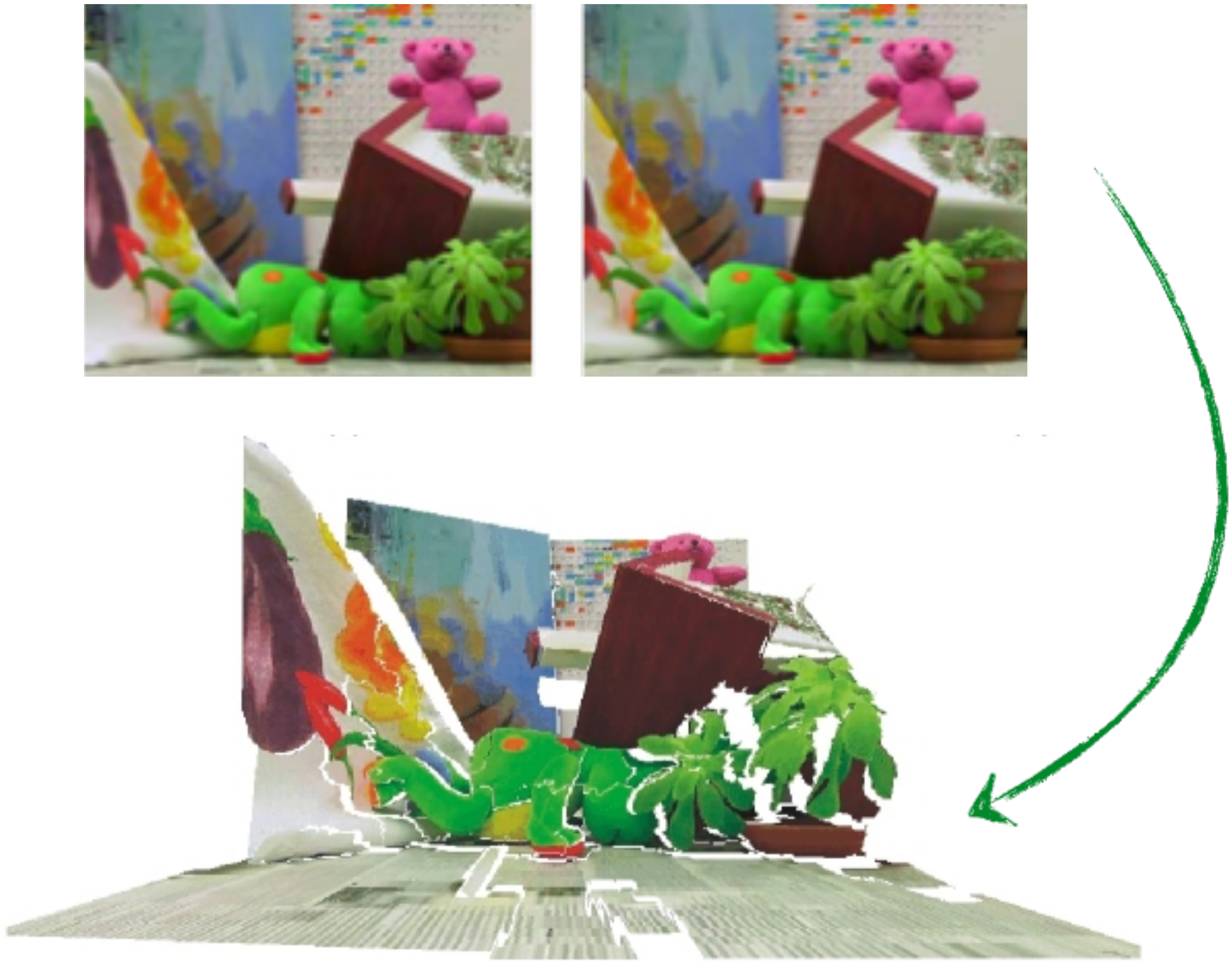
Two-View Stereo



Slides from S. Lazebnik, S. Seitz, Y. Furukawa

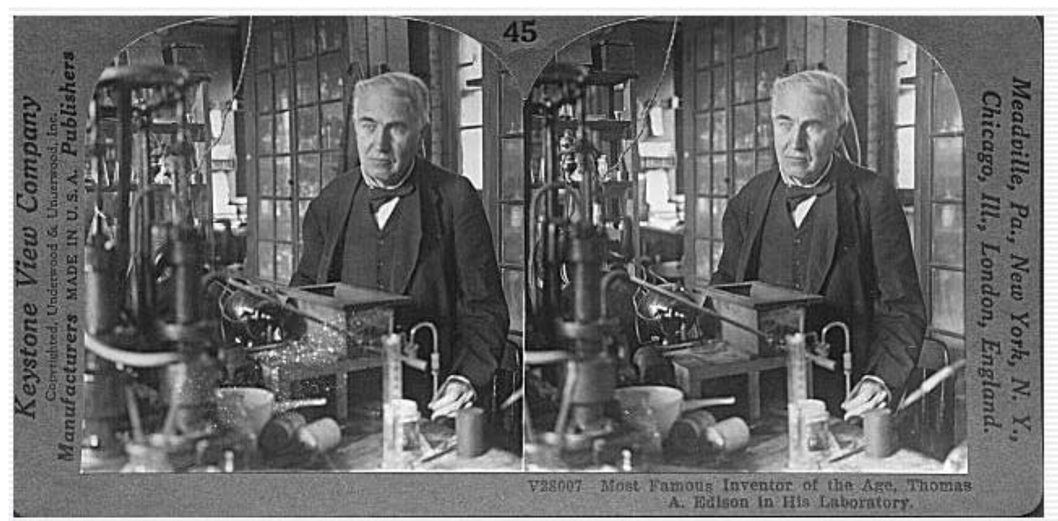
Stereo

- What cues tell us about scene depth?

Slide from L. Lazebnik.

Stereograms

- Humans can fuse pairs of images to get a sensation of depth



Stereograms: Invented by Sir Charles Wheatstone, 1838

Slide from L. Lazebnik.

Stereograms

Slide from L. Lazebnik.

Stereograms

- Humans can fuse pairs of images to get a sensation of depth

Autostereograms: www.magiceye.com

Slide from L. Lazebnik.

Stereograms

- Humans can fuse pairs of images to get a sensation of depth

Autostereograms: www.magiceye.com

Slide from L. Lazebnik.

Problem formulation

- Given a calibrated binocular stereo pair, fuse it to produce a depth image

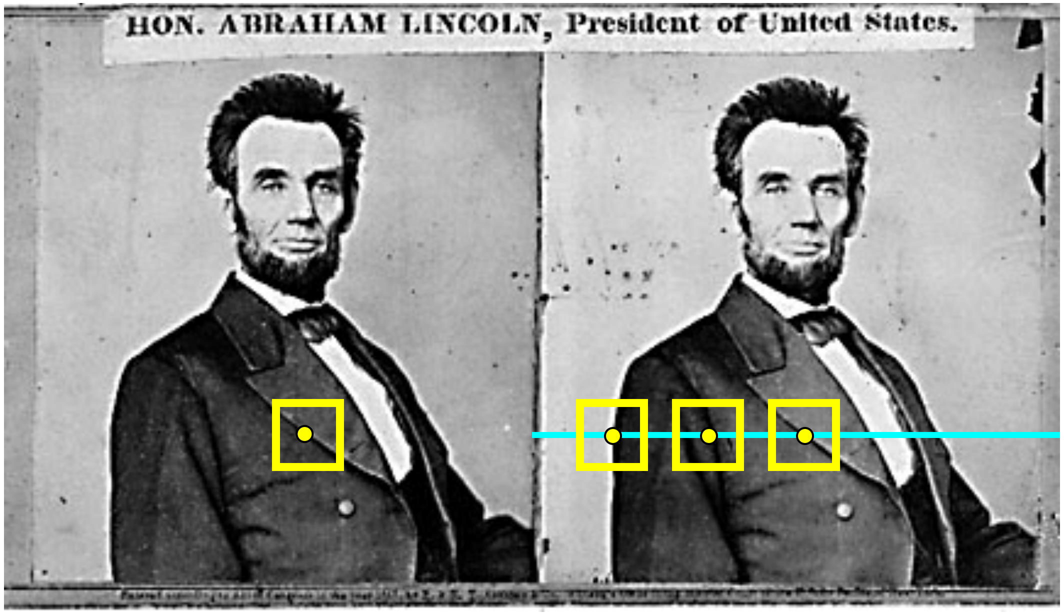
image 1

image 2

Dense depth map

Slide from L. Lazebnik.

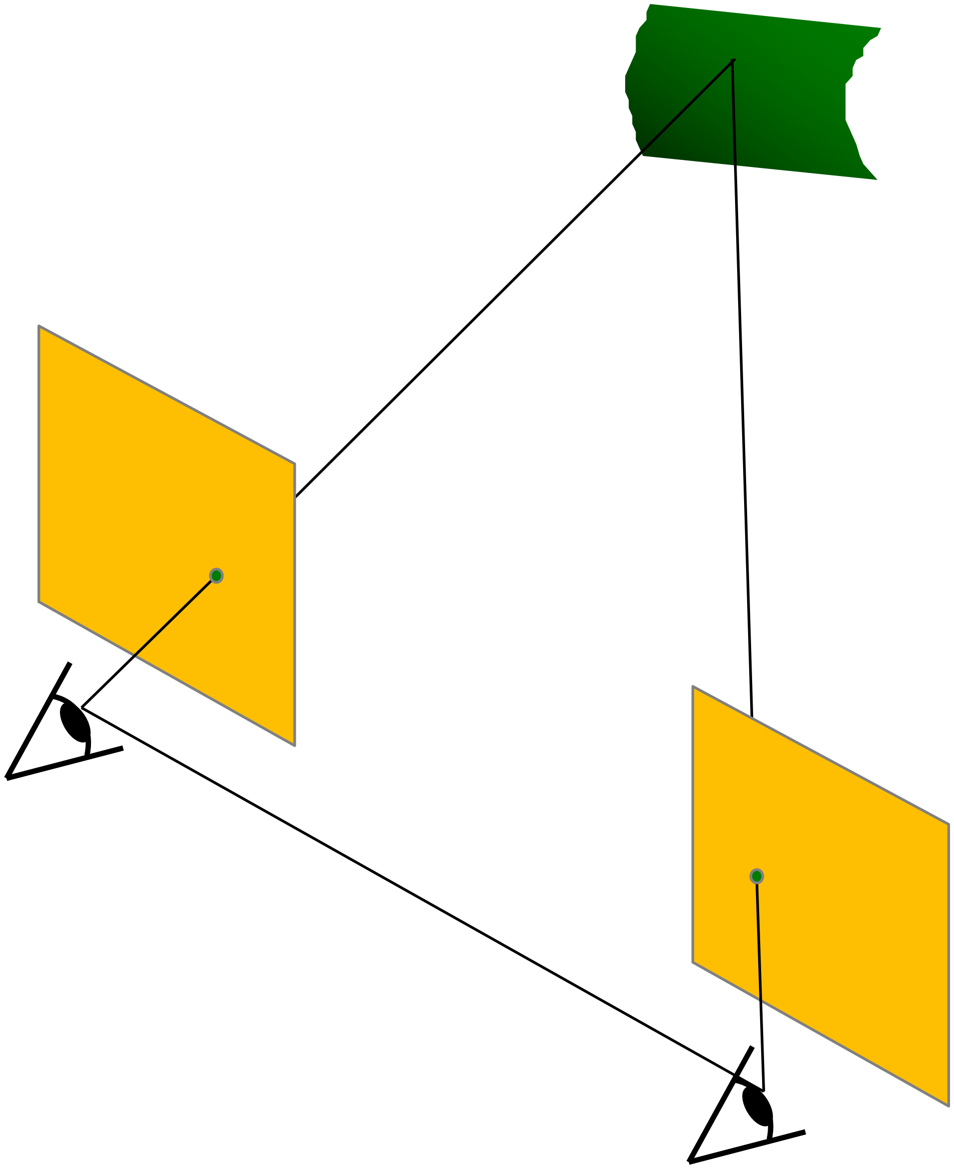
Basic stereo matching algorithm



- For each pixel in the first image
  - Find corresponding epipolar line in the right image
  - Examine all pixels on the epipolar line and pick the best match
  - Triangulate the matches to get depth information
- Simplest case: epipolar lines are corresponding scanlines
  - When does this happen?

Slide from L. Lazebnik.

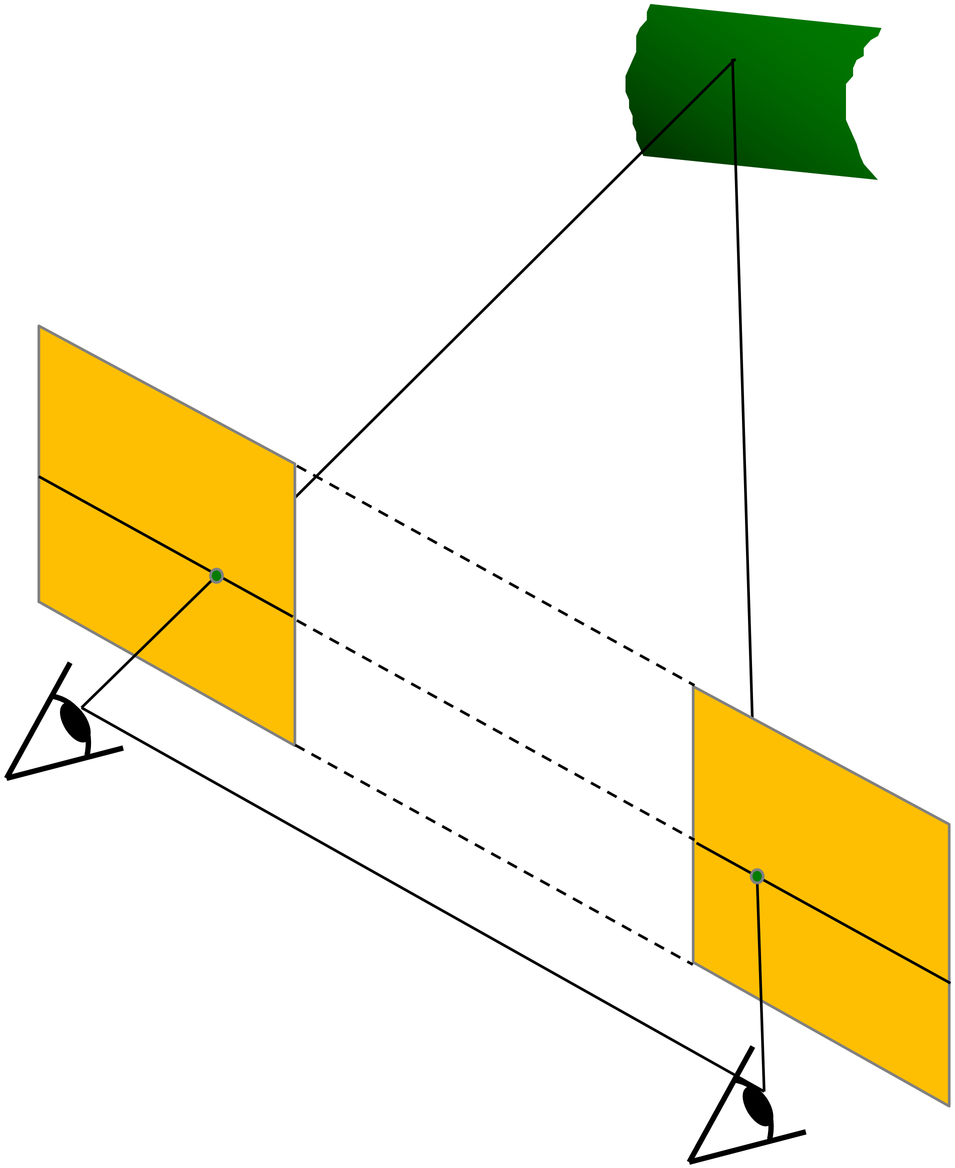
Simplest Case: Parallel images



- Image planes of cameras are parallel to each other and to the baseline
- Camera centers are at same height
- Focal lengths are the same

Slide from L. Lazebnik.

