Light, Camera and Shading

CS 543 / ECE 549 – Saurabh Gupta
Overview

• Cameras with lenses
  • Depth of field
  • Field of view
  • Lens aberrations

• Brightness of a pixel
  • Small taste of radiometry
  • In-camera transformation of light
  • Reflectance properties of surfaces
  • Lambertian reflection model
  • Shape from shading
Building a Real Camera
Home-made pinhole camera

http://www.debevec.org/Pinhole/
Shrinking the aperture

Why not make the aperture as small as possible?

- Less light gets through
- Diffraction effects…
Shrinking the aperture

Slide by Steve Seitz
Adding a lens
A lens focuses light onto the film

- Thin lens model:
  - Rays passing through the center are not deviated
    (pinhole projection model still holds)
A lens focuses light onto the film

- Thin lens model:
  - Rays passing through the center are not deviated (pinhole projection model still holds)
  - All rays parallel to the optical axis pass through the *focal point*
  - All parallel rays converge to points on the *focal plane*
Thin lens formula

- Where does the lens focus the rays coming from a given point in the scene?
Thin lens formula

- What is the relation between the focal length \( (f) \), the distance of the object from the optical center \((D)\), and the distance at which the object will be in focus \((D')\)?
Thin lens formula

Similar triangles everywhere!
Thin lens formula

Similar triangles everywhere!

\[ \frac{y'}{y} = \frac{D'}{D} \]
Thin lens formula

Similar triangles everywhere!

\[ \frac{y'}{y} = \frac{D'}{D} \]
\[ \frac{y'}{y} = \frac{(D' - f)}{f} \]

Slide by Frédéric Durand
Thin lens formula

\[ \frac{1}{D'} + \frac{1}{D} = \frac{1}{f} \]

Any point satisfying the thin lens equation is in focus.

What happens when \( D \) is very large?
Depth of Field

For a fixed focal length, there is a specific distance at which objects are “in focus”

- Other points project to a “circle of confusion” in the image
Depth of Field

http://www.cambridgeincolour.com/tutorials/depth-of-field.htm

Slide by A. Efros
Controlling depth of field

Changing the aperture size affects depth of field

- A smaller *aperture* increases the range in which the object is approximately in focus
- But small aperture reduces amount of light – need to increase *exposure*

Varying the aperture

Large aperture = small DOF

Small aperture = large DOF

Slide by A. Efros
Field of View

FOV depends on focal length and size of the camera retina

\[ \phi = \tan^{-1} \left( \frac{d/2}{f} \right) \]

Larger focal length = smaller FOV
Field of View
Field of View

- 1000 mm: 2.5°
- 500 mm: 5°
- 300 mm: 8°
- 135 mm: 18°
- 85 mm: 28°
- 50 mm: 47°
- 28 mm: 75°
- 17 mm: 104°

Slide by A. Efros
Field of View / Focal Length

Large FOV, small $f$
Camera close to car

Small FOV, large $f$
Camera far from the car

Sources: A. Efros, F. Durand
Same effect for faces

wide-angle  standard  telephoto

Source: F. Durand
Approximating an orthographic camera
The dolly zoom

• Continuously adjusting the focal length while the camera moves away from (or towards) the subject

The dolly zoom

• Continuously adjusting the focal length while the camera moves away from (or towards) the subject

• “The Vertigo shot”

Example of dolly zoom from Goodfellas (YouTube)
Example of dolly zoom from La Haine (YouTube)
Real lenses
Lens flaws: Vignetting

![Diagram showing vignetting in photography]

![Images illustrating vignetting in a photo]

Vignetting is a lens flaw where the corners of the image are darker than the center, creating a vignette effect.
Radial Distortion

- Caused by imperfect lenses
- Deviations are most noticeable near the edge of the lens

No distortion  Pin cushion  Barrel
Lens flaws: Spherical aberration

Spherical lenses don’t focus light perfectly
Rays farther from the optical axis focus closer
Lens Flaws: Chromatic Aberration

Lens has different refractive indices for different wavelengths: causes color fringing

Near Lens Center

Near Lens Outer Edge
Lens Flaws: Chromatic Aberration

Researchers tried teaching a network about objects by forcing it to assemble jigsaws.

