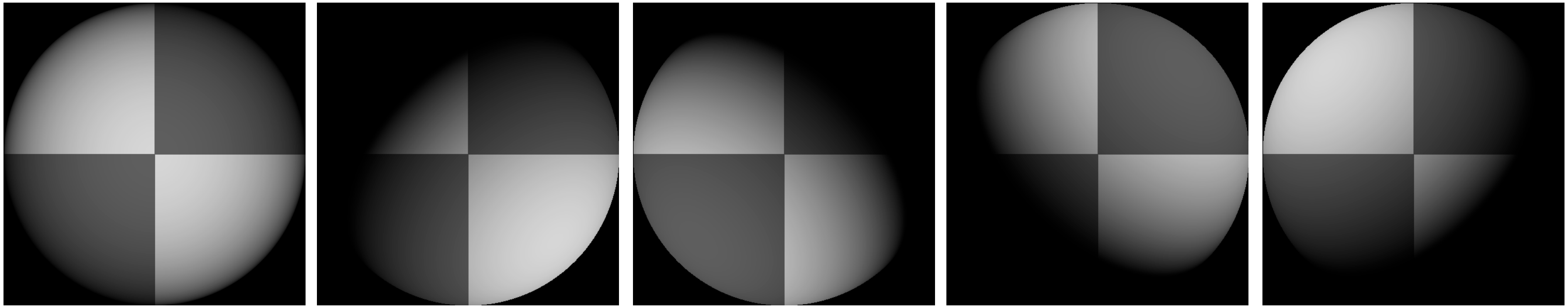
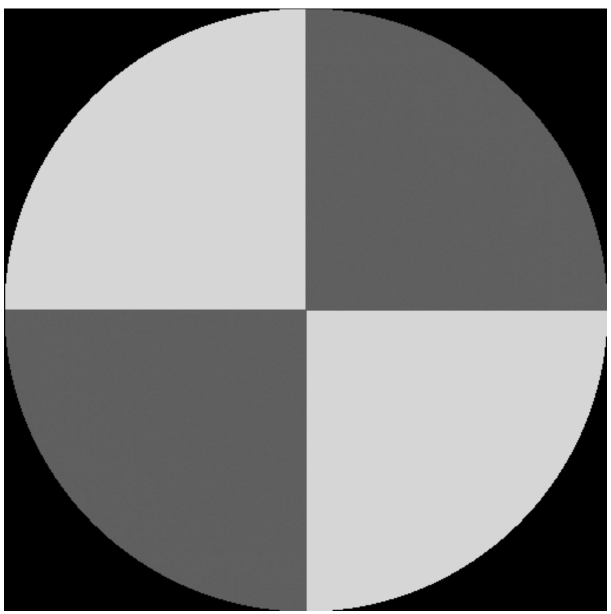
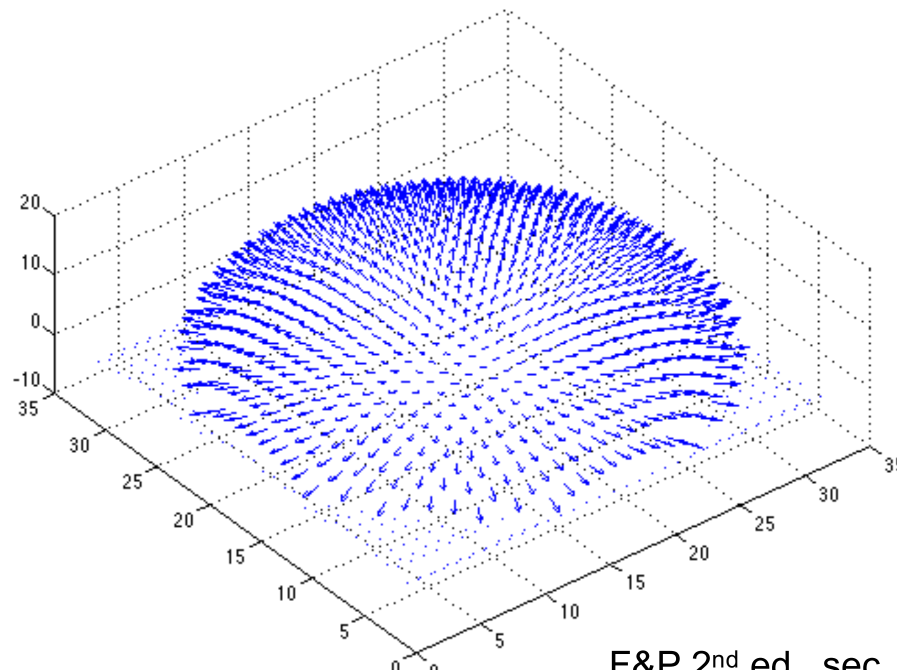
Synthetic example