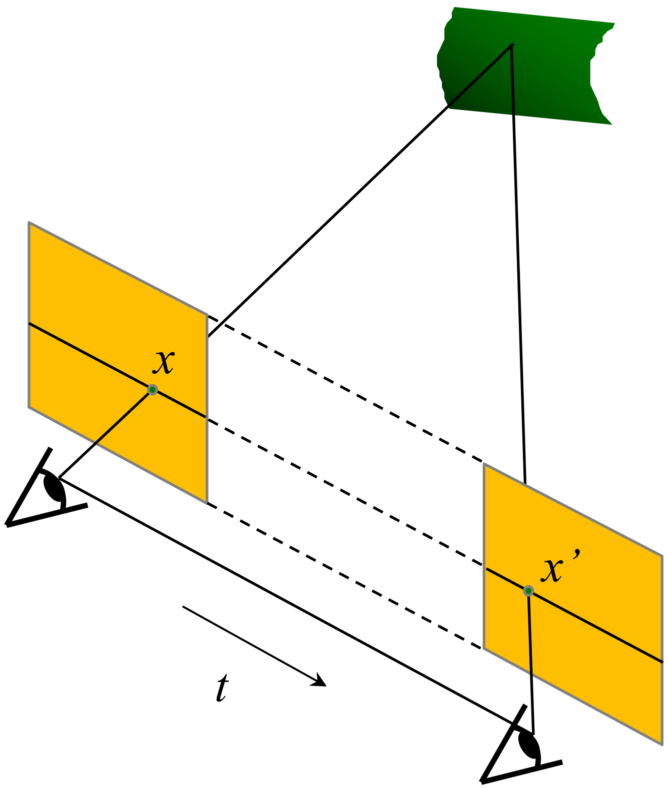
Simplest Case: Parallel images



- Image planes of cameras are parallel to each other and to the baseline
- Camera centers are at same height
- Focal lengths are the same
- Then epipolar lines fall along the horizontal scan lines of the images

Slide from L. Lazebnik.

Essential matrix for parallel images



Epipolar constraint:

$$\boldsymbol{x}'^T \boldsymbol{E}\,\boldsymbol{x} = 0, \quad \boldsymbol{E} = [\boldsymbol{t}_\times]\boldsymbol{R}$$

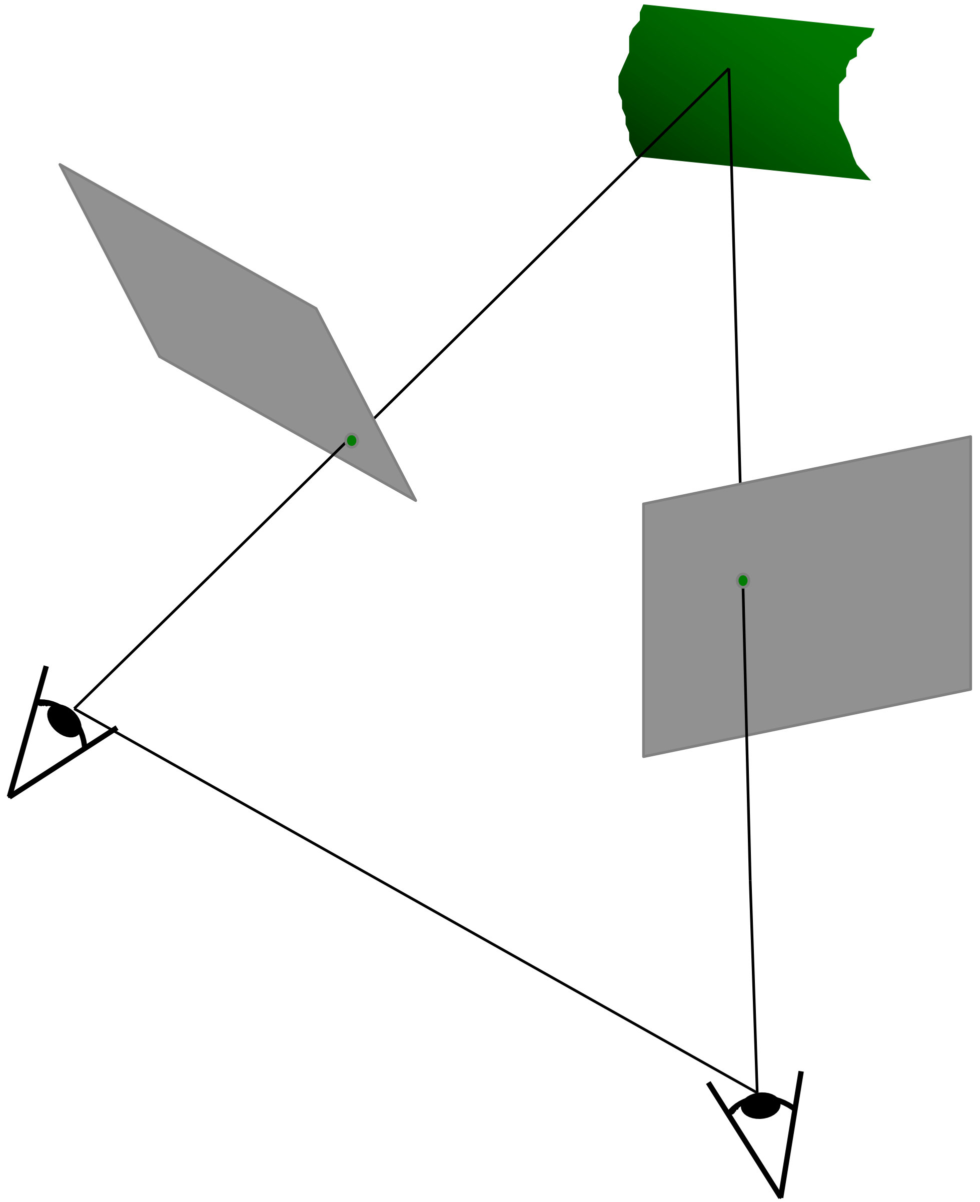
$$\boldsymbol{R} = \boldsymbol{I} \qquad \boldsymbol{t} = (T, 0, 0)$$

$$\boldsymbol{E} = [\boldsymbol{t}_\times]\boldsymbol{R} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -T \\ 0 & T & 0 \end{bmatrix}$$

$$\begin{pmatrix} u' & v' & 1 \end{pmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -T \\ 0 & T & 0 \end{bmatrix} \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = 0 \qquad \begin{pmatrix} u' & v' & 1 \end{pmatrix} \begin{pmatrix} 0 \\ -T \\ Tv \end{pmatrix} = 0 \qquad Tv' = Tv$$

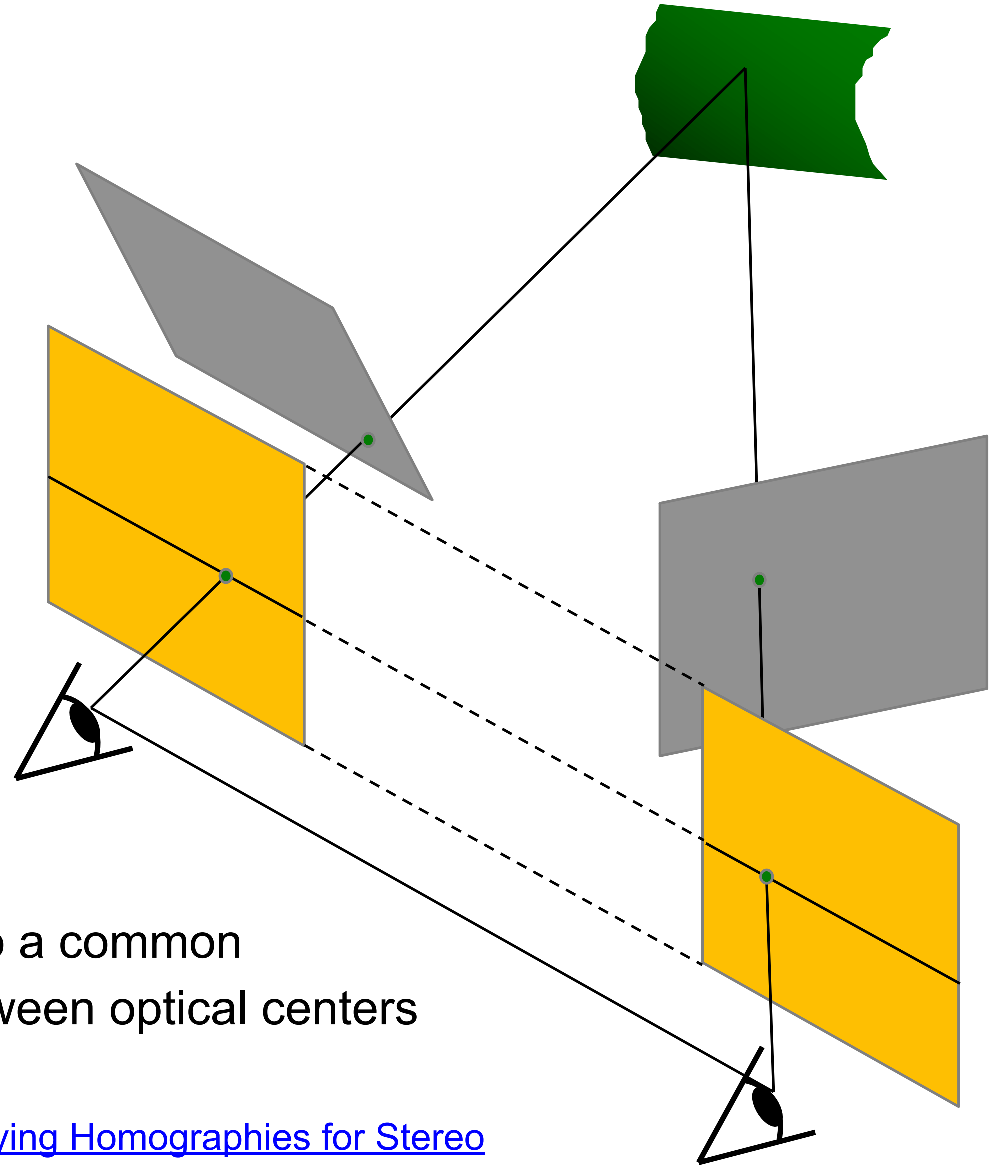
The y-coordinates of corresponding points are the same!

Stereo image rectification



Slide from L. Lazebnik.

Stereo image rectification

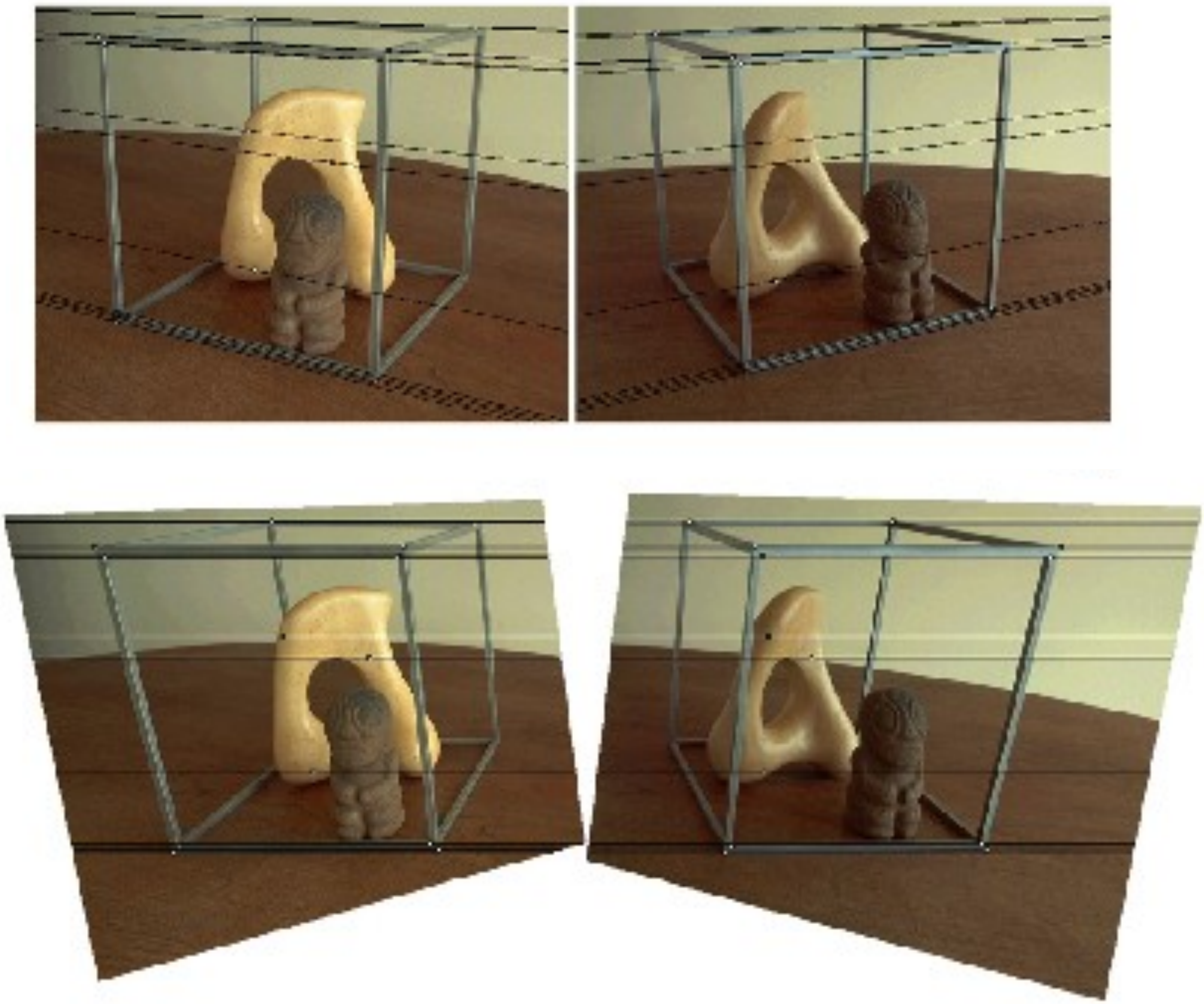


- Reproject image planes onto a common plane parallel to the line between optical centers

C. Loop and Z. Zhang. [Computing Rectifying Homographies for Stereo Vision](). CVPR 1999

Slide from L. Lazebnik.

Rectification example

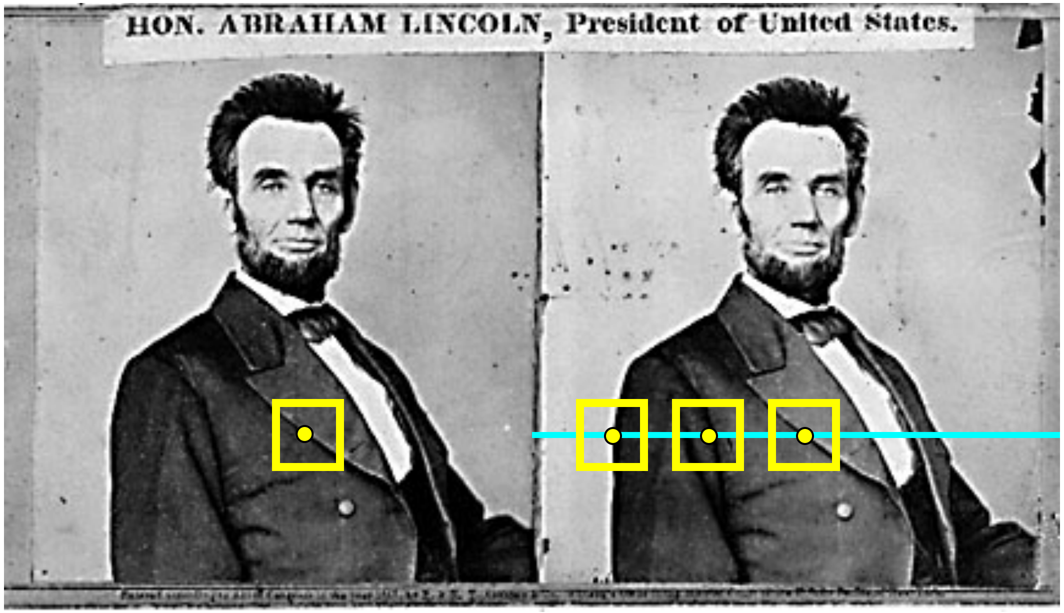


Slide from L. Lazebnik.

Another rectification example

Slide from L. Lazebnik.

