Optical flow

Many slides adapted from S. Seitz, R. Szeliski, M. Pollefeys

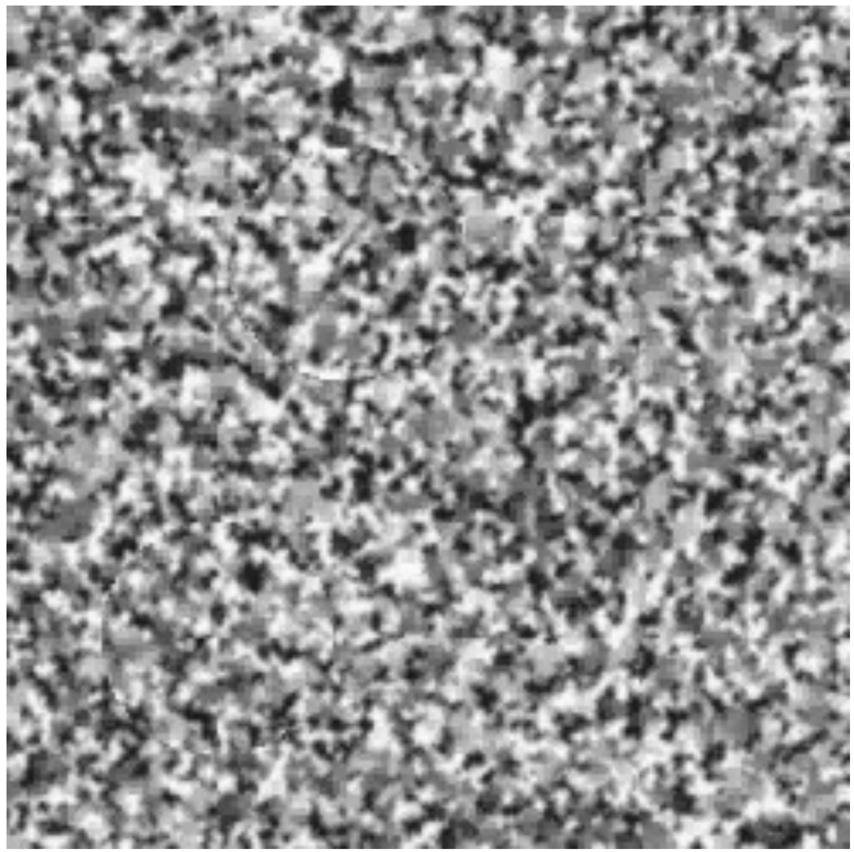
Slides from S. Lazebnik.

What direction is the object moving?

A. Left right
B. Up down
C. Top-left to bottom-right
D. Bottom left to top-right

Motion is a powerful perceptual cue

- Sometimes, it is the only cue



Motion is a powerful perceptual cue

- Even “impoverished” motion data can evoke a strong percept

G. Johansson, “Visual Perception of Biological Motion and a Model For Its Analysis",
Perception and Psychophysics 14, 201-211, 1973.