Recovered albedo



Recovered normal field



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Recovering a surface from normals

Recall the surface is written as

$$(x, y, f(x, y))$$

This means the normal has the form:

$$\mathbf{N}(x, y) = \frac{1}{\sqrt{f_x^2 + f_y^2 + 1}} \begin{pmatrix} f_x \\ f_y \\ 1 \end{pmatrix}$$

If we write the estimated vector *g* as

$$\mathbf{g}(x, y) = \begin{pmatrix} g_1(x, y) \\ g_2(x, y) \\ g_3(x, y) \end{pmatrix}$$

Then we obtain values for the partial derivatives of the surface:

$$f_x(x, y) = g_1(x, y) / g_3(x, y)$$
$$f_y(x, y) = g_2(x, y) / g_3(x, y)$$

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Recovering a surface from normals

We can now recover the surface height at any point by integration along some path, e.g.

$$f(x,y) = \int_0^x f_x(s,0)\,ds + \int_0^y f_y(x,t)\,dt + C$$

(for robustness, should take integrals over many different paths and average the results)

Integrability: for the surface f to exist, the mixed second partial derivatives must be equal:

$$\frac{\partial}{\partial y}\left(g_1(x,y)/g_3(x,y)\right) = \frac{\partial}{\partial x}\left(g_2(x,y)/g_3(x,y)\right)$$

(in practice, they should at least be similar)

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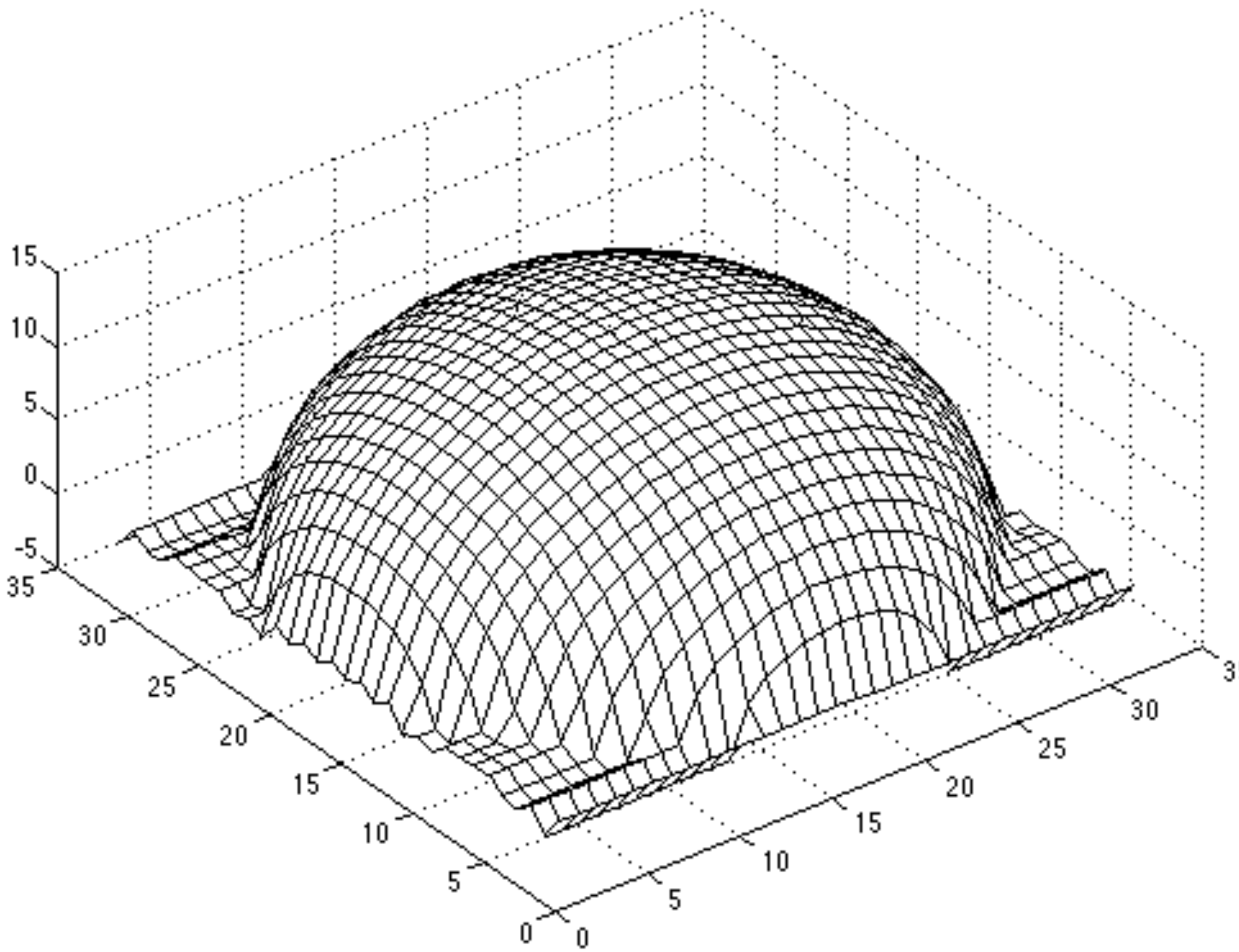
Surface recovered by integration