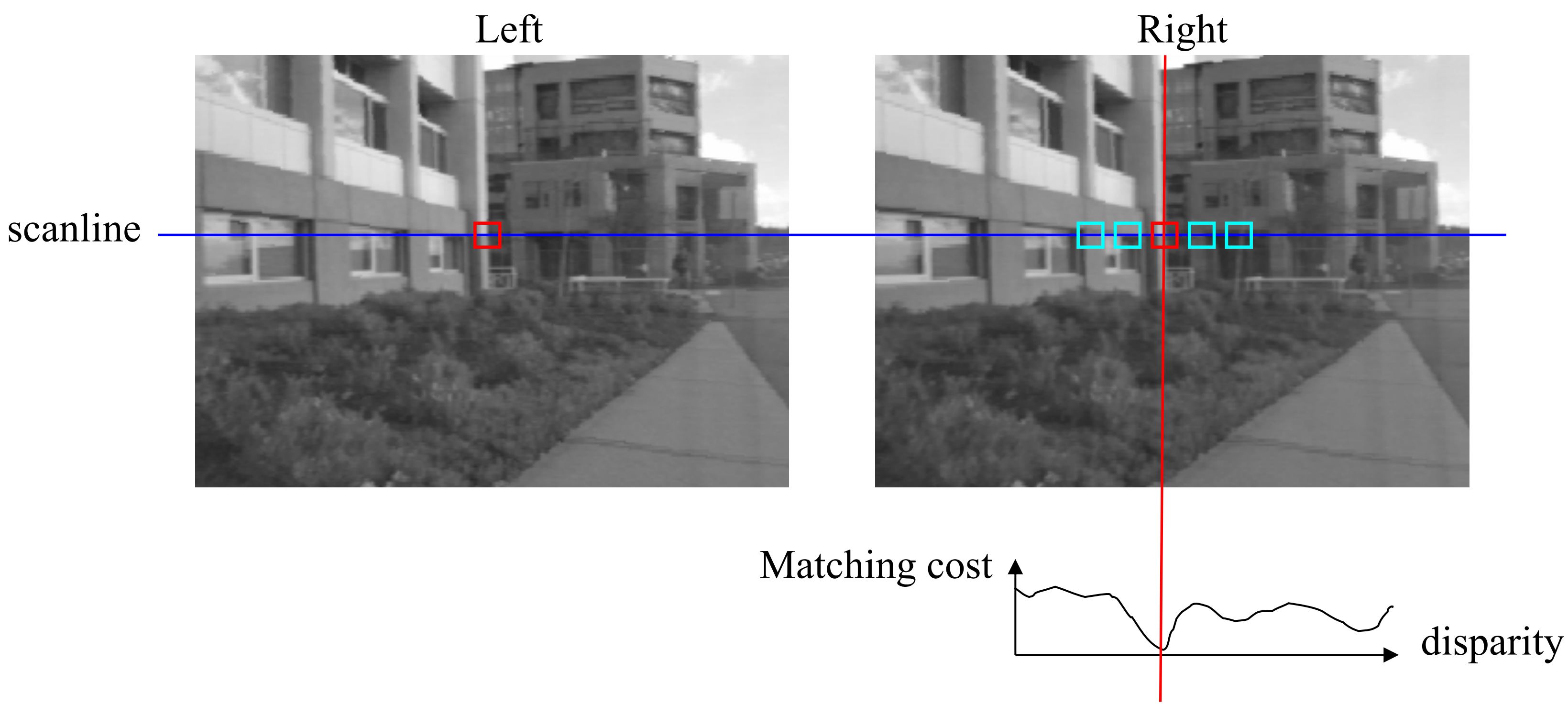
Basic stereo matching algorithm



- If necessary, rectify the two stereo images to transform epipolar lines into scanlines
- For each pixel in the first image
  - Find corresponding epipolar line in the right image
  - Examine all pixels on the epipolar line and pick the best match

Slide from L. Lazebnik.

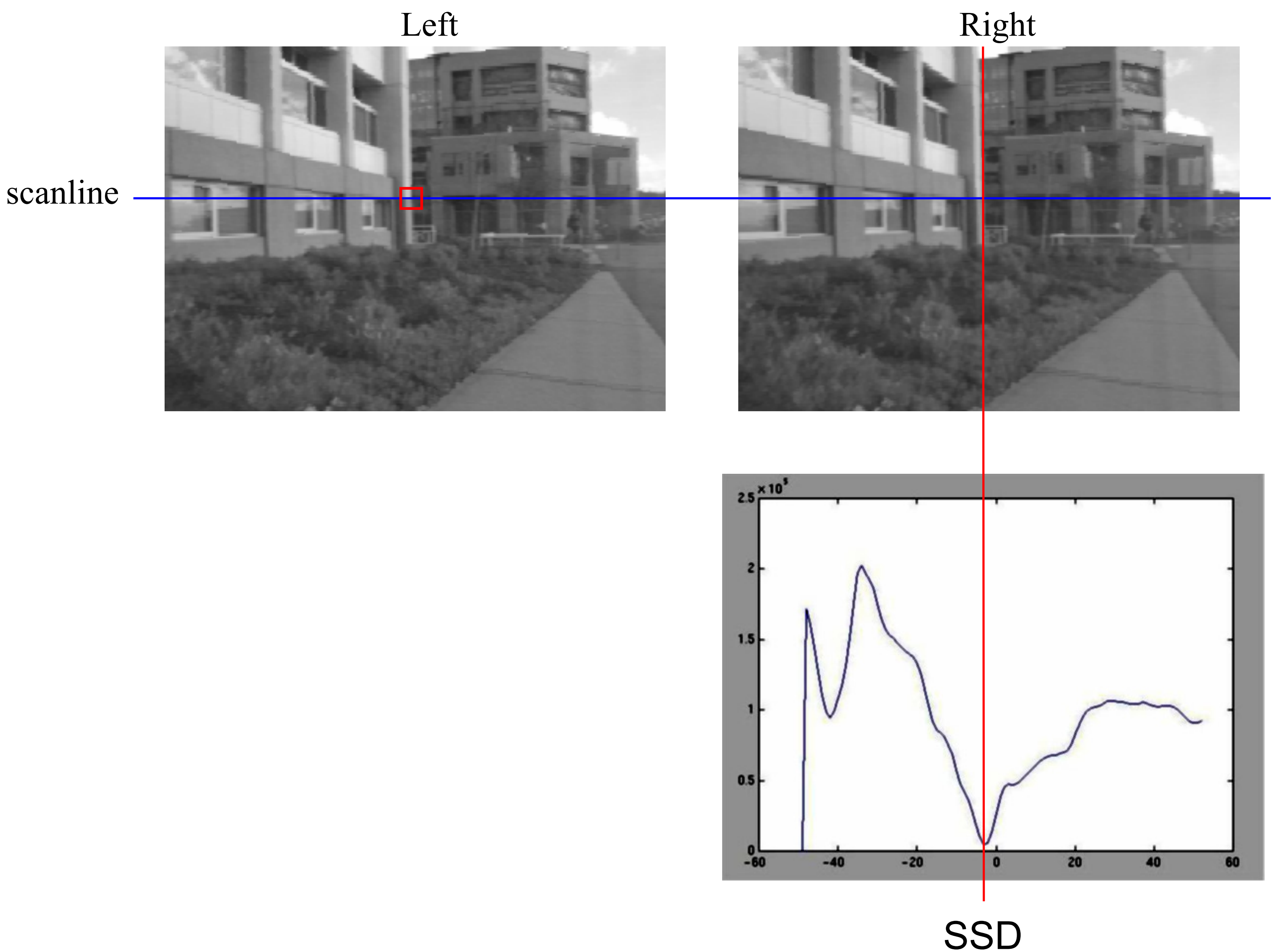
Correspondence search



- Slide a window along the right scanline and compare contents of that window with the reference window in the left image
- Matching cost: SSD or normalized correlation

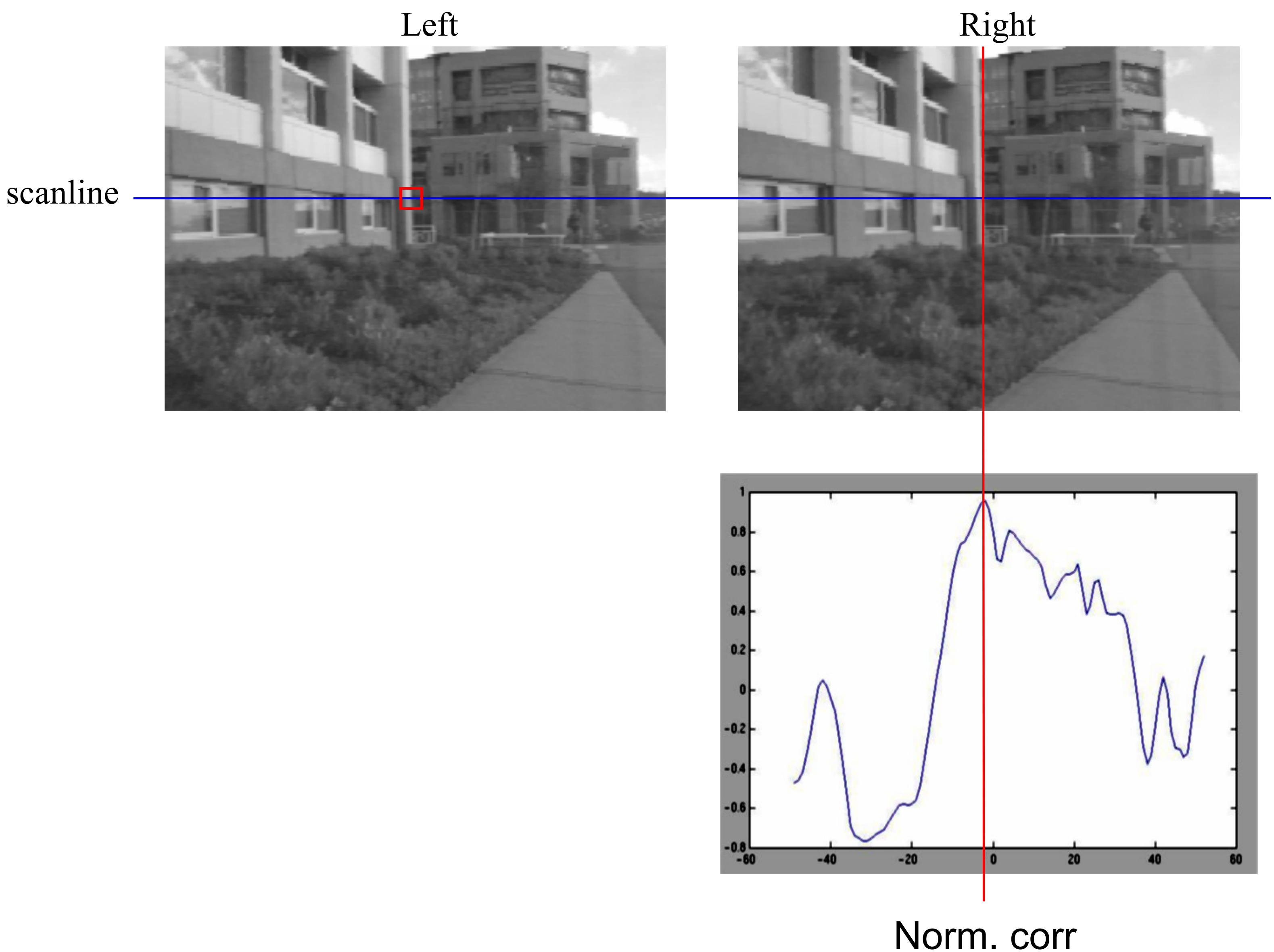
Slide from L. Lazebnik.

Correspondence search



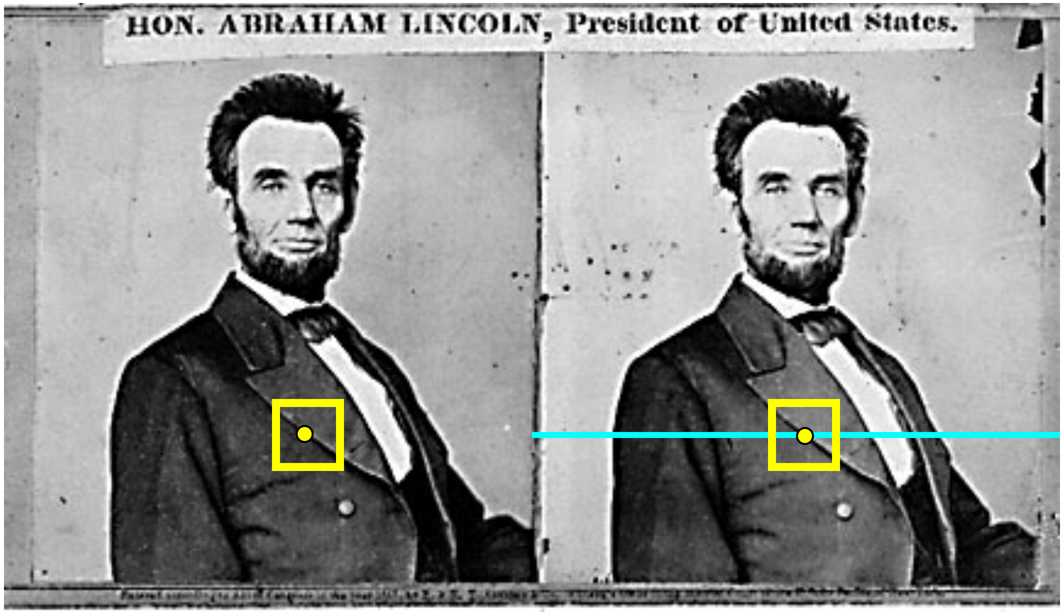
Slide from L. Lazebnik.

Correspondence search



Slide from L. Lazebnik.

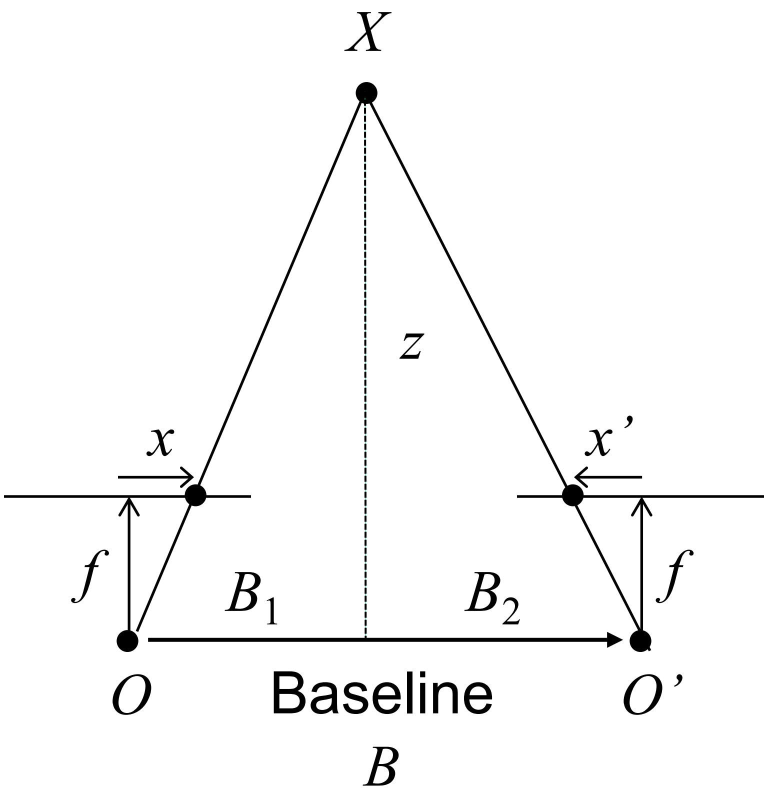
Basic stereo matching algorithm



- If necessary, rectify the two stereo images to transform epipolar lines into scanlines
- For each pixel *x* in the first image
  - Find corresponding epipolar scanline in the right image
  - Examine all pixels on the scanline and pick the best match *x'*
  - Triangulate the matches to get depth information

Slide from L. Lazebnik.