Source: L. Lazebnik

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Source: L. Lazebnik

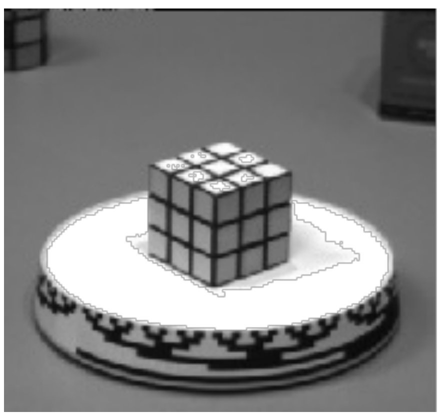
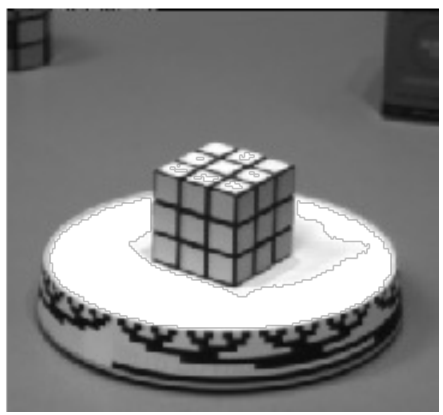
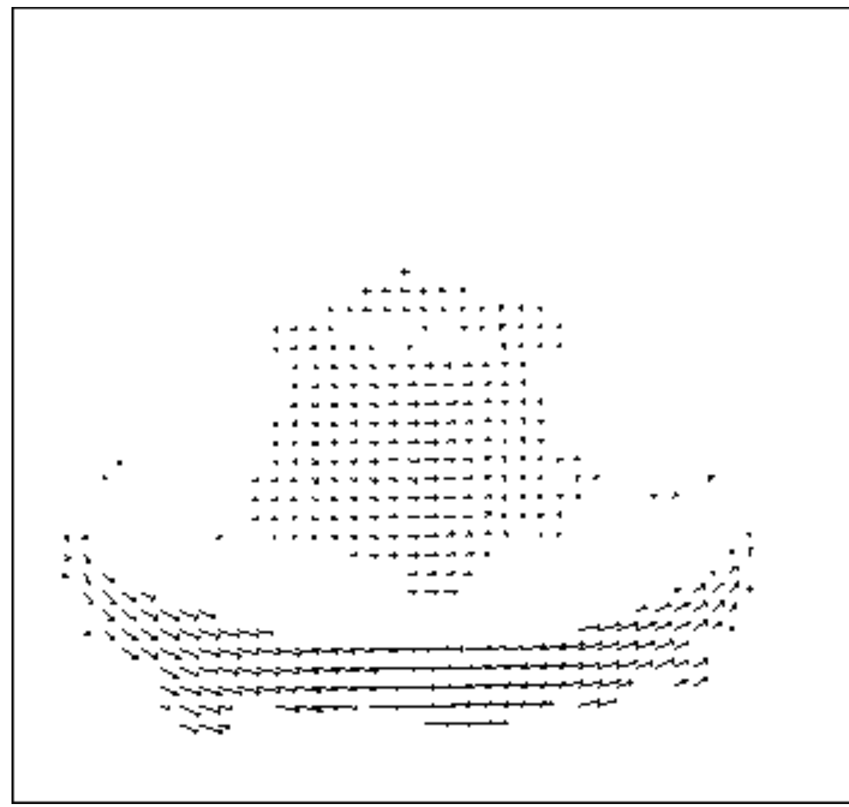
Uses of motion in computer vision

- 3D shape reconstruction
- Object segmentation
- Learning and tracking of dynamical models
- Event and activity recognition
- Self-supervised and predictive learning

Source: L. Lazebnik

Motion field

- The motion field is the projection of the 3D scene motion into the image

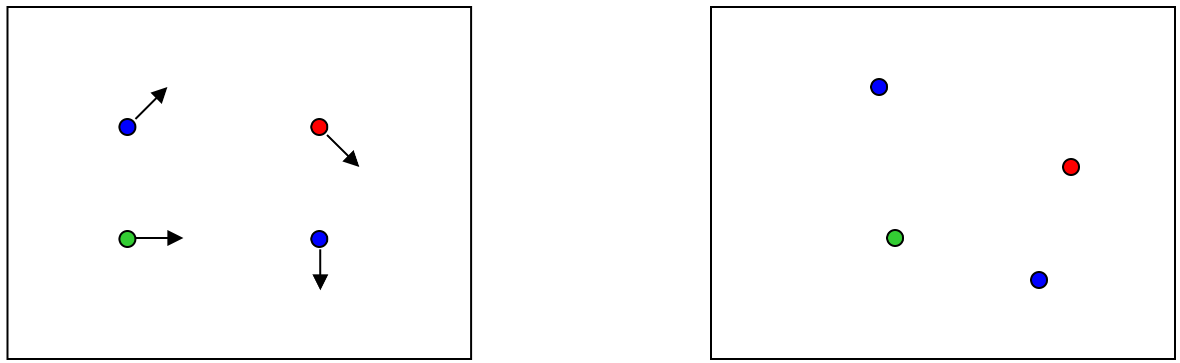
Source: L. Lazebnik

Optical flow

- **Definition**: optical flow is the *apparent* motion of brightness patterns in the image
- Ideally, optical flow would be the same as the motion field
- Have to be careful: apparent motion can be caused by lighting changes without any actual motion
  - Think of a uniform rotating sphere under fixed lighting vs. a stationary sphere under moving illumination