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Limitations

- Orthographic camera model
- Simplistic reflectance and lighting model
- No shadows
- No interreflections
- No missing data
- Integration is tricky

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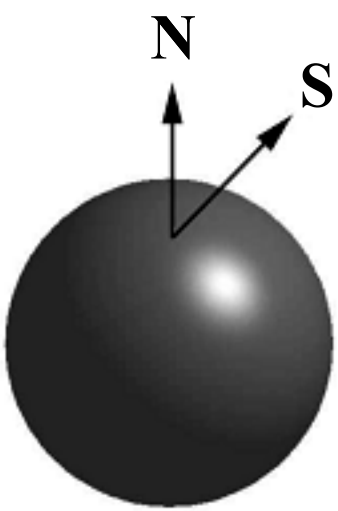
Finding the direction of the light source

$$I(x,y) = \mathbf{N}(x,y) \cdot \mathbf{S}(x,y)$$

Full 3D case:

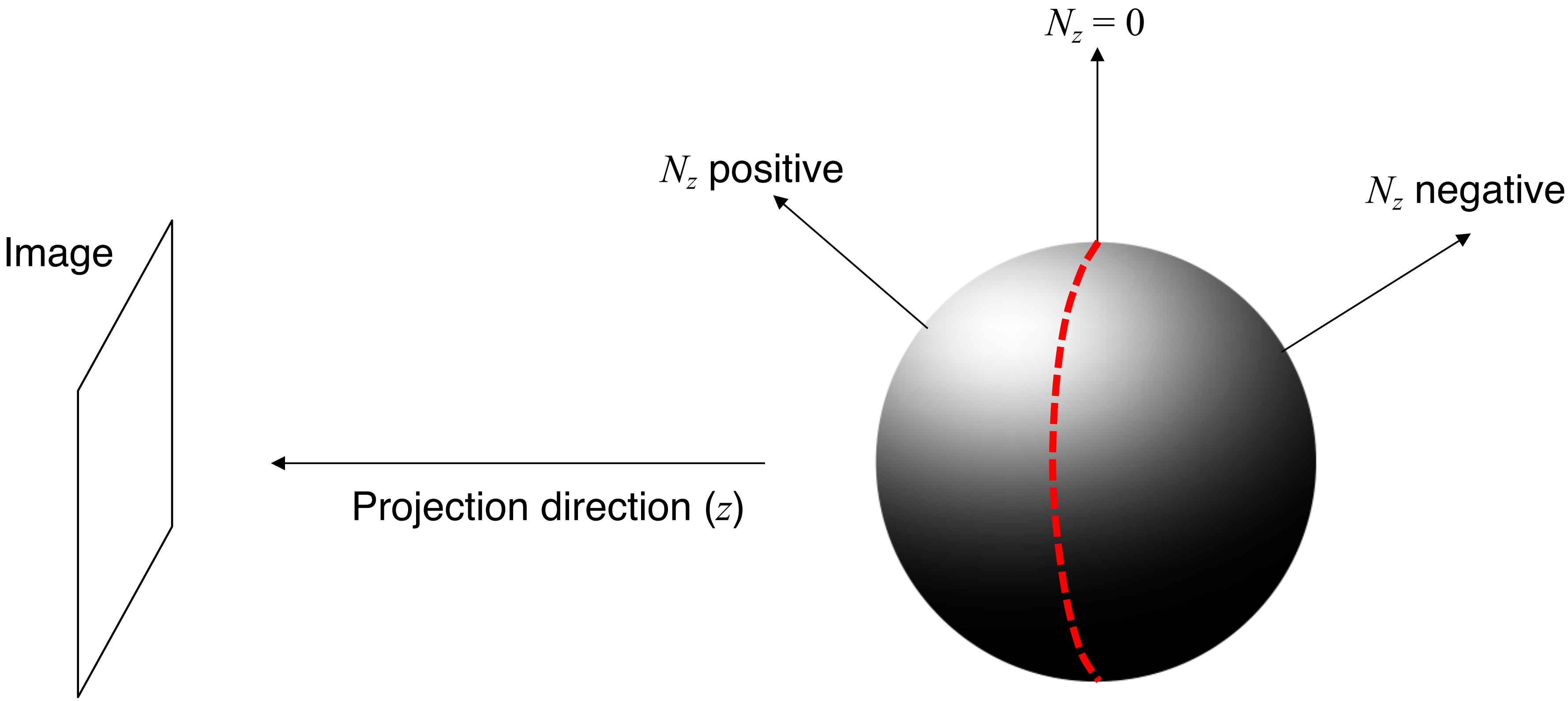


$$\begin{pmatrix} N_x(x_1,y_1) & N_y(x_1,y_1) & N_z(x_1,y_1) \\ N_x(x_2,y_2) & N_y(x_2,y_2) & N_z(x_2,y_2) \\ \vdots & \vdots & \vdots \\ N_x(x_n,y_n) & N_y(x_n,y_n) & N_z(x_n,y_n) \end{pmatrix} \begin{pmatrix} S_x \\ S_y \\ S_z \end{pmatrix} = \begin{pmatrix} I(x_1,y_1) \\ I(x_2,y_2) \\ \vdots \\ I(x_n,y_n) \end{pmatrix}$$

P. Nillius and J.-O. Eklundh, “Automatic estimation of the projected light source direction,” CVPR 2001

Finding the direction of the light source

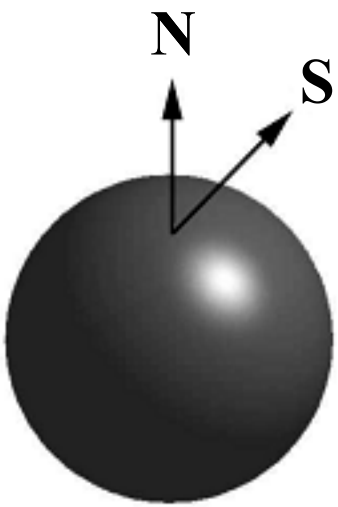
Consider points on the *occluding contour*:



P. Nillius and J.-O. Eklundh, “Automatic estimation of the projected light source direction,” CVPR 2001

Finding the direction of the light source