Depth from disparity



$$\frac{x}{f} = \frac{B_1}{z} \qquad \frac{-x'}{f} = \frac{B_2}{z}$$

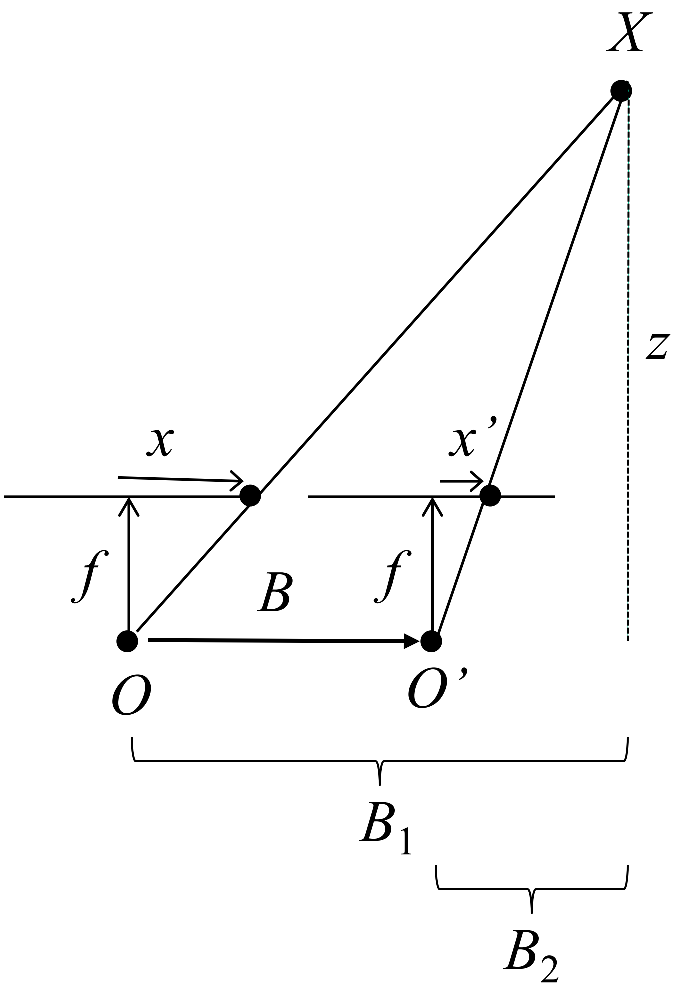
$$\frac{x - x'}{f} = \frac{B_1 + B_2}{z}$$

$$disparity = x - x' = \frac{B \cdot f}{z}$$

Disparity is inversely proportional to depth!

Slide from L. Lazebnik.

Depth from disparity



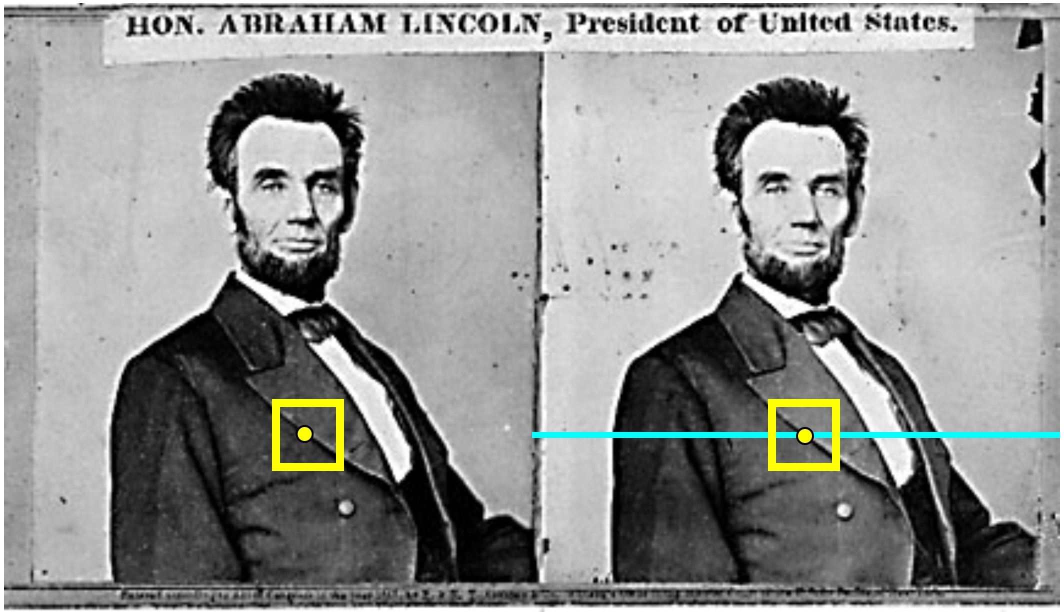
$$\frac{x}{f} = \frac{B_1}{z} \qquad \frac{x'}{f} = \frac{B_2}{z}$$

$$\frac{x - x'}{f} = \frac{B_1 - B_2}{z}$$

$$disparity = x - x' = \frac{B \cdot f}{z}$$

Slide from L. Lazebnik.

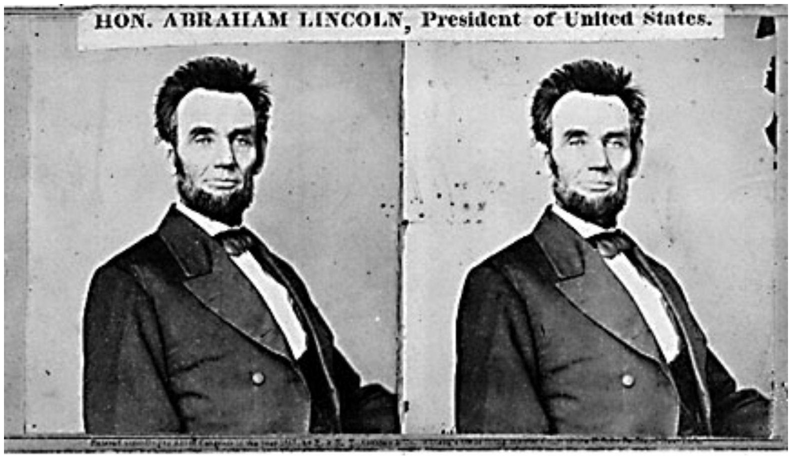
Basic stereo matching algorithm



- If necessary, rectify the two stereo images to transform epipolar lines into scanlines
- For each pixel x in the first image
  - Find corresponding epipolar scanline in the right image
  - Examine all pixels on the scanline and pick the best match x'
  - Compute disparity $x-x'$ and set $\text{depth}(x) = B*f/(x-x')$

Slide from L. Lazebnik.

Failures of correspondence search



Textureless surfaces

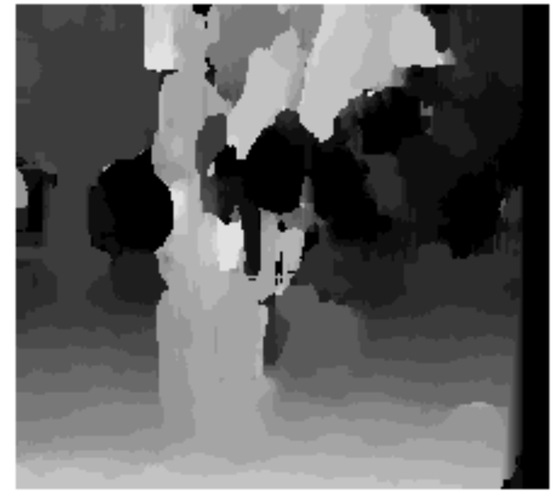
Occlusions, repetition

Non-Lambertian surfaces, specularities

Slide from L. Lazebnik.

Effect of window size

W = 3



W = 20

- Smaller window
  - \+ More detail
  - – More noise

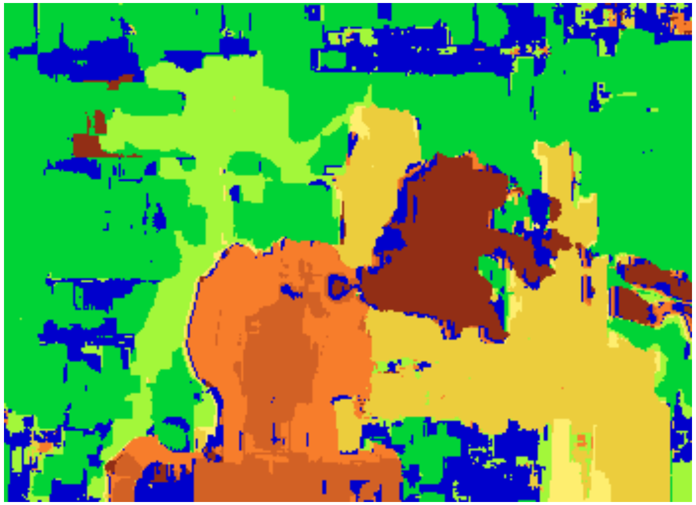
- Larger window
  - \+ Smoother disparity maps
  - – Less detail

Slide from L. Lazebnik.

Results with window search

Data

Window-based matching



Ground truth

Slide from L. Lazebnik.

Better methods exist...

Graph cuts

Ground truth

Y. Boykov, O. Veksler, and R. Zabih, Fast Approximate Energy Minimization via Graph Cuts, PAMI 2001

For the latest and greatest: http://www.middlebury.edu/stereo/

Slide from L. Lazebnik.

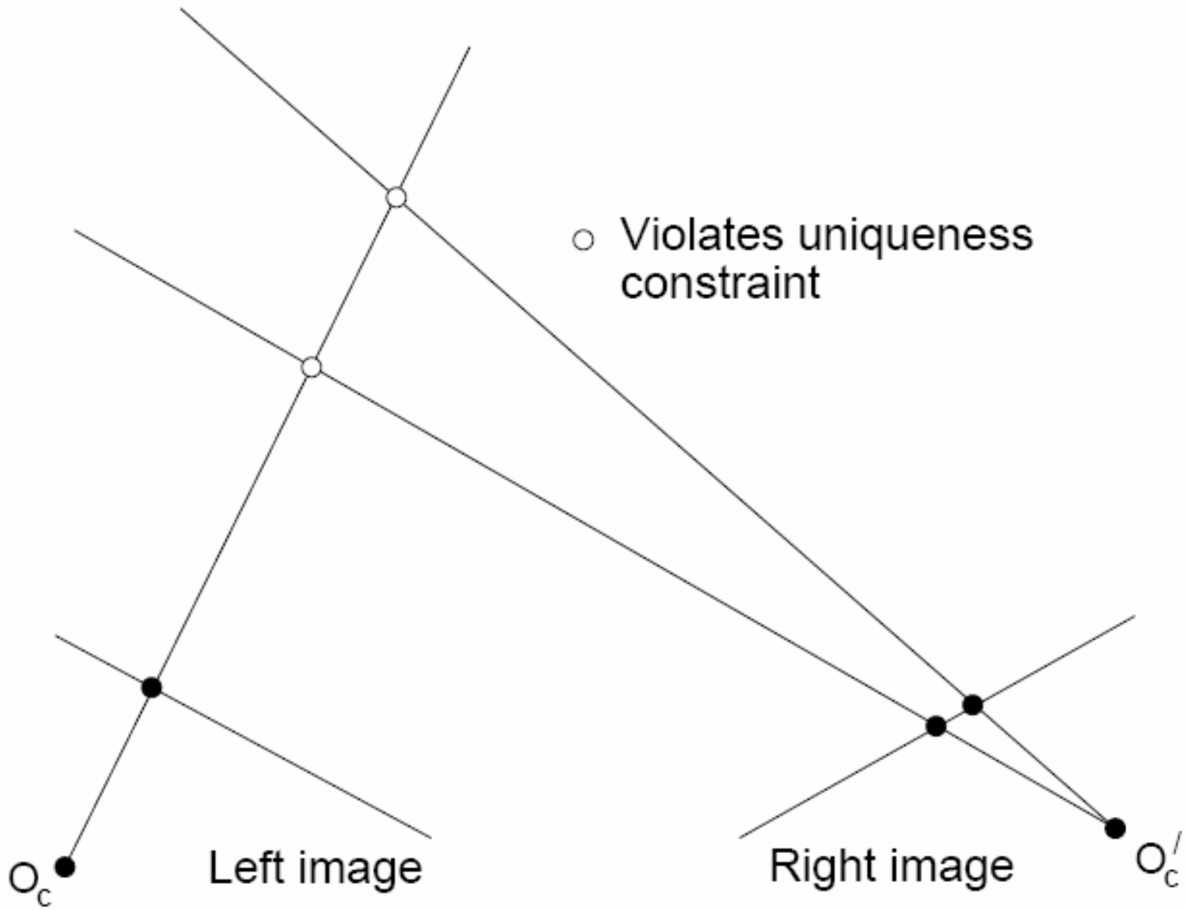
How can we improve window-based matching?

- The similarity constraint is **local** (each reference window is matched independently)
- Need to enforce **non-local** correspondence constraints

Slide from L. Lazebnik.

Non-local constraints

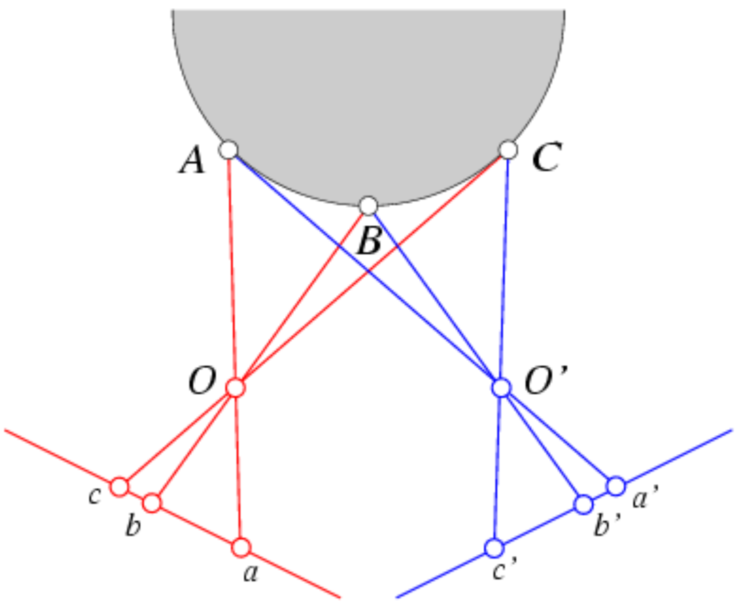
- Uniqueness
  - For any point in one image, there should be at most one matching point in the other image



Slide from L. Lazebnik.

Non-local constraints

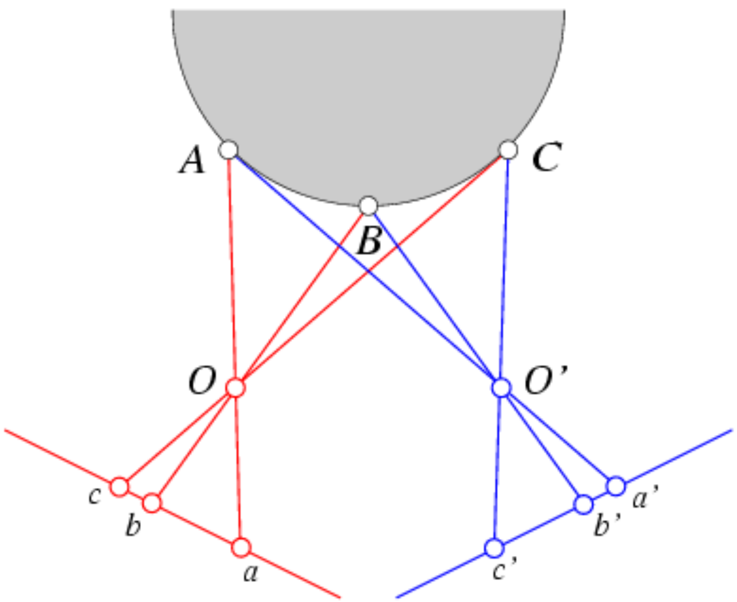
- Uniqueness
  - For any point in one image, there should be at most one matching point in the other image
- Ordering
  - Corresponding points should be in the same order in both views

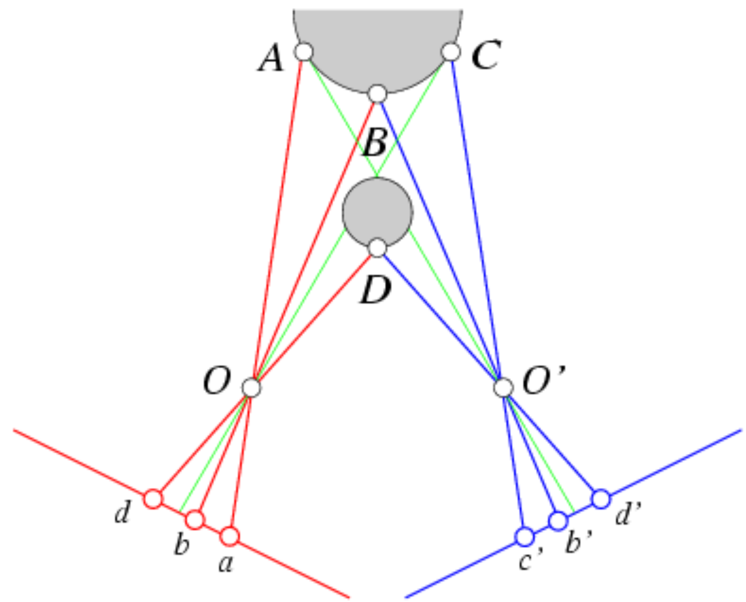


Slide from L. Lazebnik.

