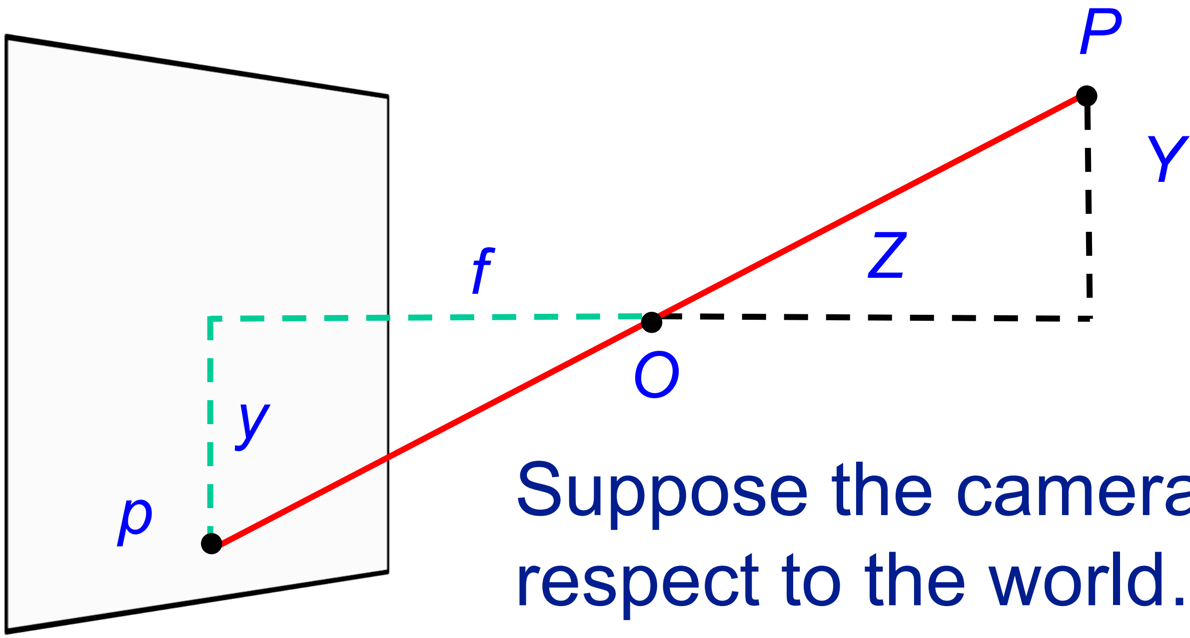
Dynamic Perspective

CS 543 / ECE 549 – Saurabh Gupta
Spring 2021, UIUC

http://saurabhg.web.illinois.edu/teaching/ece549/sp2021/

Many slides adapted from J. Malik.

Perspective Projection



$$(X, Y, Z) \rightarrow \left(\frac{fX}{Z}, \frac{fY}{Z}\right)$$

Suppose the camera moves with respect to the world…

- Point P (X, Y, Z) in the world moves relative to the camera, its projection in the image (x, y) moves as well.
- This movement in the image plane is called optical flow.
- Suppose the image of the point (x, y) moves to $(x + \Delta x, y + \Delta y)$ in time Δt, then $\left(\frac{\Delta x}{\Delta t}, \frac{\Delta y}{\Delta t}\right)$ are the two components of the optical flow.

Outline

- Relate optical flow to camera motion
- Special cases

How does a point X in the scene move?

- Assume that the camera moves with a translational velocity $t = (t_x, t_y, t_z)$ and angular velocity $\omega = (\omega_x, \omega_y, \omega_z)$.
- Linear velocity of point $P = (X, Y, Z)$ is given by $\dot{P} = -t \ - \omega \times P$.

$$\begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{bmatrix} = -\begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix} - \begin{bmatrix} \omega_y Z - \omega_z Y \\ \omega_z X - \omega_x Z \\ \omega_x Y - \omega_y X \end{bmatrix}$$