$$I(x,y) = \mathbf{N}(x,y) \cdot \mathbf{S}(x,y)$$



Full 3D case:

$$\begin{pmatrix} N_x(x_1,y_1) & N_y(x_1,y_1) & N_z(x_1,y_1) \\ N_x(x_2,y_2) & N_y(x_2,y_2) & N_z(x_2,y_2) \\ \vdots & \vdots & \vdots \\ N_x(x_n,y_n) & N_y(x_n,y_n) & N_z(x_n,y_n) \end{pmatrix} \begin{pmatrix} S_x \\ S_y \\ S_z \end{pmatrix} = \begin{pmatrix} I(x_1,y_1) \\ I(x_2,y_2) \\ \vdots \\ I(x_n,y_n) \end{pmatrix}$$

For points on the *occluding contour,* $N_z = 0$:

$$\begin{pmatrix} N_x(x_1,y_1) & N_y(x_1,y_1) \\ N_x(x_2,y_2) & N_y(x_2,y_2) \\ \vdots & \vdots \\ N_x(x_n,y_n) & N_y(x_n,y_n) \end{pmatrix} \begin{pmatrix} S_x \\ S_y \end{pmatrix} = \begin{pmatrix} I(x_1,y_1) \\ I(x_2,y_2) \\ \vdots \\ I(x_n,y_n) \end{pmatrix}$$

P. Nillius and J.-O. Eklundh, “Automatic estimation of the projected light source direction,” CVPR 2001

Finding the direction of the light source

P. Nillius and J.-O. Eklundh, “Automatic estimation of the projected light source direction,” CVPR 2001

Application: Detecting composite photos

Fake photo

Real photo

M. K. Johnson and H. Farid, [Exposing Digital Forgeries by Detecting Inconsistencies in Lighting](), ACM Multimedia and Security Workshop, 2005.