Source: L. Lazebnik

Estimating optical flow

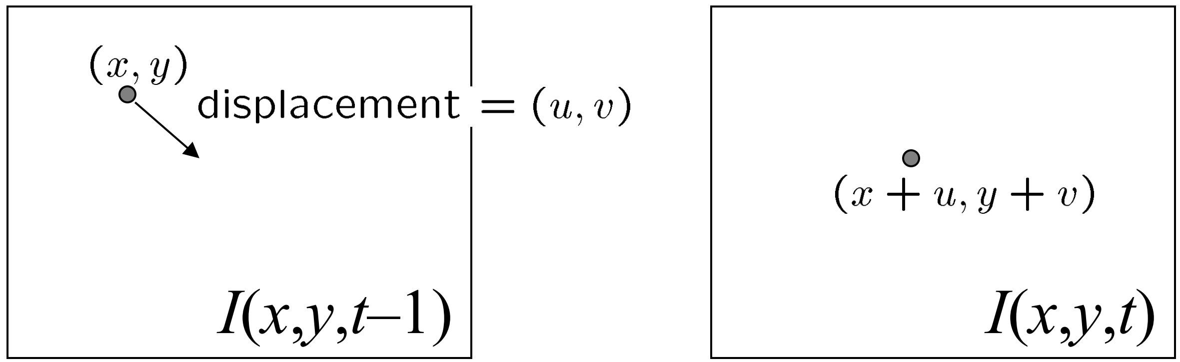


$I(x,y,t-1)$ $\quad$ $I(x,y,t)$

- Given two subsequent frames, estimate the apparent motion field $u(x,y)$ and $v(x,y)$ between them
- Key assumptions
  - **Brightness constancy:** projection of the same point looks the same in every frame
  - **Small motion:** points do not move very far
  - **Spatial coherence:** points move like their neighbors

Source: L. Lazebnik

The brightness constancy constraint



Brightness Constancy Equation:

$$I(x, y, t-1) = I(x + u(x, y), y + v(x, y), t)$$

Linearizing the right side using Taylor expansion:

$$I(x, y, t-1) \approx I(x, y, t) + I_x\, u(x, y) + I_y\, v(x, y)$$

Hence, $I_x\, u + I_y\, v + I_t \approx 0$

Source: L. Lazebnik

The brightness constancy constraint

$$I_x u + I_y v + I_t = 0$$

- How many equations and unknowns per pixel?
  - One equation, two unknowns
- What does this constraint mean?

$$\nabla I \cdot (u, v) + I_t = 0$$

- The component of the flow perpendicular to the gradient (i.e., parallel to the edge) is unknown!