Non-local constraints

- Uniqueness
  - For any point in one image, there should be at most one matching point in the other image
- Ordering
  - Corresponding points should be in the same order in both views





Ordering constraint doesn't hold

Slide from L. Lazebnik.

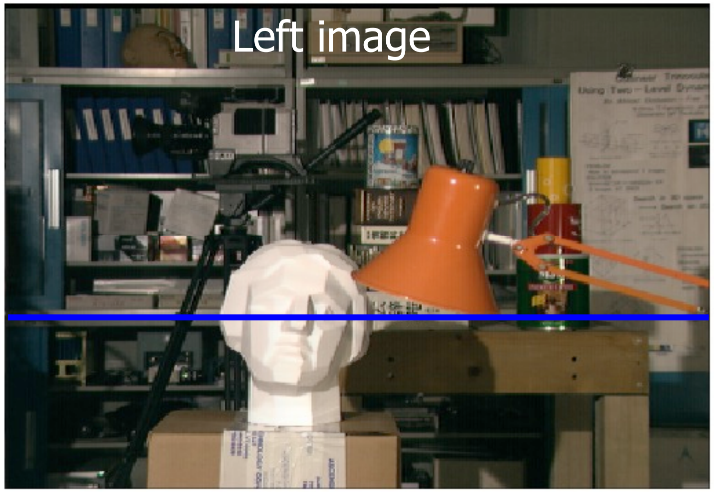
Non-local constraints

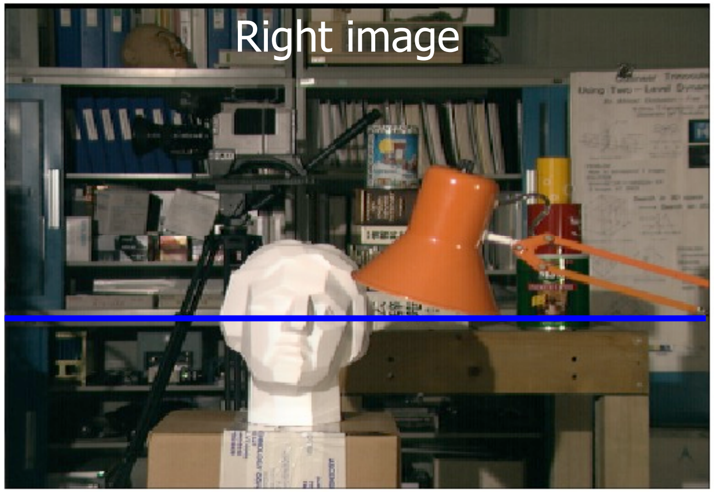
- Uniqueness
  - For any point in one image, there should be at most one matching point in the other image
- Ordering
  - Corresponding points should be in the same order in both views
- Smoothness
  - We expect disparity values to change slowly (for the most part)

Slide from L. Lazebnik.

Scanline stereo

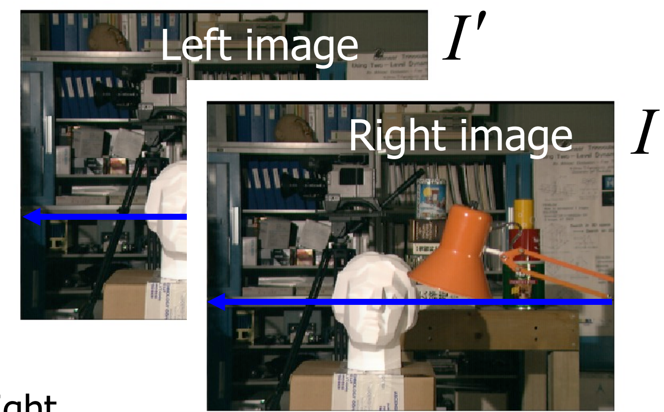
- Try to coherently match pixels on the entire scanline
- Different scanlines are still optimized independently

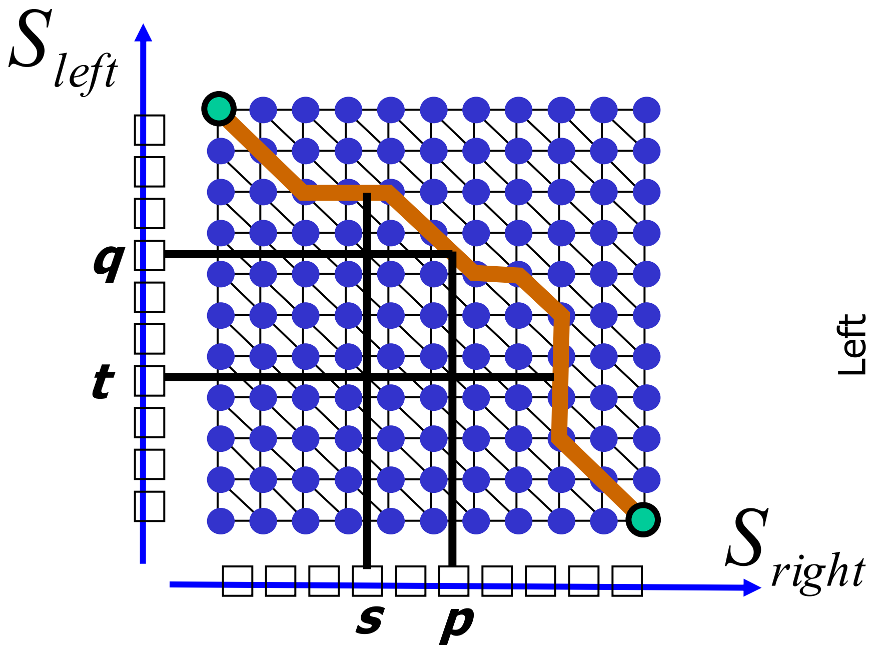


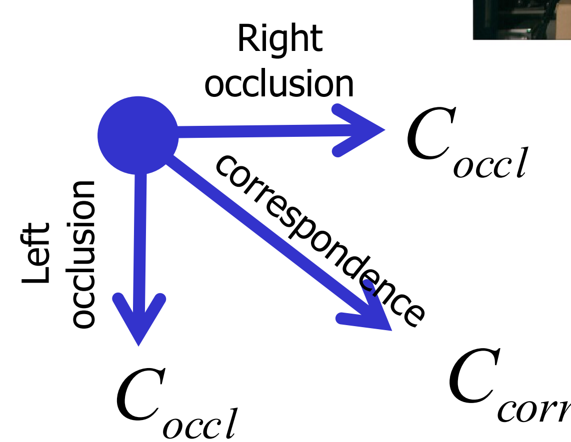


Slide from L. Lazebnik.

“Shortest paths” for scan-line stereo







Can be implemented with dynamic programming

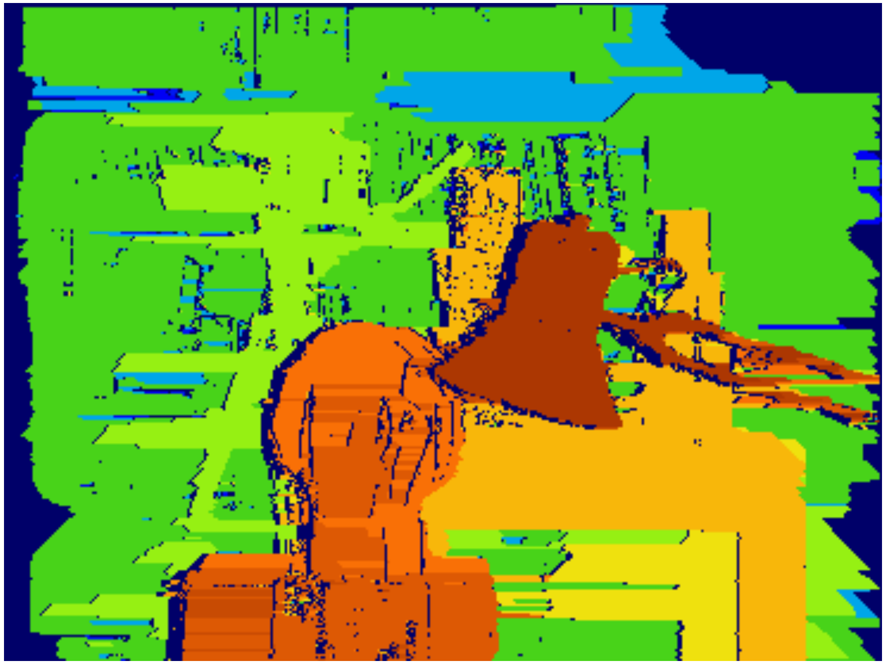
Ohta & Kanade ’85, Cox et al. ‘96

Slide from L. Lazebnik.

Slide credit: Y. Boykov

Coherent stereo on 2D grid

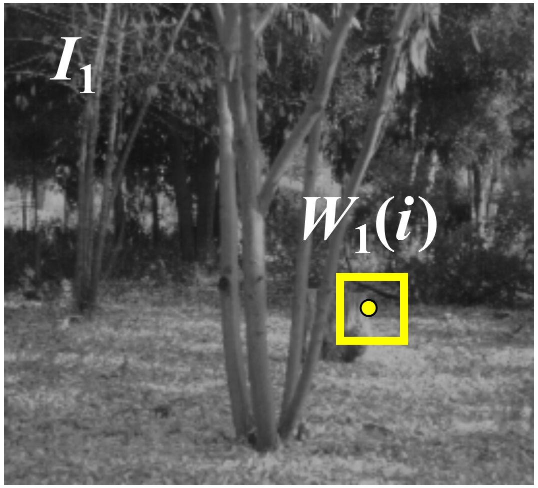
- Scanline stereo generates streaking artifacts

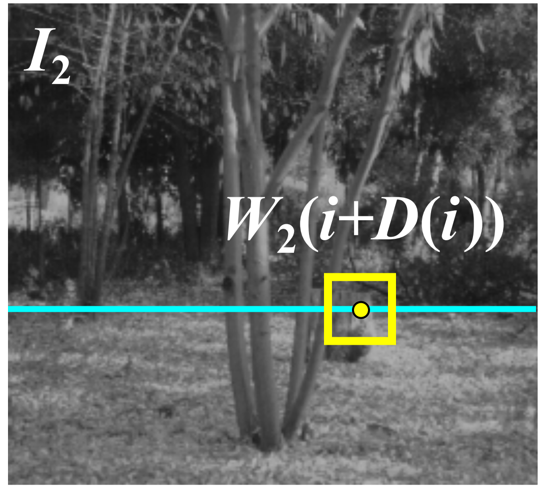


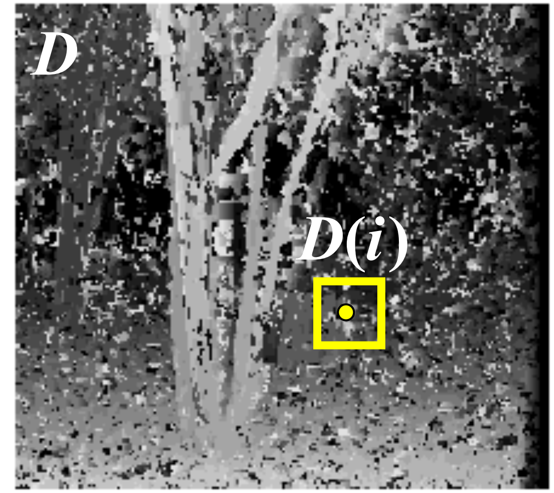
- Can’t use dynamic programming to find spatially coherent disparities/ correspondences on a 2D grid

Slide from L. Lazebnik.

Stereo matching as global optimization







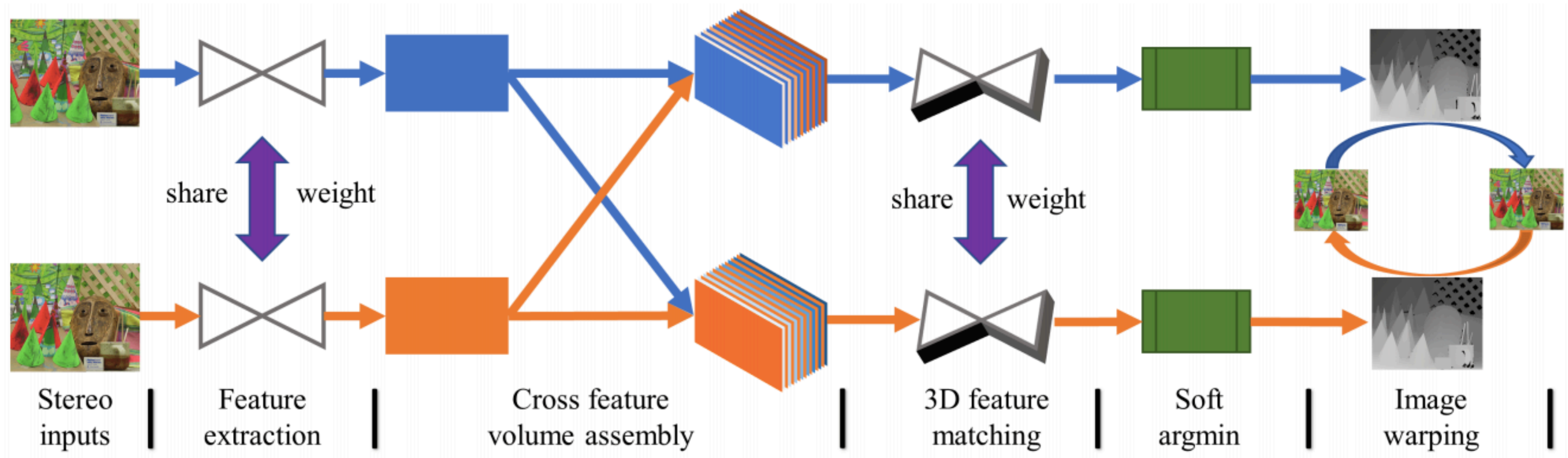
$$E(D) = \underbrace{\sum_i \left(W_1(i) - W_2(i + D(i))\right)^2}_{\textit{data term}} + \lambda \underbrace{\sum_{\text{neighbors } i,j} \rho\left(D(i) - D(j)\right)}_{\textit{smoothness term}}$$

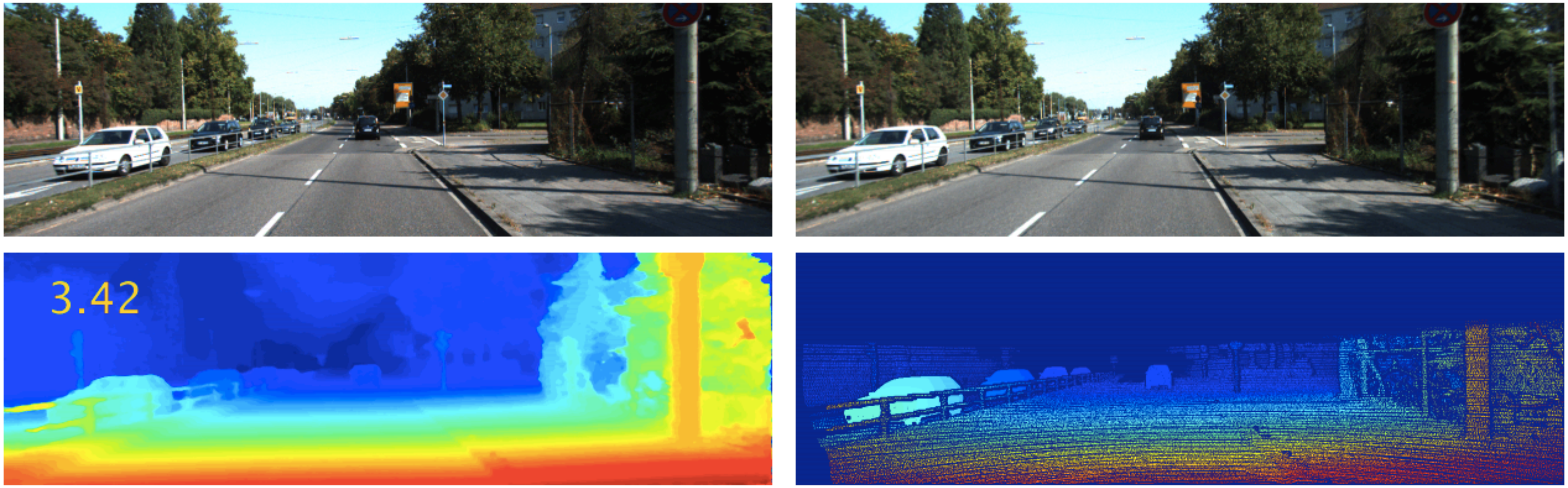
- Energy functions of this form can be minimized using *graph cuts*

Y. Boykov, O. Veksler, and R. Zabih, Fast Approximate Energy Minimization via Graph Cuts, PAMI 2001

Slide from L. Lazebnik.