Initial layout, with sampled patches in red

Image layout is discarded

We can recover image layout automatically

Slide Credit: C. Doersch
Rolling Shutter

Rolling Shutter: pixels read in sequence
Can get global reading, but $$$
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Image formation

What determines the brightness of an image pixel?

Distribution and properties of light sources

Sensor properties

Exposure

Surface reflectance properties

Surface shape and orientation

Optics

Slide by L. Fei-Fei
Fundamental radiometric relation

$L$: Radiance emitted from $P$ toward $P'$
- Energy carried by a ray (Watts per sq. meter per steradian)

$E$: Irradiance falling on $P'$ from the lens
- Energy arriving at a surface (Watts per sq. meter)

What is the relationship between $E$ and $L$?
Fundamental radiometric relation

- Image irradiance is linearly related to scene radiance.
- Irradiance is proportional to the area of the lens and inversely proportional to the squared distance between the lens and the image plane.
- The irradiance falls off as the angle between the viewing ray and the optical axis increases.

\[
E = \left[ \frac{\pi}{4} \left( \frac{d}{f} \right)^2 \cos^4 \alpha \right] L
\]
Relation between Image Irradiance $E$ and Scene Radiance $L$

- Solid angles of the double cone (orange and green):

$$d\omega_i = d\omega_s \quad \frac{dA_i \cos \alpha}{(f / \cos \alpha)^2} = \frac{dA_s \cos \theta}{(z / \cos \alpha)^2}$$

- Solid angle subtended by lens:

$$d\omega_L = \frac{\pi d^2}{4} \frac{\cos \alpha}{(z / \cos \alpha)^2}$$

$$\frac{dA_s}{dA_i} = \frac{\cos \alpha}{\cos \theta} \left(\frac{z}{f}\right)^2$$

Slide from S Narasimhan.
Relation between Image Irradiance $E$ and Scene Radiance $L$

- Flux received by lens from $dA_s = \text{Flux projected onto image } dA_i$

$$L (dA_s \cos \theta) d\omega_L = E dA_i \quad \text{(3)}$$

- From (1), (2), and (3):

$$E = L \frac{\pi}{4} \left( \frac{d}{f} \right)^2 \cos \alpha^4$$

- Image irradiance is proportional to Scene Radiance!
- Small field of view $\Rightarrow$ Effects of 4th power of cosine are small.
From light rays to pixel values

\[ X = E \cdot \Delta t \]

- Camera response function: the mapping \( f \) from irradiance to pixel values
  - Useful if we want to estimate material properties
  - Enables us to create high dynamic range (HDR) images
Basic models of reflection

Specular: light bounces off at the incident angle
• E.g., mirror

Diffuse: light scatters in all directions
• E.g., brick, cloth, rough wood
Other possible effects

transparency

light source

refraction

light source

Slide from D. Hoiem
Other possible effects

subsurface scattering

light source

Slide from D. Hoiem
Other possible effects

**fluorescence**

![Fluorescent minerals](https://en.wikipedia.org/wiki/Fluorescence#/media/File:Fluorescent_minerals_hg.jpg)

**phosphorescence**

![Phosphorescent objects](https://en.wikipedia.org/wiki/Fluorescence#/media/File:Fluorescent_minerals_hg.jpg)

Light source

\[ \lambda_1 \]

\[ \lambda_2 \]

\[ t=1 \]

\[ t>1 \]

Slide from D. Hoiem
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Lambertian reflectance model

Some light is absorbed (function of albedo $\rho$)
Remaining light is scattered (diffuse reflection)
Examples: soft cloth, concrete, matte paints

$\text{absorption} = (1 - \rho)$
Specular Reflection

Reflected direction depends on light orientation and surface normal
- E.g., mirrors are fully specular
Most surfaces have both specular and diffuse components.

Specularity = spot where specular reflection dominates (typically reflects light source).