Now, lets consider the effect of projection

$$\begin{bmatrix}\dot{X}\\ \dot{Y}\\ \dot{Z}\end{bmatrix} = -\begin{bmatrix}t_x\\ t_y\\ t_z\end{bmatrix} - \begin{bmatrix}\omega_y Z - \omega_z Y\\ \omega_z X - \omega_x Z\\ \omega_x Y - \omega_y X\end{bmatrix}$$

- Assume, $f = 1$, $x = \frac{X}{Z}, y = \frac{Y}{Z}$.
- $\dot{x} = \frac{\dot{X}Z - \dot{Z}X}{Z^2}, \dot{y} = \frac{\dot{Y}Z - \dot{Z}Y}{Z^2}$
- Substitute $\dot{X}, \dot{Y}, \dot{Z}$, from equation above:

$$\begin{bmatrix}u\\ v\end{bmatrix} = \begin{bmatrix}\dot{x}\\ \dot{y}\end{bmatrix} = \frac{1}{Z}\begin{bmatrix}-1 & 0 & x\\ 0 & -1 & y\end{bmatrix}\begin{bmatrix}t_x\\ t_y\\ t_z\end{bmatrix} + \begin{bmatrix}xy & -(1+x^2) & y\\ (1+y^2) & -xy & -x\end{bmatrix}\begin{bmatrix}\omega_x\\ \omega_y\\ \omega_z\end{bmatrix}$$

Dynamic Perspective Equations

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \frac{1}{Z}\begin{bmatrix} -1 & 0 & x \\ 0 & -1 & y \end{bmatrix}\begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix} + \begin{bmatrix} xy & -(1+x^2) & y \\ (1+y^2) & -xy & -x \end{bmatrix}\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$$

Translation Component

Rotation Component

Optical flow for pure rotation

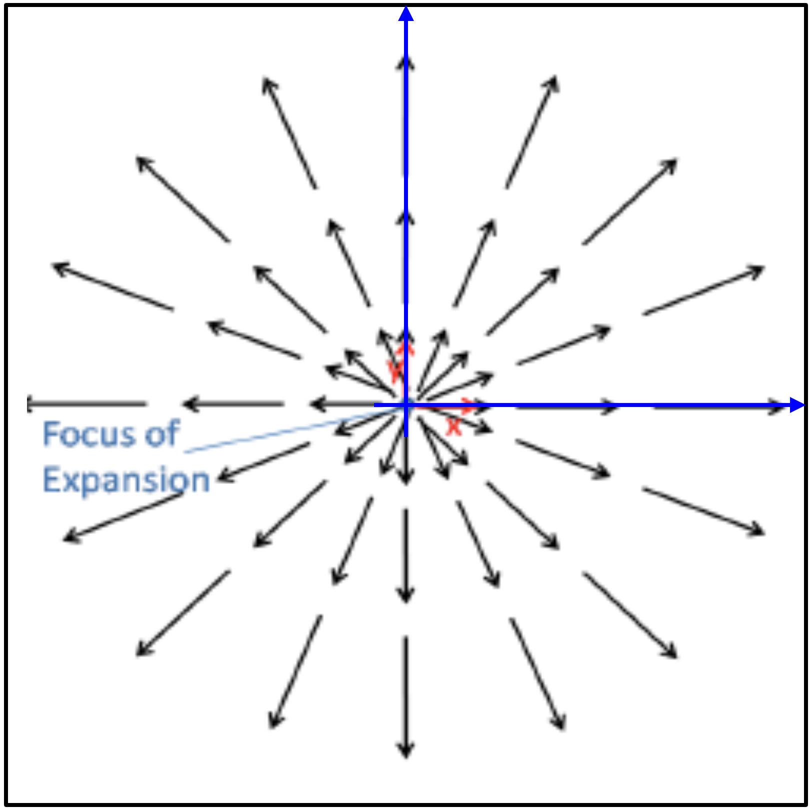
$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \frac{1}{Z}\begin{bmatrix} -1 & 0 & x \\ 0 & -1 & y \end{bmatrix}\begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix} + \begin{bmatrix} xy & -(1+x^2) & y \\ (1+y^2) & -xy & -x \end{bmatrix}\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$$

- $\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} xy & -(1+x^2) & y \\ (1+y^2) & -xy & -x \end{bmatrix}\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$
- We can determine ω from the flow field.
- Flow field is independent of $Z(x, y)$.