Stereo matching as a prediction problem

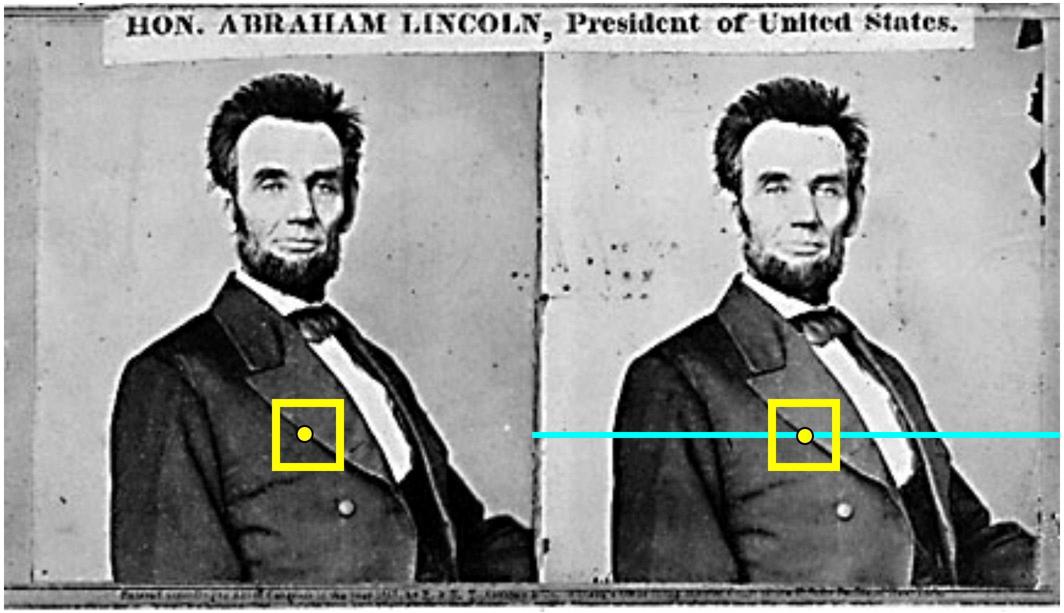




Y. Zhong, Y. Dai, and H. Li, [Self-Supervised Learning for Stereo Matching with Self-Improving Ability](), arXiv 2017

Slide from L. Lazebnik.

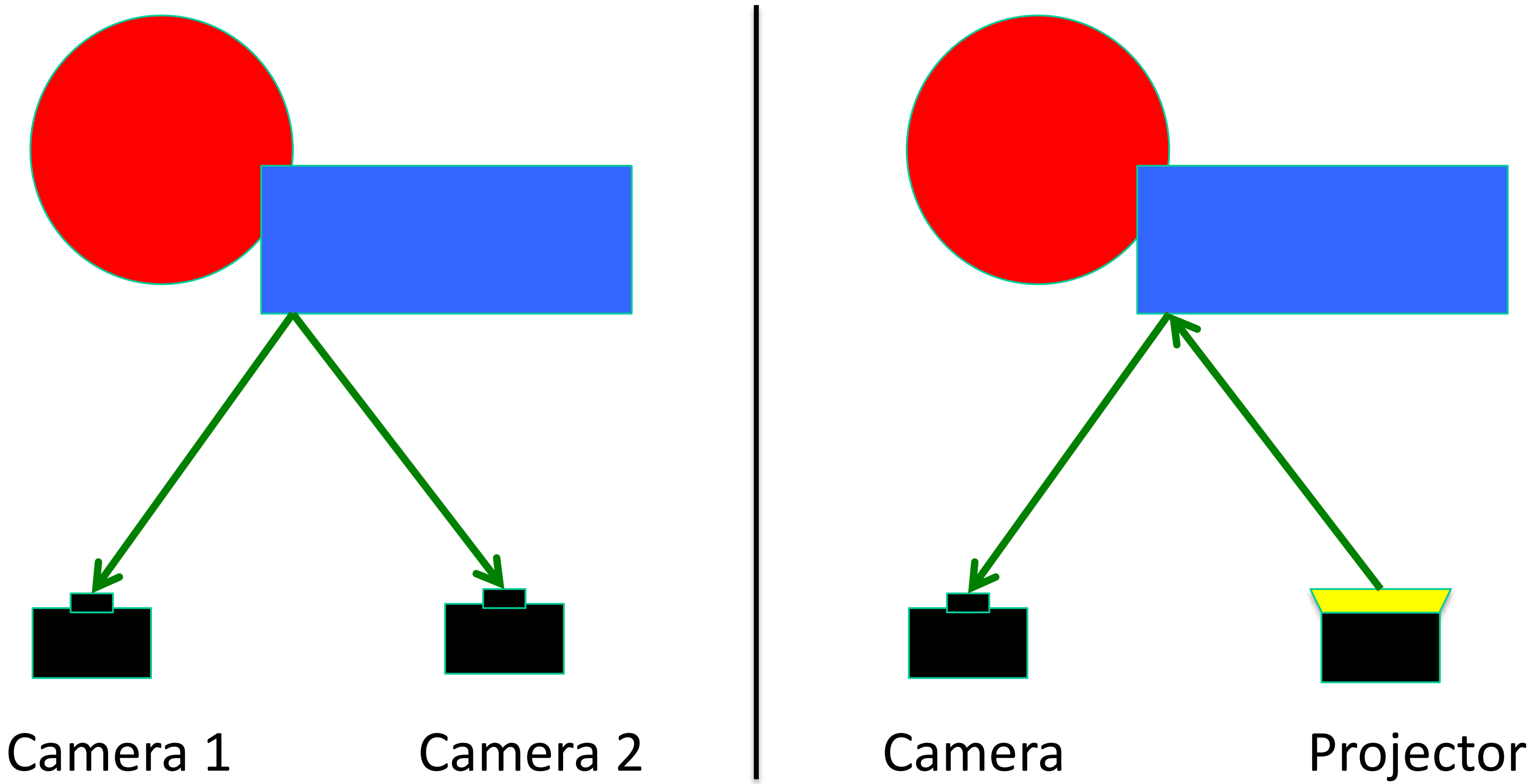
Review: Basic stereo matching algorithm



- For each pixel x in the reference image
  - Find corresponding epipolar scanline in the other image
  - Examine all pixels on the scanline and pick the best match x'
  - Compute disparity $x - x'$ and set depth(x) = $B*f/(x - x')$

Slide from L. Lazebnik.

Depth from Triangulation



Passive Stereopsis

Active Stereopsis

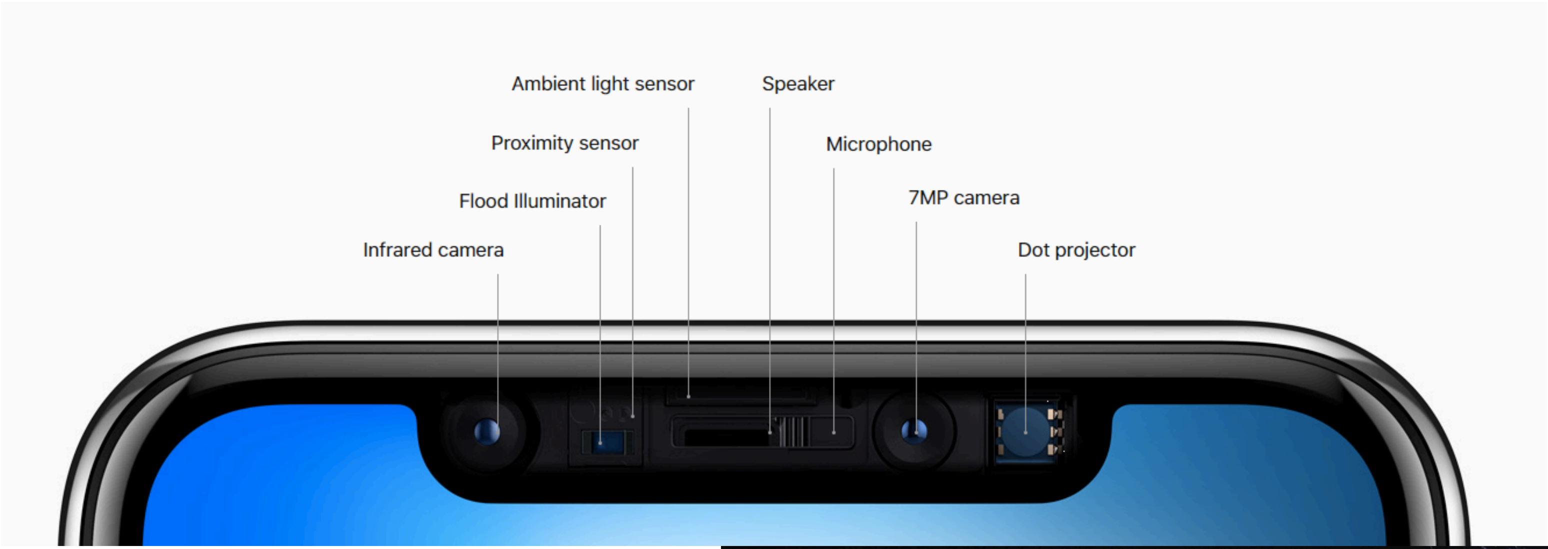
Active sensing simplifies the problem of estimating point correspondences

Kinect: Structured infrared light

http://bbzippo.wordpress.com/2010/11/28/kinect-in-infrared/

Slide from L. Lazebnik.

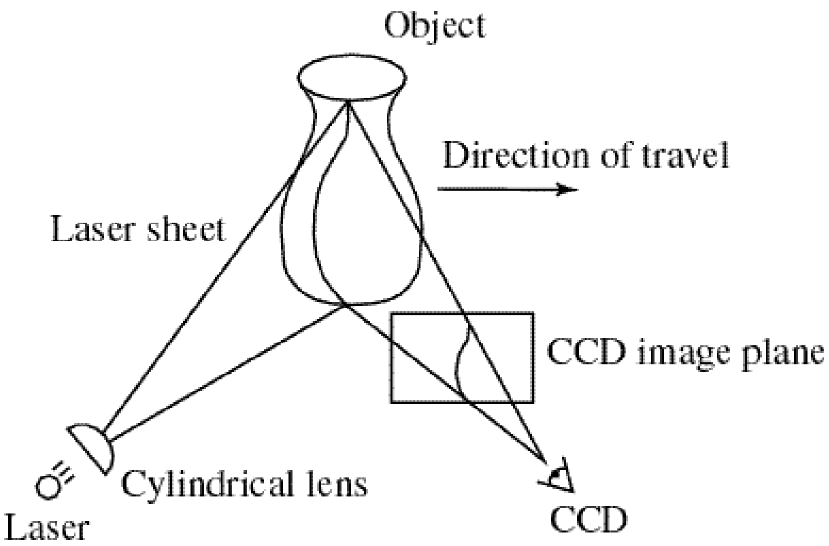
Apple TrueDepth

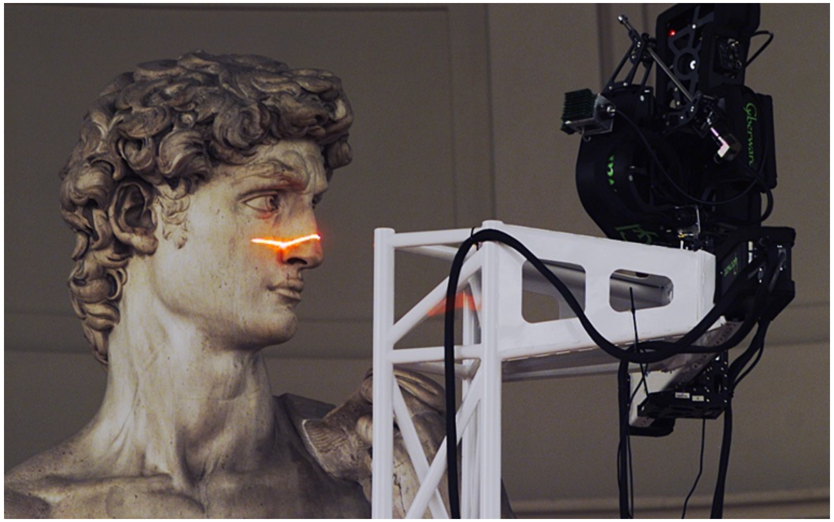


https://www.cnet.com/news/apple-face-id-truedepth-how-it-works/

Slide from L. Lazebnik.

Laser scanning





Digital Michelangelo Project
Levoy et al.
http://graphics.stanford.edu/projects/mich/

Optical triangulation

- Project a single stripe of laser light
- Scan it across the surface of the object
- This is a very precise version of structured light scanning

Source: S. Seitz

Laser scanned models

The Digital Michelangelo Project, Levoy et al.

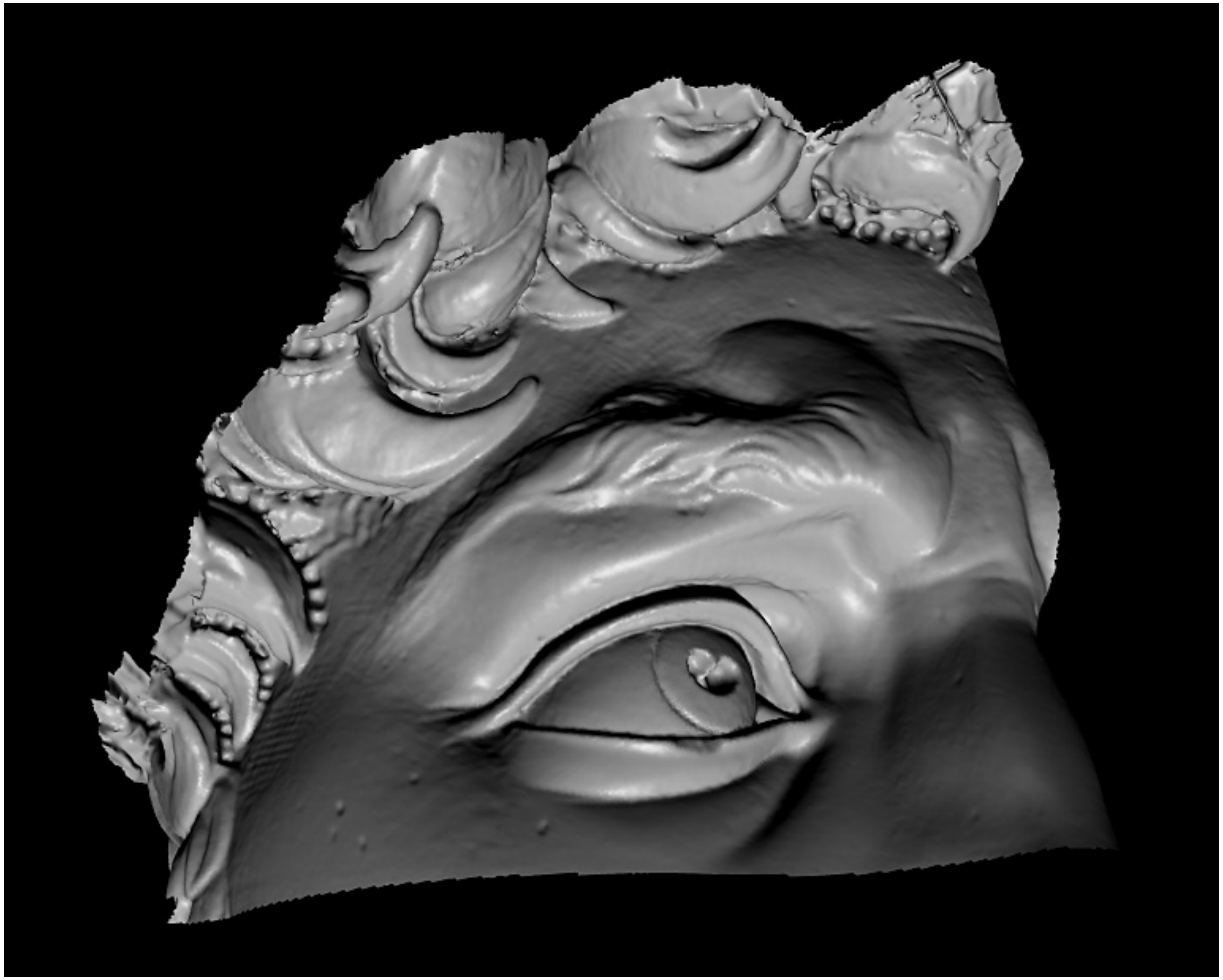
Source: S. Seitz

Laser scanned models

The Digital Michelangelo Project, Levoy et al.