Source: L. Lazebnik

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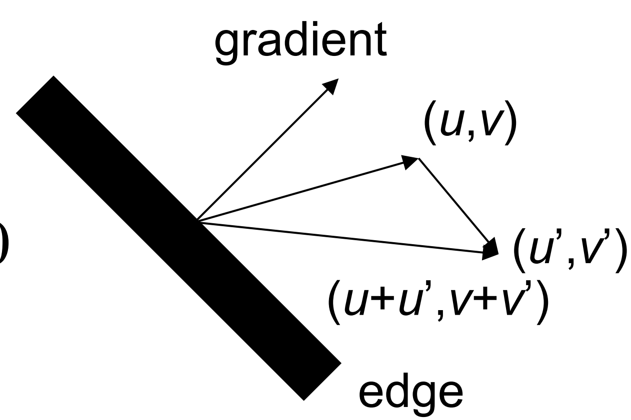
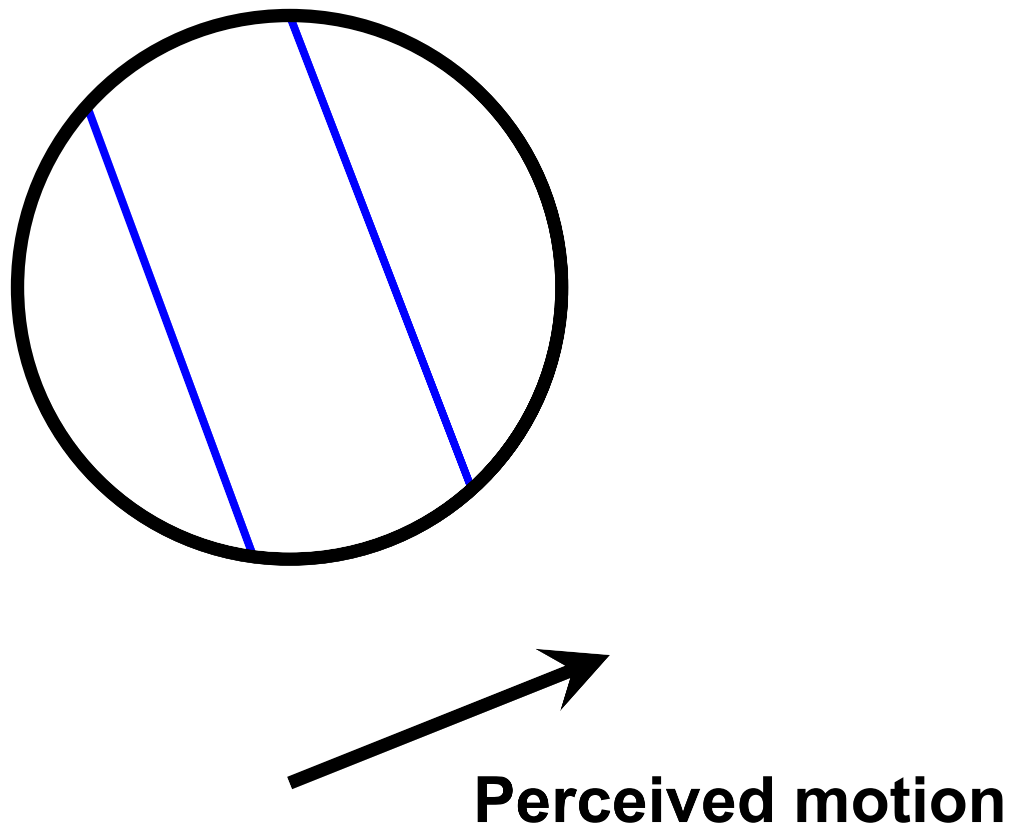
If (u, v) satisfies the equation,
so does ($u+u'$, $v+v'$) if $\nabla I \cdot (u', v') = 0$