Optical flow for pure translation along Z-axis

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \frac{1}{Z}\begin{bmatrix} -1 & 0 & x \\ 0 & -1 & y \end{bmatrix}\begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix} + \begin{bmatrix} xy & -(1+x^2) & y \\ (1+y^2) & -xy & -x \end{bmatrix}\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$$

- $\begin{bmatrix} u \\ v \end{bmatrix} = \frac{t_z}{Z(x,y)}\begin{bmatrix} x \\ y \end{bmatrix}$
- Optical flow vector is a scalar multiple of position vector.
- Scale factor ambiguity, if $t_z \rightarrow kt_z$, and $Z \rightarrow kZ$, optical flow remains unchanged.
- But, you can get time to collision, Z/t_z.




STOP


STOP

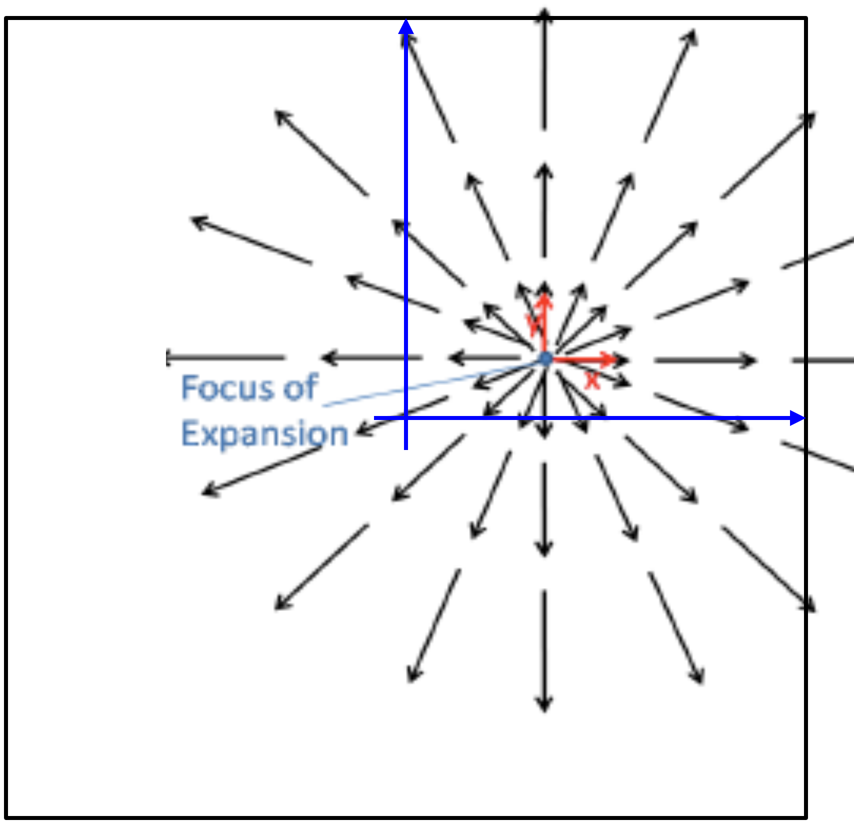

STOP


STOP

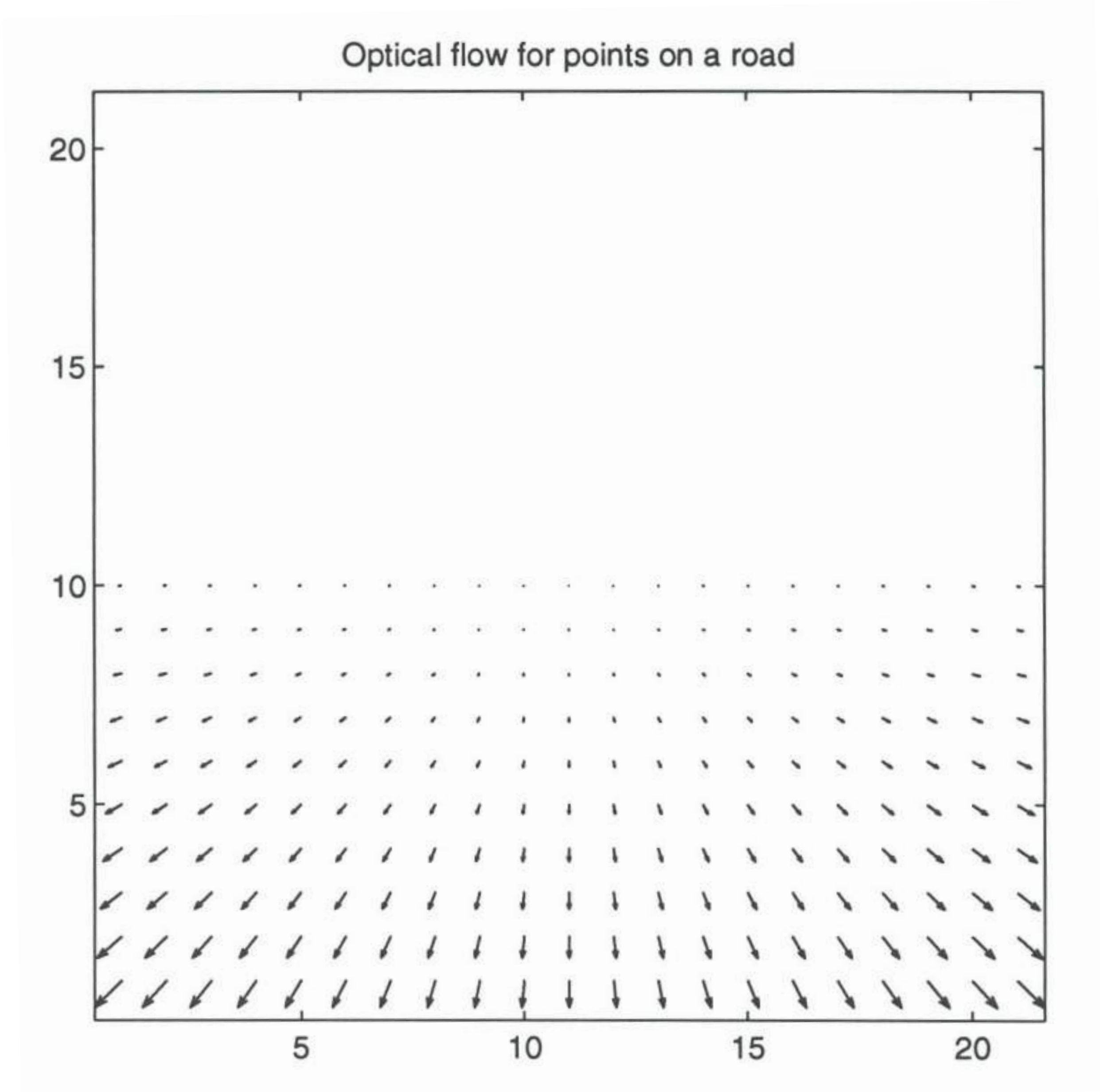
Optical flow for pure translation along Z-axis

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \frac{1}{Z}\begin{bmatrix} -1 & 0 & x \\ 0 & -1 & y \end{bmatrix}\begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix} + \begin{bmatrix} xy & -(1+x^2) & y \\ (1+y^2) & -xy & -x \end{bmatrix}\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$$