Source: S. Seitz

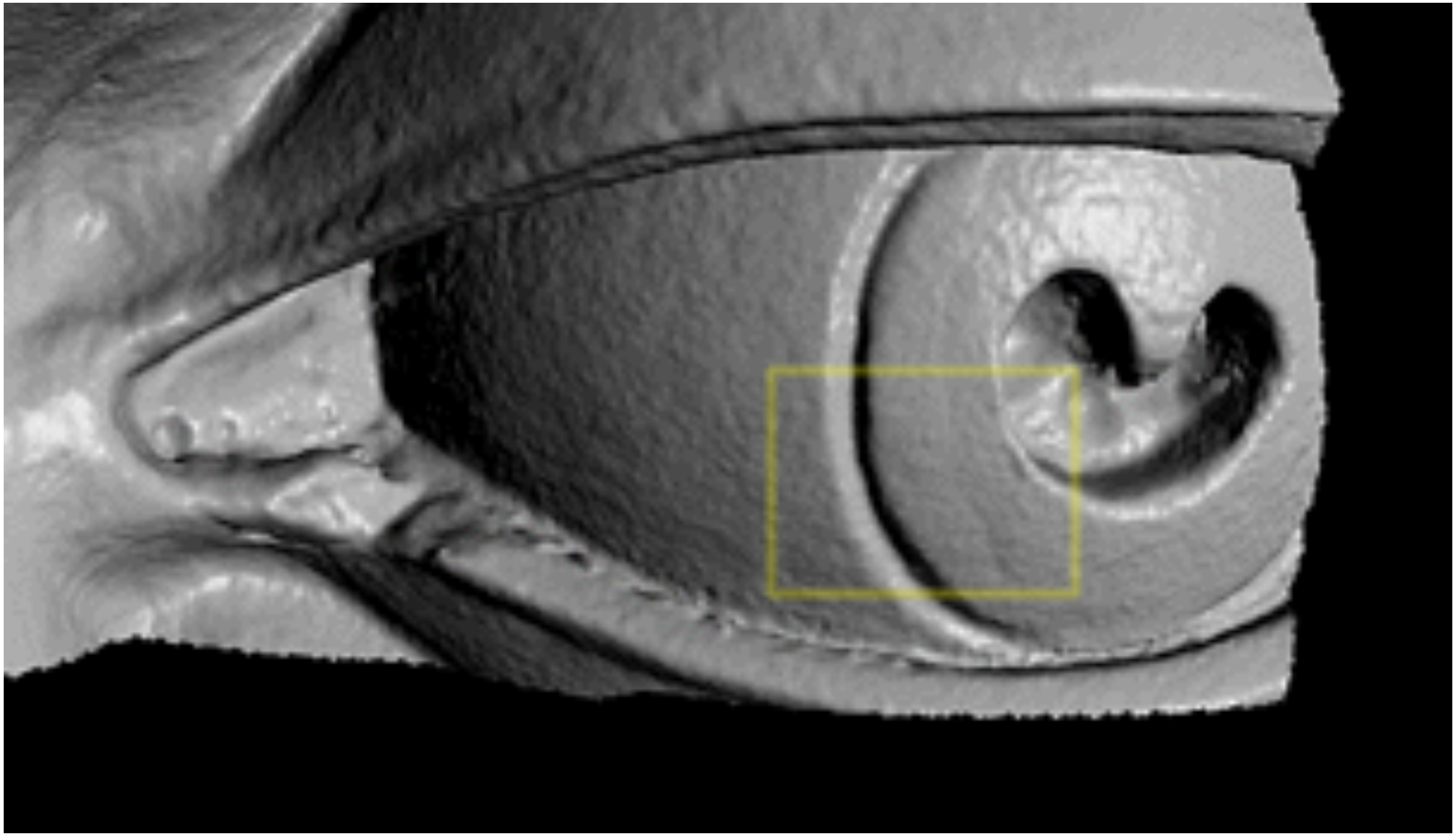
Laser scanned models



The Digital Michelangelo Project, Levoy et al.

Source: S. Seitz

Laser scanned models

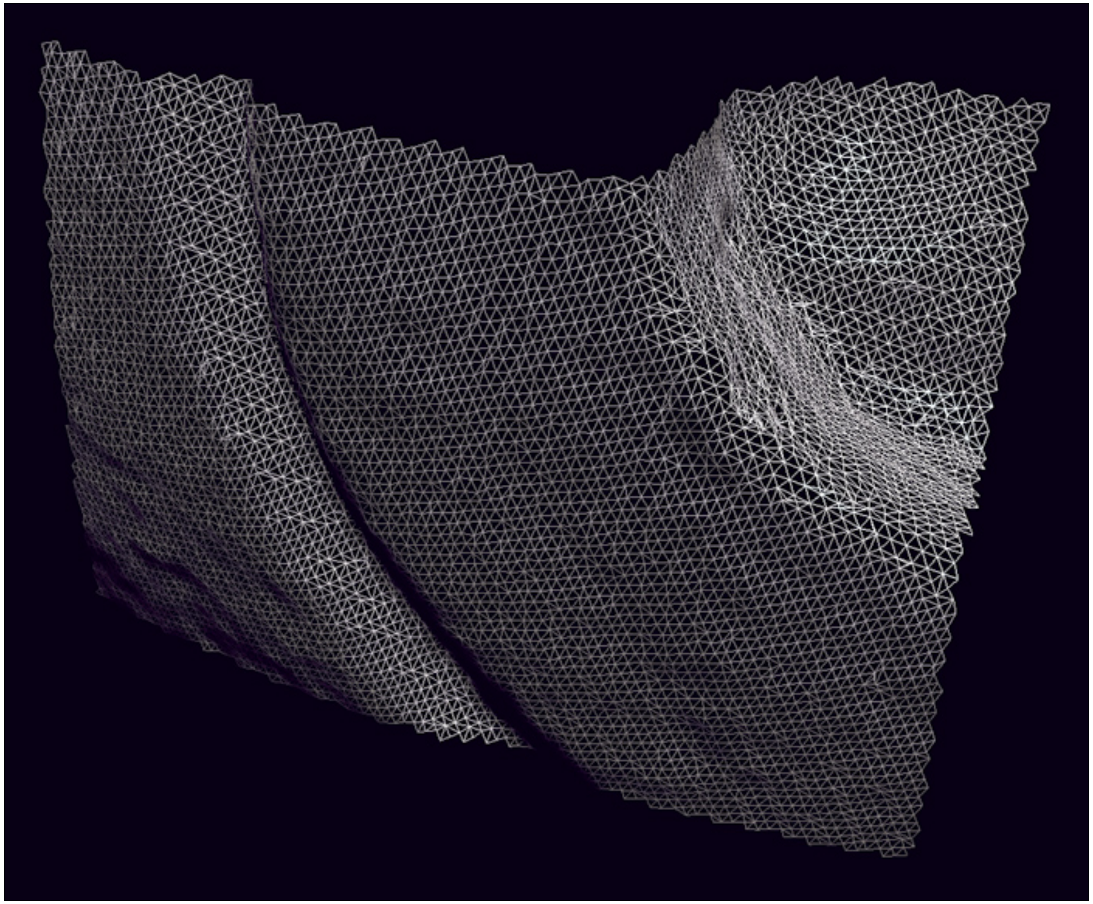


The Digital Michelangelo Project, Levoy et al.

Source: S. Seitz

Laser scanned models

1.0 mm resolution (56 million triangles)



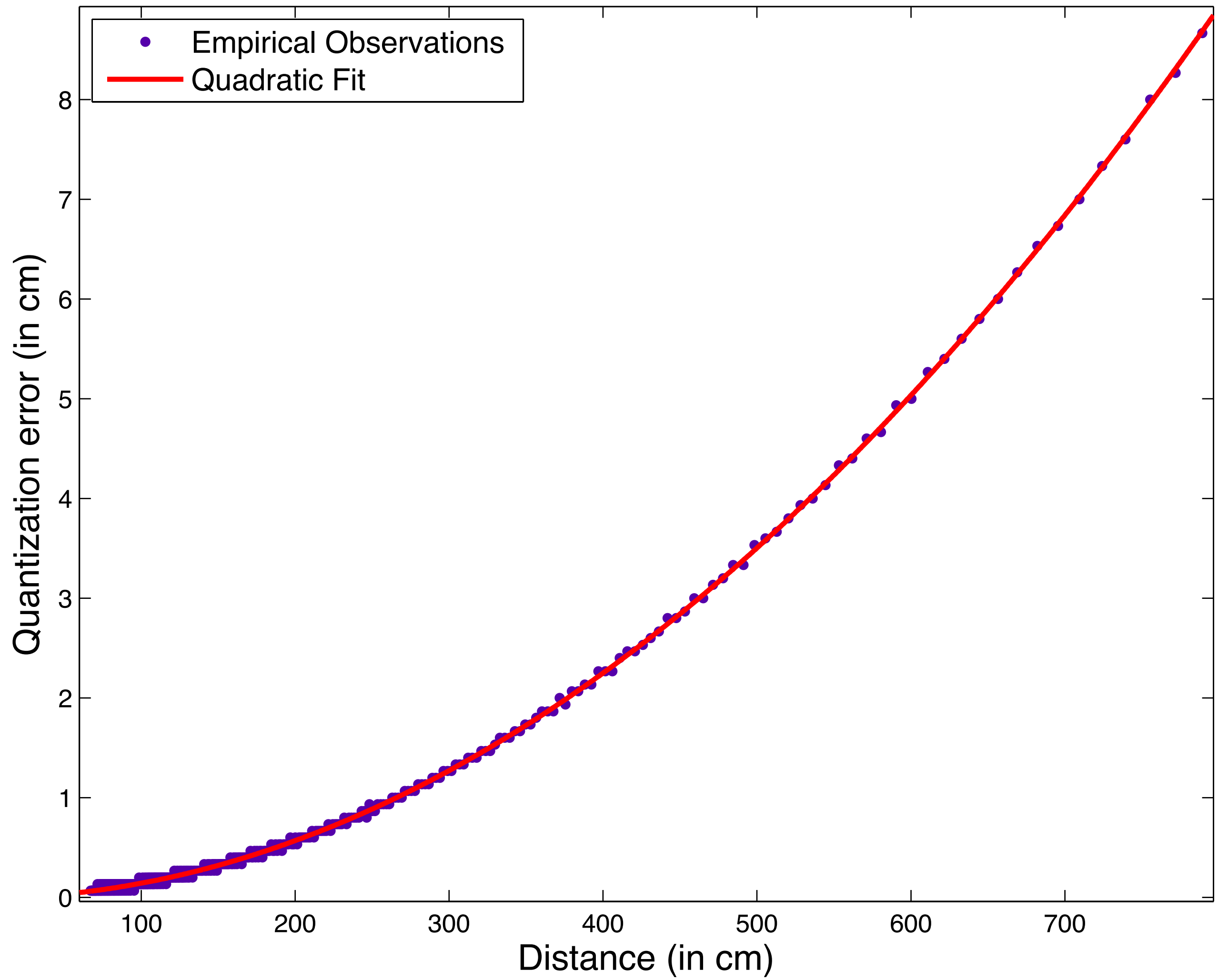
The Digital Michelangelo Project, Levoy et al.

Source: S. Seitz

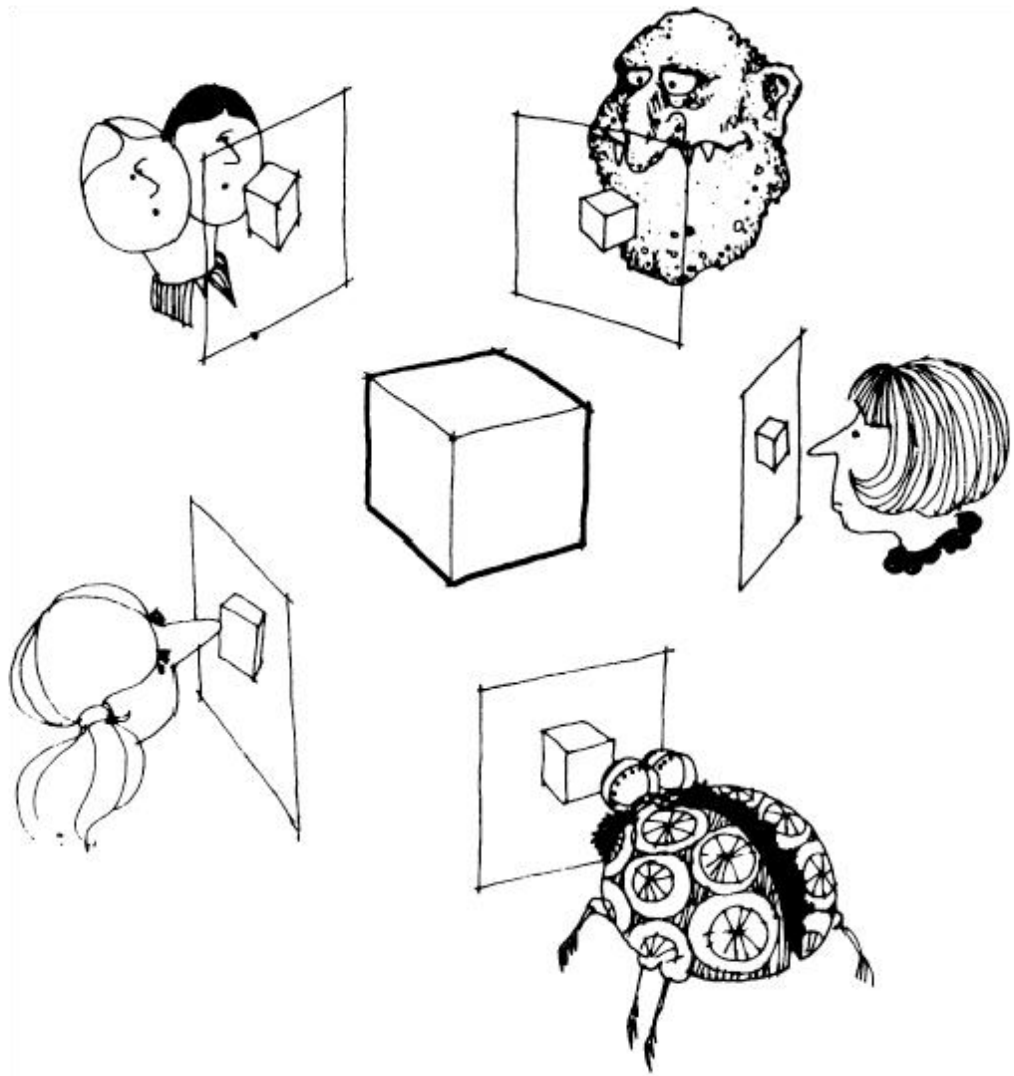
Stereo error(distance)