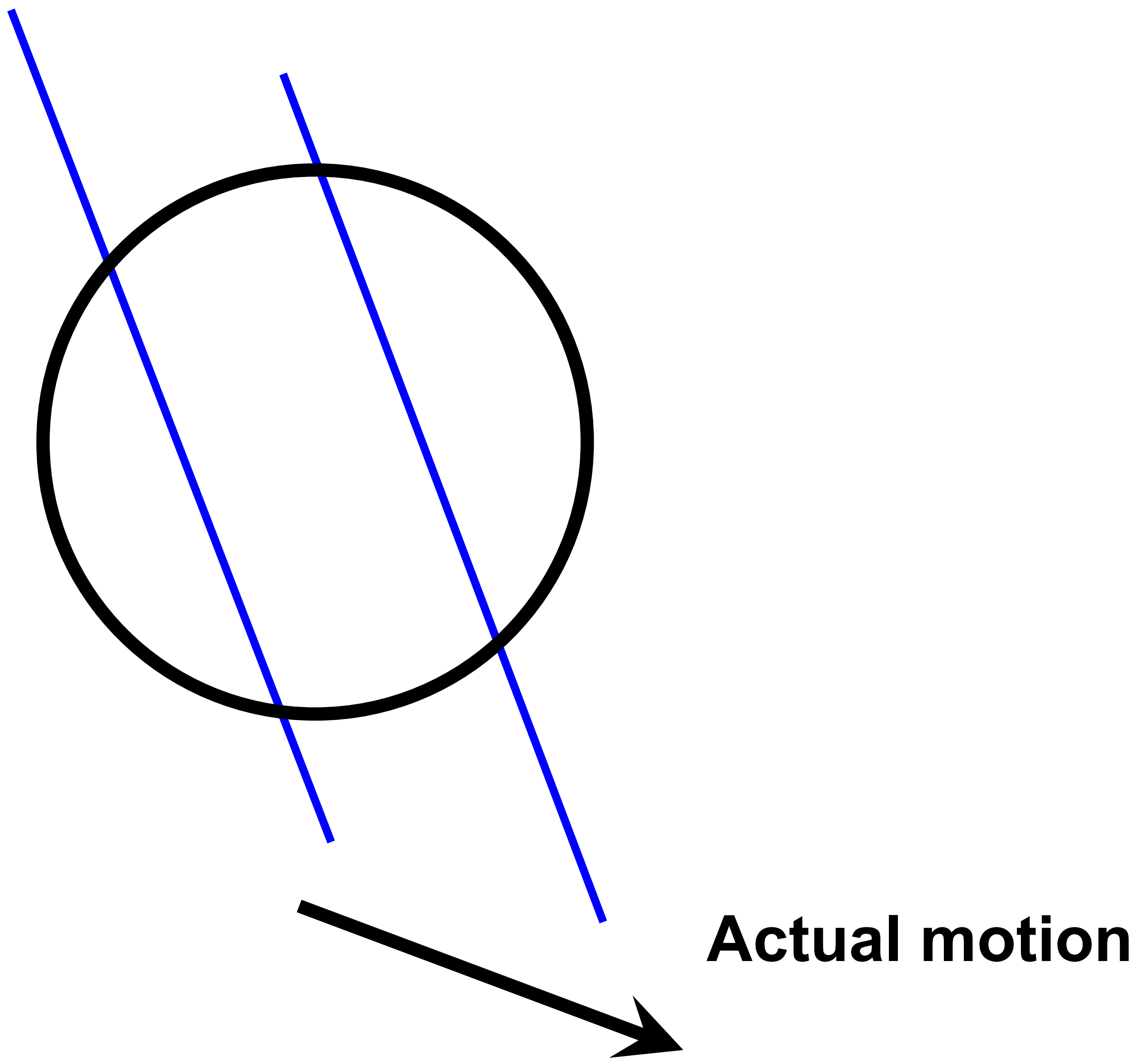
Source: L. Lazebnik

The aperture problem



Source: L. Lazebnik

The aperture problem



Source: L. Lazebnik

The barber pole illusion

http://en.wikipedia.org/wiki/Barberpole_illusion

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Solving the aperture problem

- How to get more equations for a pixel?
- **Spatial coherence constraint:** assume the pixel's neighbors have the same (u,v)
  - E.g., if we use a 5x5 window, that gives us 25 equations per pixel

$$\nabla I(\mathbf{x}_i)\cdot[u,v]+I_t(\mathbf{x}_i)=0$$

$$\begin{bmatrix} I_x(\mathbf{x}_1) & I_y(\mathbf{x}_1) \\ I_x(\mathbf{x}_2) & I_y(\mathbf{x}_2) \\ \vdots & \vdots \\ I_x(\mathbf{x}_n) & I_y(\mathbf{x}_n) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = -\begin{bmatrix} I_t(\mathbf{x}_1) \\ I_t(\mathbf{x}_2) \\ \vdots \\ I_t(\mathbf{x}_n) \end{bmatrix}$$

B. Lucas and T. Kanade. An iterative image registration technique with an application to stereo vision. In *Proceedings of the International Joint Conference on Artificial Intelligence*, pp. 674–679, 1981.

Source: L. Lazebnik

Lucas-Kanade flow

- Linear least squares problem:

$$\begin{bmatrix} I_x(\mathbf{x}_1) & I_y(\mathbf{x}_1) \\ I_x(\mathbf{x}_2) & I_y(\mathbf{x}_2) \\ \vdots & \vdots \\ I_x(\mathbf{x}_n) & I_y(\mathbf{x}_n) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = -\begin{bmatrix} I_t(\mathbf{x}_1) \\ I_t(\mathbf{x}_2) \\ \vdots \\ I_t(\mathbf{x}_n) \end{bmatrix}$$

- When is this system solvable?

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$$\underset{n\times 2}{\mathbf{A}} \; \underset{2\times 1}{\mathbf{d}} = \underset{n\times 1}{\mathbf{b}}$$

- Solution given by $(\mathbf{A}^T\mathbf{A})\mathbf{d} = \mathbf{A}^T\mathbf{b}$

$$\begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}$$

(summations are over all pixels in the window)

M = A^{T}A is the second moment matrix!