- $u = \frac{-t_x + x t_z}{Z(x,y)}$, $\mathrm{v} = \frac{-t_y + y t_z}{Z(x,y)}$

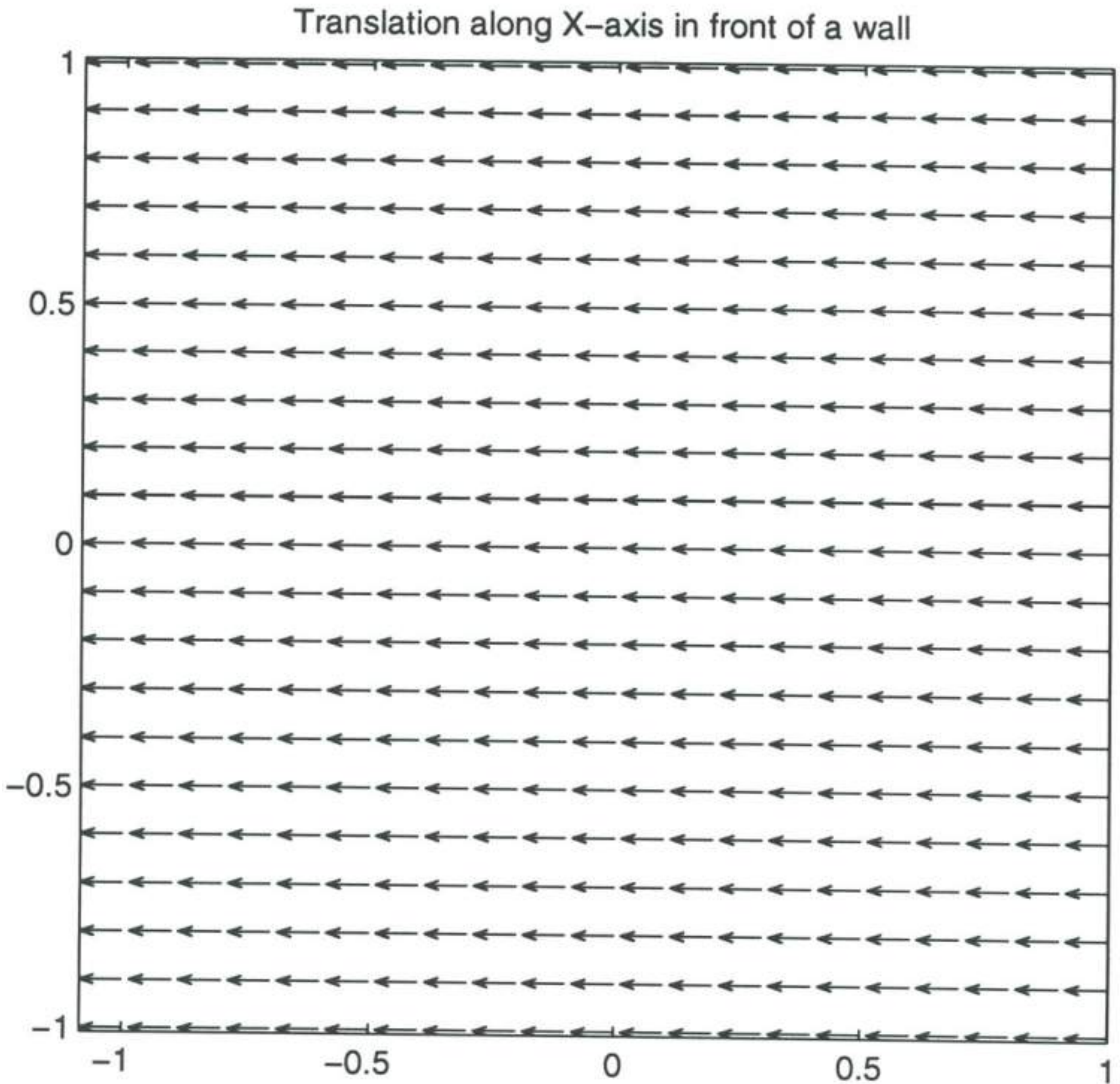


Optical flow for points on a road



Slide by J. Malik

Translating along X-axis in front of a wall



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Recap

- Relate optical flow to camera motion

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \frac{1}{Z}\begin{bmatrix} -1 & 0 & x \\ 0 & -1 & y \end{bmatrix}\begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix} + \begin{bmatrix} xy & -(1+x^2) & y \\ (1+y^2) & -xy & -x \end{bmatrix}\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$$

- Special cases
  - Pure rotation / pure translation / time to collision