Error in distance estimate increases quadratically with the distance

$$
\begin{aligned}
Z &= \text{distance} \\
d &= \text{disparity} \\
Z &= \frac{C}{d} \\
\delta Z &= \frac{-Z^2}{C}\delta d \\
|\delta Z| &= \frac{Z^2}{C}|\delta d| \\
\text{error} &\propto \text{distance}^2
\end{aligned}
$$

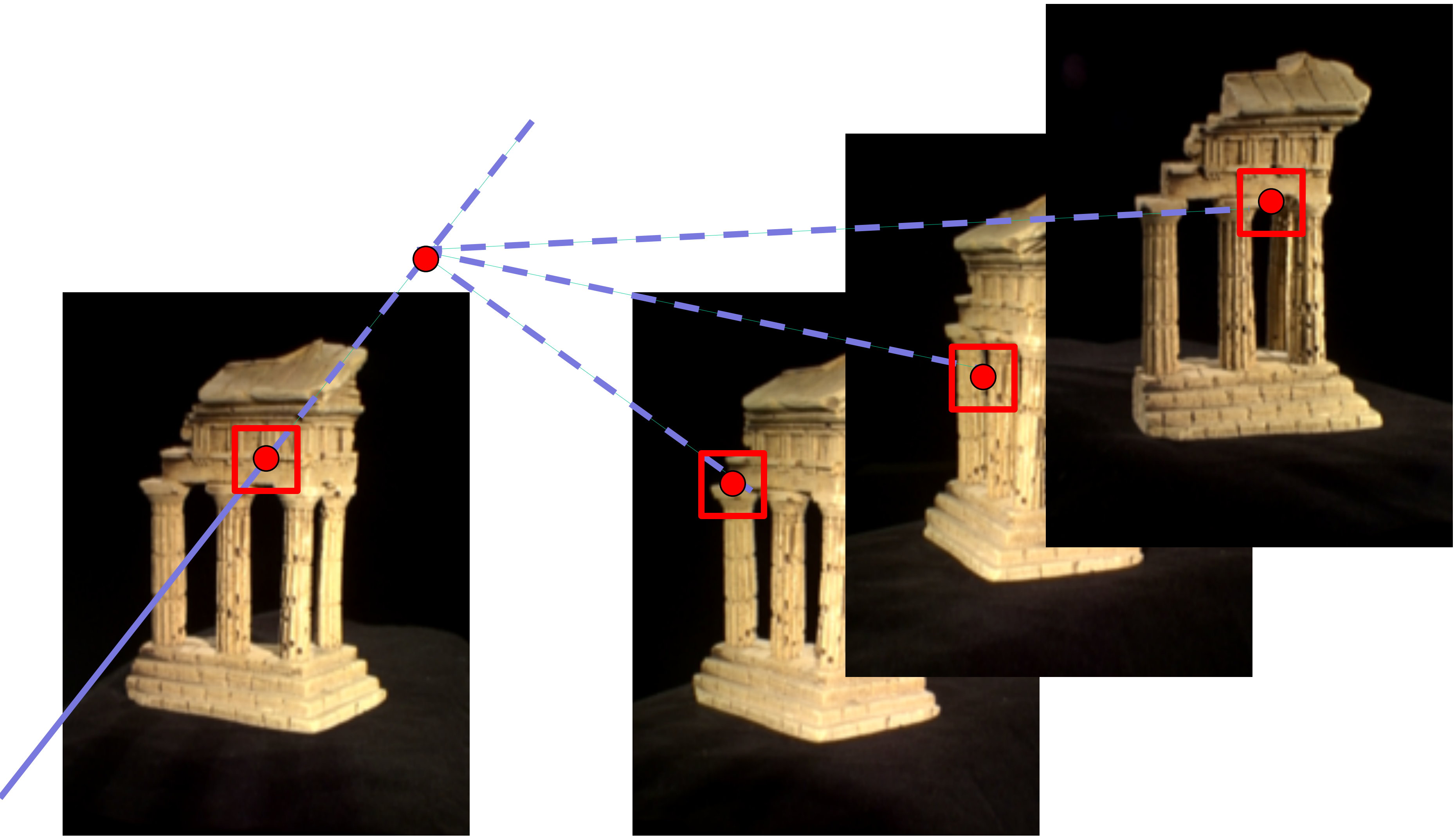


Multi-view stereo



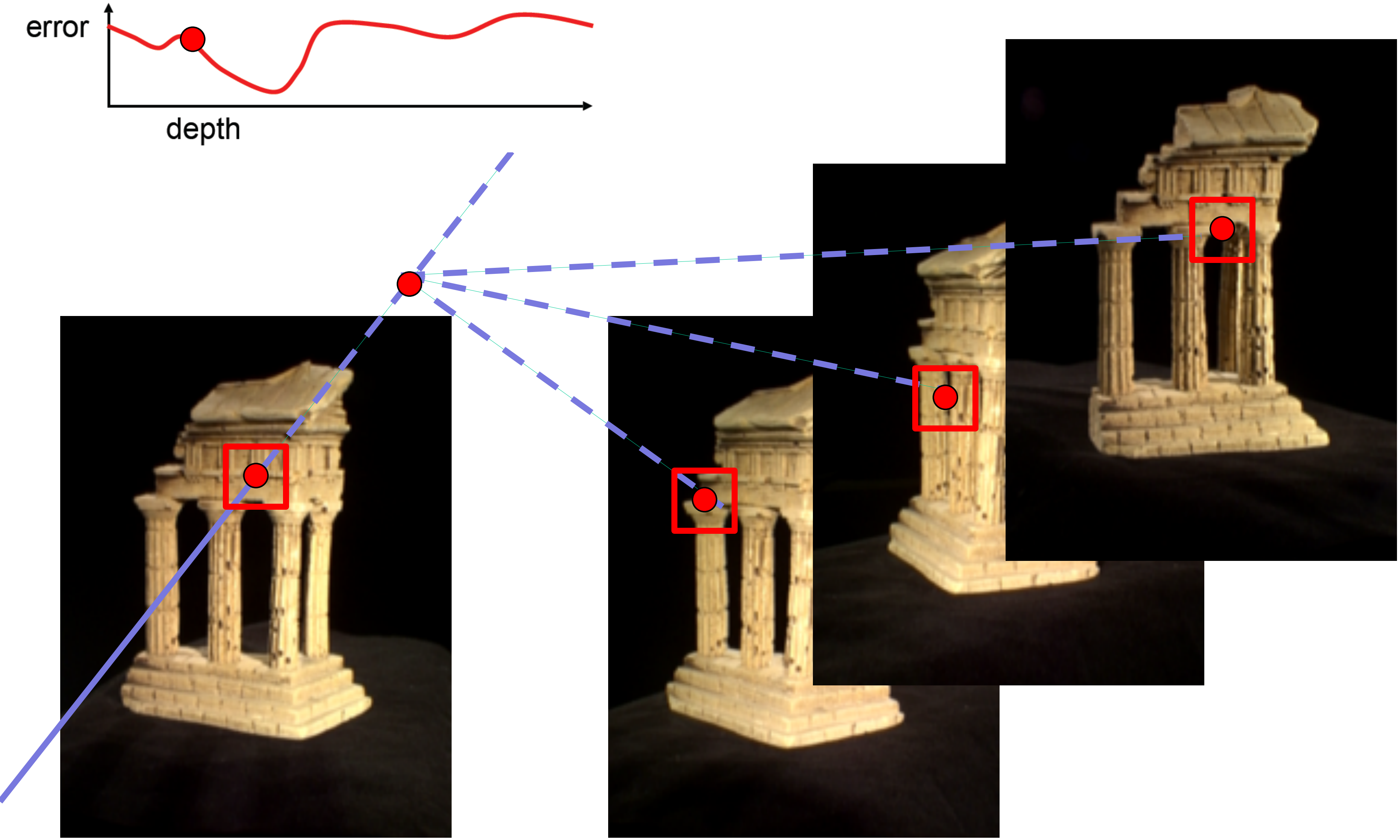
Slide from L. Lazebnik.

Multi-view stereo: Basic idea



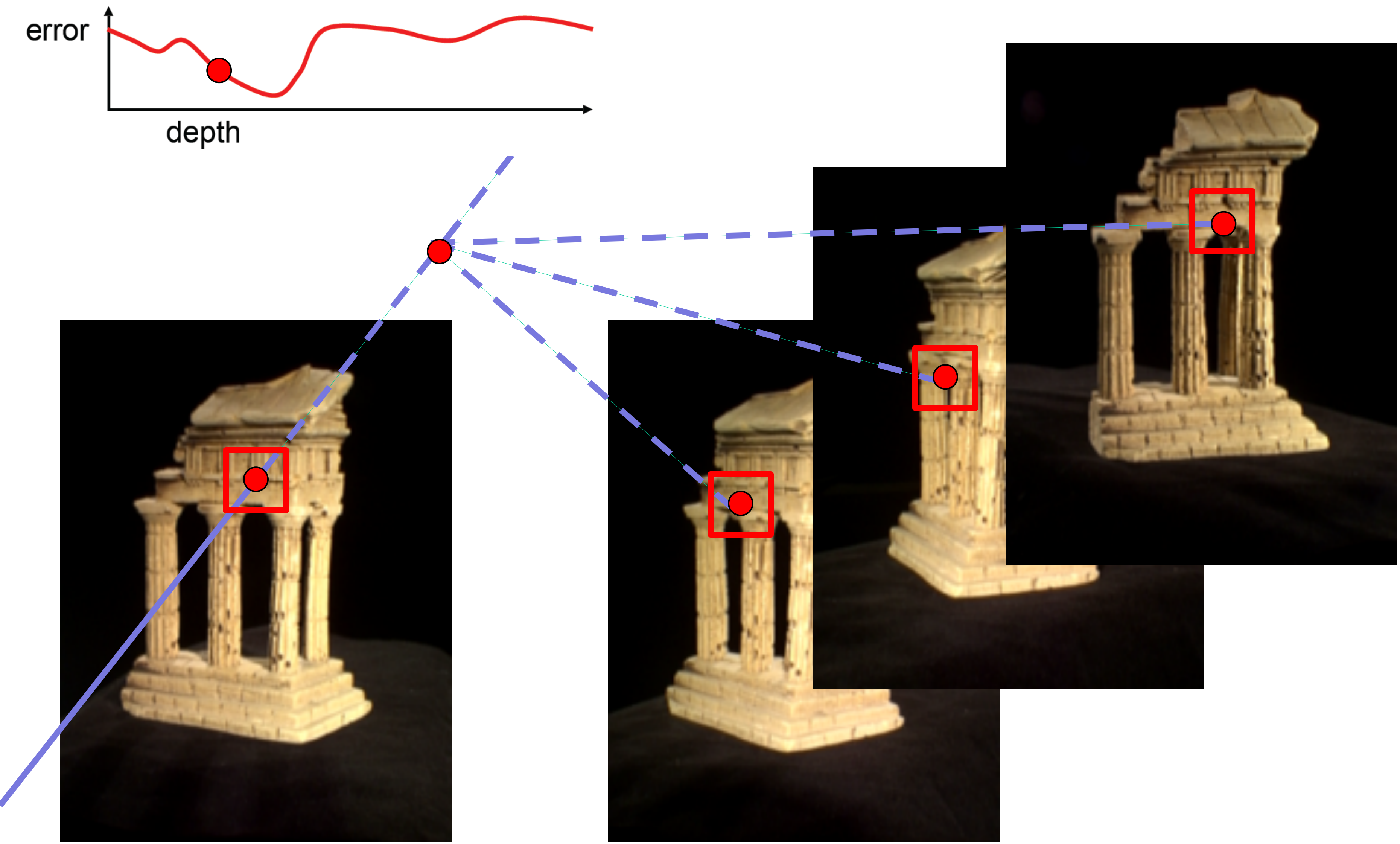
Source: Y. Furukawa

Multi-view stereo: Basic idea



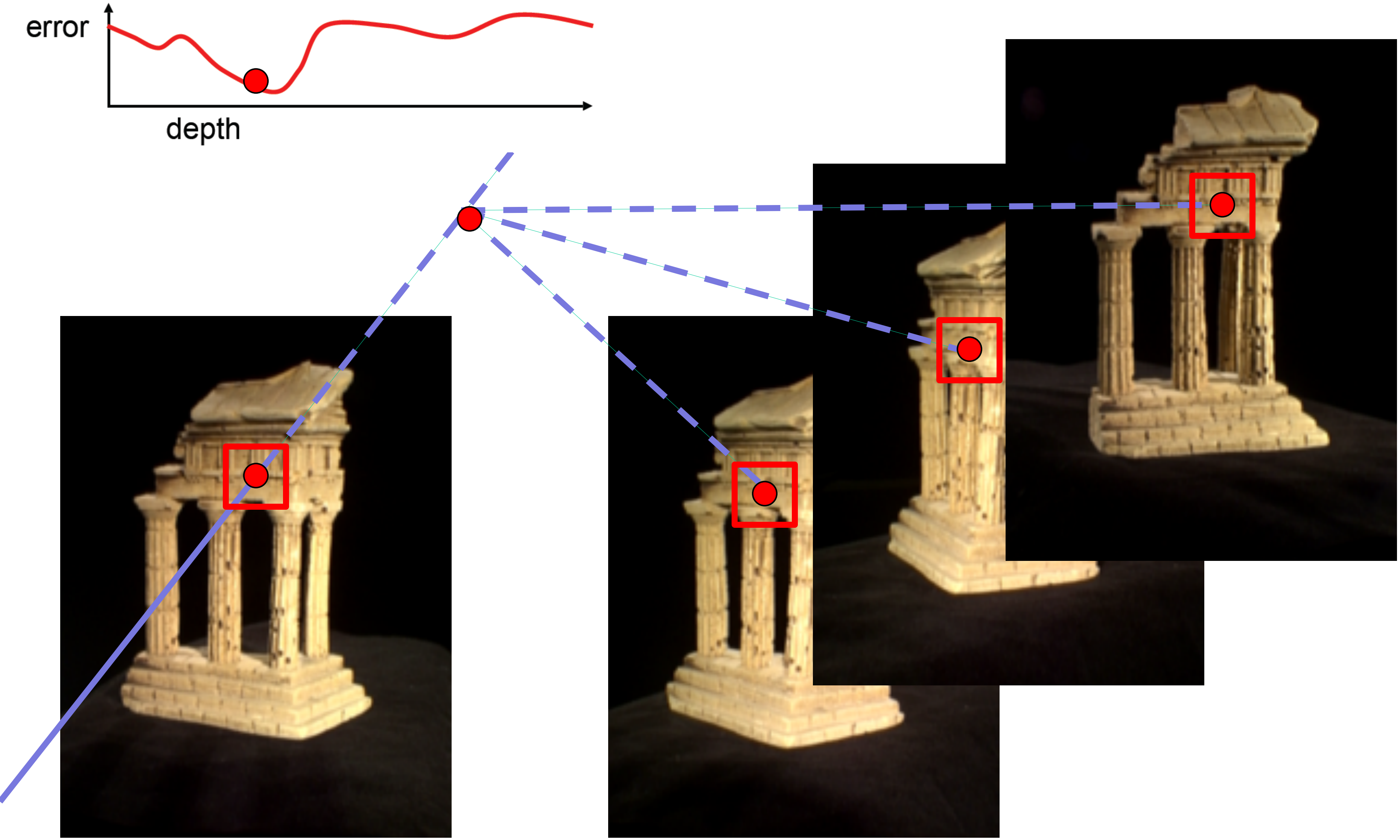
Source: Y. Furukawa

Multi-view stereo: Basic idea



Source: Y. Furukawa

Multi-view stereo: Basic idea



Source: Y. Furukawa

Towards Internet-Scale Multi-View Stereo

YouTube video, CMVS software

Y. Furukawa, B. Curless, S. Seitz and R. Szeliski, Towards Internet-scale Multi-view Stereo, CVPR 2010.

Applications

Source: N. Snavely

Applications

Source: N. Snavely