Optical flow

Many slides adapted from S. Seitz, R. Szeliski, M. Pollefeys

Slides from S. Lazebnik.
Motion is a powerful perceptual cue

- Sometimes, it is the only cue
Motion is a powerful perceptual cue

- Even “impoverished” motion data can evoke a strong percept

Motion is a powerful perceptual cue

- Even “impoverished” motion data can evoke a strong percept


Source: L. Lazebnik
Uses of motion in computer vision

- 3D shape reconstruction
- Object segmentation
- Learning and tracking of dynamical models
- Event and activity recognition
- Self-supervised and predictive learning

Source: L. Lazebnik
Motion field

- The motion field is the projection of the 3D scene motion into the image

Source: L. Lazebnik
Optical flow

• **Definition**: optical flow is the *apparent* motion of brightness patterns in the image
• Ideally, optical flow would be the same as the motion field
• Have to be careful: apparent motion can be caused by lighting changes without any actual motion
  • Think of a uniform rotating sphere under fixed lighting vs. a stationary sphere under moving illumination

Source: L. Lazebnik
Estimating optical flow

Given two subsequent frames, estimate the apparent motion field $u(x,y)$ and $v(x,y)$ between them.

Key assumptions

- **Brightness constancy**: projection of the same point looks the same in every frame
- **Small motion**: points do not move very far
- **Spatial coherence**: points move like their neighbors

Source: L. Lazebnik
The brightness constancy constraint

Brightness Constancy Equation:

\[ I(x, y, t - 1) = I(x + u(x, y), y + v(x, y), t) \]

Linearizing the right side using Taylor expansion:

\[ I(x, y, t - 1) \approx I(x, y, t) + I_x u(x, y) + I_y v(x, y) \]

Hence,

\[ I_x u + I_y v + I_t \approx 0 \]

Source: L. Lazebnik
The brightness constancy constraint

\[ I_x u + I_y v + I_t = 0 \]

• How many equations and unknowns per pixel?
  • One equation, two unknowns

• What does this constraint mean?
  \[ \nabla I \cdot (u, v) + I_t = 0 \]

• The component of the flow perpendicular to the gradient (i.e., parallel to the edge) is unknown!

Source: L. Lazebnik
The brightness constancy constraint

\[ I_x u + I_y v + I_t = 0 \]

• How many equations and unknowns per pixel?
  • One equation, two unknowns

• What does this constraint mean?

\[ \nabla I \cdot (u, v) + I_t = 0 \]

• The component of the flow perpendicular to the gradient (i.e., parallel to the edge) is unknown!

If \((u, v)\) satisfies the equation, so does \((u+u', v+v')\) if \(\nabla I \cdot (u', v') = 0\)

Source: L. Lazebnik
The aperture problem

Perceived motion

Source: L. Lazebnik
The aperture problem

Source: L. Lazebnik
The barber pole illusion

http://en.wikipedia.org/wiki/Barberpole_illusion

Source: L. Lazebnik
The barber pole illusion

Source: L. Lazebnik

http://en.wikipedia.org/wiki/Barberpole_illusion
Solving the aperture problem

• How to get more equations for a pixel?
• **Spatial coherence constraint**: assume the pixel’s neighbors have the same \((u, v)\)
  • E.g., if we use a 5x5 window, that gives us 25 equations per pixel

\[
\nabla I(x_i) \cdot [u, v] + I_t(x_i) = 0
\]

\[
\begin{bmatrix}
I_x(x_1) & I_y(x_1) \\
I_x(x_2) & I_y(x_2) \\
\vdots & \vdots \\
I_x(x_n) & I_y(x_n)
\end{bmatrix}
\begin{bmatrix}
u \\
v
\end{bmatrix}
= -
\begin{bmatrix}
I_t(x_1) \\
I_t(x_2) \\
\vdots \\
I_t(x_n)
\end{bmatrix}
\]

Lucas-Kanade flow

- Linear least squares problem:

\[
\begin{bmatrix}
I_x(x_1) & I_y(x_1) \\
I_x(x_2) & I_y(x_2) \\
\vdots & \vdots \\
I_x(x_n) & I_y(x_n)
\end{bmatrix}
\begin{bmatrix}
u \\
v \\
\vdots \\
v
\end{bmatrix} =
\begin{bmatrix}
I_t(x_1) \\
I_t(x_2) \\
\vdots \\
I_t(x_n)
\end{bmatrix}
\]

- When is this system solvable?

Lucas-Kanade flow

- Linear least squares problem:

\[
\begin{bmatrix}
I_x(x_1) & I_y(x_1) \\
I_x(x_2) & I_y(x_2) \\
\vdots & \vdots \\
I_x(x_n) & I_y(x_n)
\end{bmatrix}
\begin{bmatrix}
u \\
v
\end{bmatrix} = 
\begin{bmatrix}
I_t(x_1) \\
I_t(x_2) \\
\vdots \\
I_t(x_n)
\end{bmatrix}
\]

\[
A \ d = b
\]

\[
\begin{array}{ccc}
t \\
2 \\
2 \times \end{array} 
\begin{array}{ccc}
t \\
1 \\
1 \times \end{array}
\]

- Solution given by \((A^T A)d = A^T b\)

\[
\begin{bmatrix}
\sum I_x I_x & \sum I_x I_y \\
\sum I_x I_y & \sum I_y I_y
\end{bmatrix}
\begin{bmatrix}
u \\
v
\end{bmatrix} = 
\begin{bmatrix}
\sum I_x I_t \\
\sum I_y I_t
\end{bmatrix}
\]

\[
M = A^T A \text{ is the second moment matrix!}
\]

(summations are over all pixels in the window)

Recall: second moment matrix

- Estimation of optical flow is well-conditioned precisely for regions with high “cornerness”:

Source: L. Lazebnik
Conditions for solvability

- “Bad” case: single straight edge

Source: L. Lazebnik
Conditions for solvability

• “Good” case

Source: L. Lazebnik
Lucas-Kanade flow example

Input frames

Output

Source: L. Lazebnik

Source: MATLAB Central File Exchange
Errors in Lucas-Kanade

- The motion is large (larger than a pixel)
- A point does not move like its neighbors
- Brightness constancy does not hold

Source: L. Lazebnik
“Flower garden” example

Source: L. Lazebnik

* From Khurram Hassan-Shafique CAP5415 Computer Vision 2003
“Flower garden” example

Lucas-Kanade fails in areas of large motion

Source: L. Lazebnik

* From Khurram Hassan-Shafique CAP5415 Computer Vision 2003
Multi-resolution estimation

Source: L. Lazebnik

* From Khurram Hassan-Shafique CAP5415 Computer Vision 2003
Multi-resolution estimation

With multi-resolution estimation

Source: L. Lazebnik

* From Khurram Hassan-Shafique CAP5415 Computer Vision 2003
Fixing the errors in Lucas-Kanade

- The motion is large (larger than a pixel)
  - Multi-resolution estimation, iterative refinement
  - Feature matching
- A point does not move like its neighbors
  - Motion segmentation

Figure 11: (a) The optic flow from multi-scale gradient method. (b) Segmentation obtained by clustering optic flow into affine motion regions. (c) Segmentation from consistency checking by image warping. Representing moving images with layers.

Fixing the errors in Lucas-Kanade

- The motion is large (larger than a pixel)
  - Multi-resolution estimation, iterative refinement
  - Feature matching
- A point does not move like its neighbors
  - Motion segmentation
- Brightness constancy does not hold
  - Feature matching

Source: L. Lazebnik