Typically, specular component is small.
BRDF: Bidirectional Reflectance Distribution Function

- Model of local reflection that tells how bright a surface appears when viewed from one direction when light falls on it from another.
- Definition: ratio of the radiance in the emitted direction to irradiance in the incident direction.

Slide credit: S. Savarese
BRDFs can be incredibly complicated...
Diffuse reflection

- Light is reflected equally in all directions
  - Dull, matte surfaces like chalk or latex paint
  - Microfacets scatter incoming light randomly

- Brightness of the surface depends on the incidence of illumination

brighter  darker
Diffuse reflection: Lambert’s law

\[ B = \rho \mathbf{N} \cdot \mathbf{S} \]
\[ = \rho \| \mathbf{S} \| \cos \theta \]

- \( B \): radiosity (total power leaving the surface per unit area)
- \( \rho \): albedo (fraction of incident irradiance reflected by the surface)
- \( \mathbf{N} \): unit normal
- \( \mathbf{S} \): source vector (magnitude proportional to intensity of the source)
Specular reflection

- Radiation arriving along a source direction leaves along the **specular direction** (source direction reflected about normal)
- On real surfaces, energy usually goes into a lobe of directions
- **Phong model:** reflected energy falls off with $\cos^n(\delta \theta)$

Moving the light source

Changing the exponent

Slide from L. Lazebnik
Specular reflection

Slide from L. Lazebnik
Lambertian + Specular Model

- $I(x) = \text{Ambient Term} + \text{Diffuse Term} + \text{Specular Term}$
Photometric stereo (shape from shading)

• Can we reconstruct the shape of an object based on shading cues?
Photometric stereo

Assume:

- A Lambertian object
- A *local shading model* (each point on a surface receives light only from sources visible at that point)
- A set of *known* light source directions
- A set of pictures of an object, obtained in exactly the same camera/object configuration but using different sources
- Orthographic projection

Goal: reconstruct object shape and albedo
Example 1

Recovered albedo

Recovered normal field

Recovered surface model

Slide from L. Lazebnik

F&P 2nd ed., sec. 2.2.4
Example 2

Input

Recovered albedo

Recovered normal field

Recovered surface model

Slide from L. Lazebnik
Image model

- **Known:** source vectors $S_j$ and pixel values $I_j(x,y)$
- **Unknown:** surface normal $N(x,y)$ and albedo $\rho(x,y)$
Image model

- **Known:** source vectors $S_j$ and pixel values $I_j(x,y)$
- **Unknown:** surface normal $N(x,y)$ and albedo $\rho(x,y)$
- Assume that the response function of the camera is a linear scaling by a factor of $k$
- Lambert’s law:

\[
I_j(x,y) = k \rho(x,y) (N(x,y) \cdot S_j)
= (\rho(x,y)N(x,y)) \cdot (kS_j)
= g(x, y) \cdot V_j
\]
Least squares problem

• For each pixel, set up a linear system:

\[
\begin{bmatrix}
I_1(x,y) \\
I_2(x,y) \\
\vdots \\
I_n(x,y)
\end{bmatrix}
\begin{bmatrix}
V_1^T \\
V_2^T \\
\vdots \\
V_n^T
\end{bmatrix}
= \begin{bmatrix}
\mathbf{g}(x,y)
\end{bmatrix}
\]

\(I_1(x,y)\) \(\cdots\) \(I_n(x,y)\)
\((n \times 1)\)
known
\((n \times 3)\)
known
\((3 \times 1)\)
unknown

• Obtain least-squares solution for \(\mathbf{g}(x,y)\)
  (which we defined as \(\mathbf{N}(x,y) \rho(x,y)\))

• Since \(\mathbf{N}(x,y)\) is the unit normal, \(\rho(x,y)\) is given by the
  magnitude of \(\mathbf{g}(x,y)\)

• Finally, \(\mathbf{N}(x,y) = \mathbf{g}(x,y) / \rho(x,y)\)
Synthetic example

Recovered albedo

Recovered normal field

Slide from L. Lazebnik
Recovering a surface from normals

Recall the surface is written as

\[(x, y, f(x, y))\]