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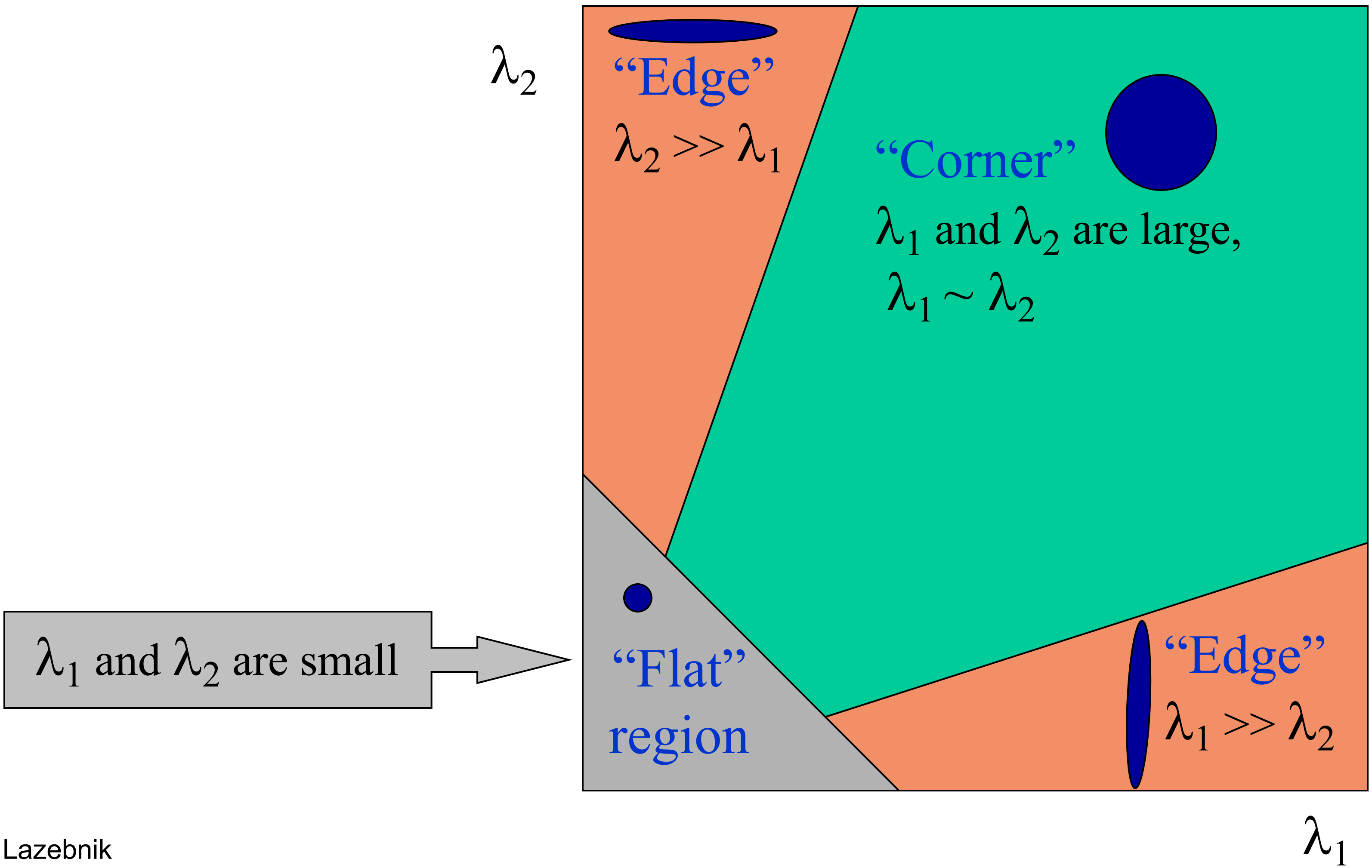
Source: L. Lazebnik

Recall: second moment matrix

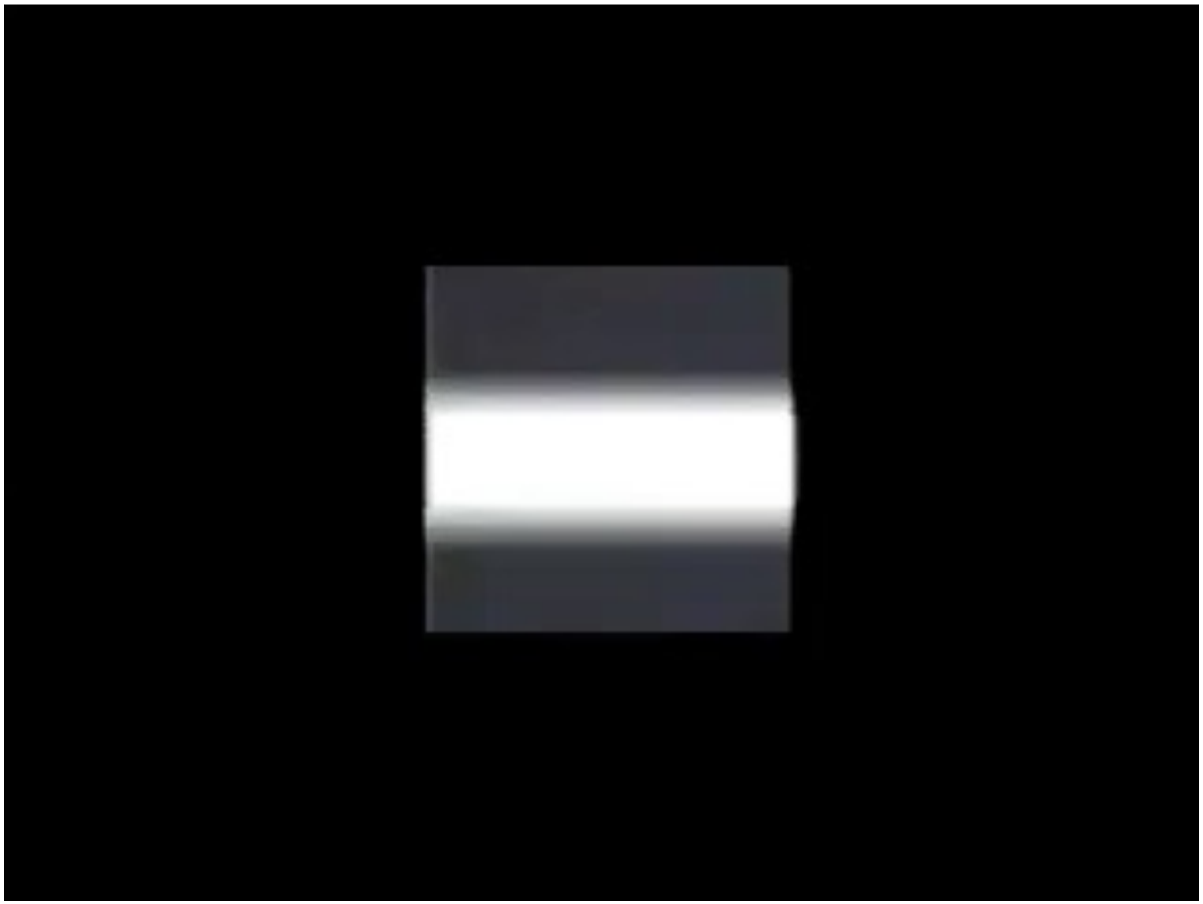
- Estimation of optical flow is well-conditioned precisely for regions with high "cornerness":