This means the normal has the form:

\[
\mathbf{N}(x, y) = \frac{1}{\sqrt{f_x^2 + f_y^2 + 1}} \begin{pmatrix} f_x \\ f_y \\ 1 \end{pmatrix}
\]

If we write the estimated vector \( g \) as

\[
g(x, y) = \begin{pmatrix} g_1(x, y) \\ g_2(x, y) \\ g_3(x, y) \end{pmatrix}
\]

Then we obtain values for the partial derivatives of the surface:

\[
f_x(x, y) = \frac{g_1(x, y)}{g_3(x, y)}
\]

\[
f_y(x, y) = \frac{g_2(x, y)}{g_3(x, y)}
\]
Recovering a surface from normals

We can now recover the surface height at any point by integration along some path, e.g.

$$f(x, y) = \int_0^x f_x(s, 0) \, ds + \int_0^y f_y(x, t) \, dt + C$$

(for robustness, should take integrals over many different paths and average the results)

**Integrability:** for the surface $f$ to exist, the mixed second partial derivatives must be equal:

$$\frac{\partial}{\partial y} \left( \frac{g_1(x, y)}{g_3(x, y)} \right) = \frac{\partial}{\partial x} \left( \frac{g_2(x, y)}{g_3(x, y)} \right)$$

(in practice, they should at least be similar)
Surface recovered by integration
Limitations

• Orthographic camera model
• Simplistic reflectance and lighting model
• No shadows
• No interreflections
• No missing data
• Integration is tricky
Finding the direction of the light source

\[ I(x,y) = \mathbf{N}(x,y) \cdot \mathbf{S}(x,y) \]

**Full 3D case:**

\[
\begin{pmatrix}
  N_x(x_1, y_1) & N_y(x_1, y_1) & N_z(x_1, y_1) \\
  N_x(x_2, y_2) & N_y(x_2, y_2) & N_z(x_2, y_2) \\
  \vdots & \vdots & \vdots \\
  N_x(x_n, y_n) & N_y(x_n, y_n) & N_z(x_n, y_n)
\end{pmatrix}
\begin{pmatrix}
  S_x \\
  S_y \\
  S_z
\end{pmatrix}
= 
\begin{pmatrix}
  I(x_1, y_1) \\
  I(x_2, y_2) \\
  \vdots \\
  I(x_n, y_n)
\end{pmatrix}
\]

Finding the direction of the light source

Consider points on the *occluding contour*:

\[ N_z = 0 \]

\[ N_z \text{ positive} \]

\[ N_z \text{ negative} \]

Finding the direction of the light source

\[ I(x,y) = \mathbf{N}(x,y) \cdot \mathbf{S}(x,y) \]

Full 3D case:

\[
\begin{pmatrix}
N_x(x_1, y_1) & N_y(x_1, y_1) & N_z(x_1, y_1) \\
N_x(x_2, y_2) & N_y(x_2, y_2) & N_z(x_2, y_2) \\
\vdots & \vdots & \vdots \\
N_x(x_n, y_n) & N_y(x_n, y_n) & N_z(x_n, y_n)
\end{pmatrix}
\begin{pmatrix}
S_x \\
S_y \\
S_z
\end{pmatrix} =
\begin{pmatrix}
I(x_1, y_1) \\
I(x_2, y_2) \\
\vdots \\
I(x_n, y_n)
\end{pmatrix}
\]

For points on the \textit{occluding contour}, \( N_z = 0 \):

\[
\begin{pmatrix}
N_x(x_1, y_1) & N_y(x_1, y_1) \\
N_x(x_2, y_2) & N_y(x_2, y_2) \\
\vdots & \vdots \\
N_x(x_n, y_n) & N_y(x_n, y_n)
\end{pmatrix}
\begin{pmatrix}
S_x \\
S_y
\end{pmatrix} =
\begin{pmatrix}
I(x_1, y_1) \\
I(x_2, y_2) \\
\vdots \\
I(x_n, y_n)
\end{pmatrix}
\]

Finding the direction of the light source

Application: Detecting composite photos

Fake photo

Real photo