Source: L. Lazebnik

Conditions for solvability

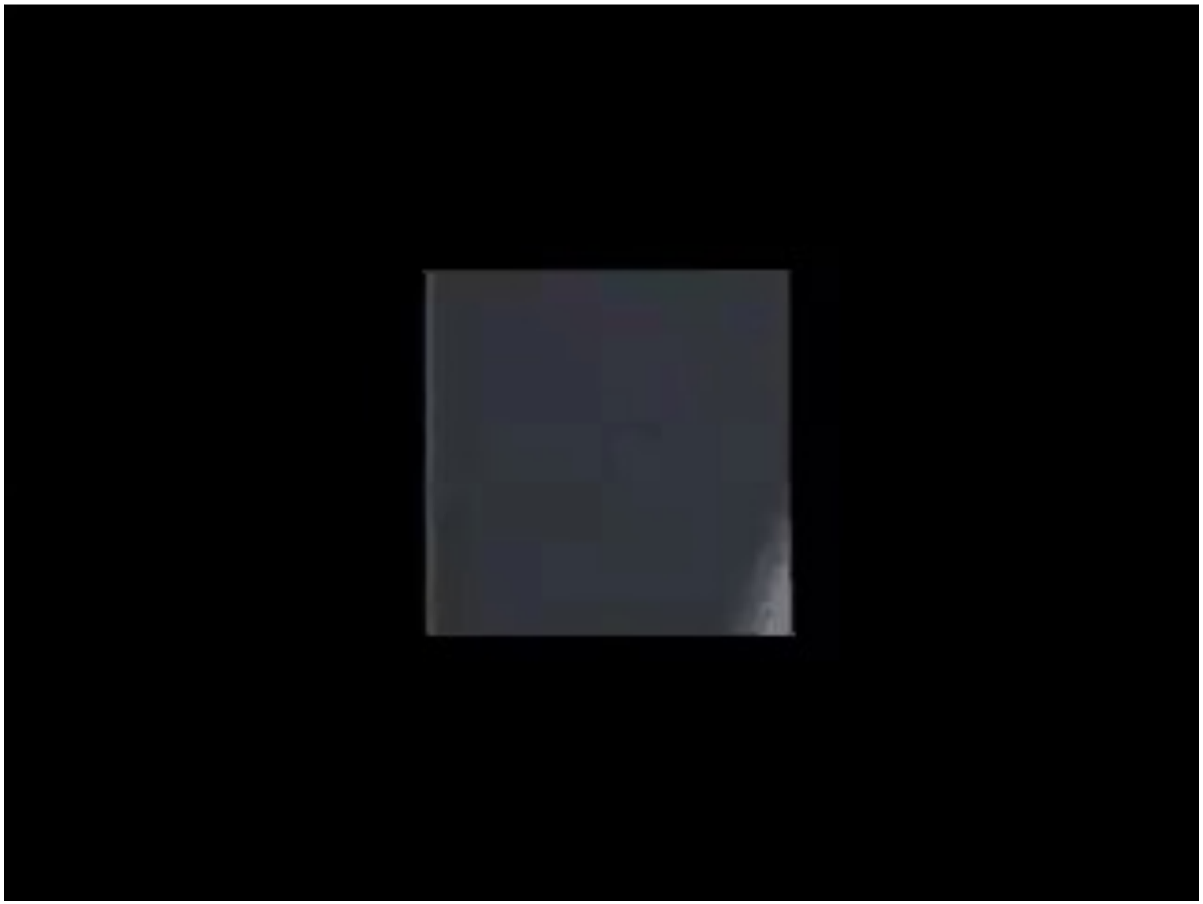
- "Bad" case: single straight edge



Source: L. Lazebnik

Conditions for solvability

- “Good” case

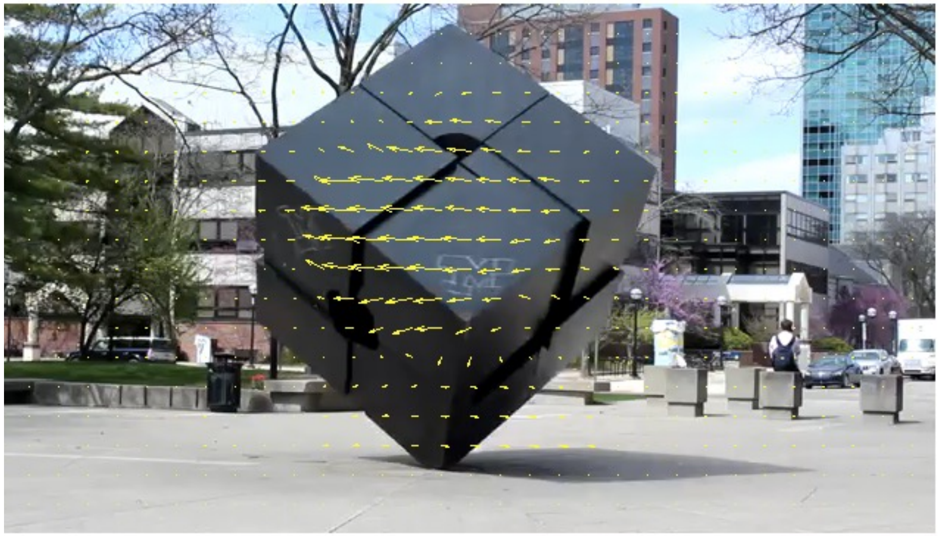


Source: L. Lazebnik

Lucas-Kanade flow example

Input frames

Output



Source: [MATLAB Central File Exchange]

Source: L. Lazebnik

Errors in Lucas-Kanade

- The motion is large (larger than a pixel)
- A point does not move like its neighbors
- Brightness constancy does not hold

Source: L. Lazebnik

“Flower garden” example

*** From Khurram Hassan-Shafique CAP5415 Computer Vision 2003**

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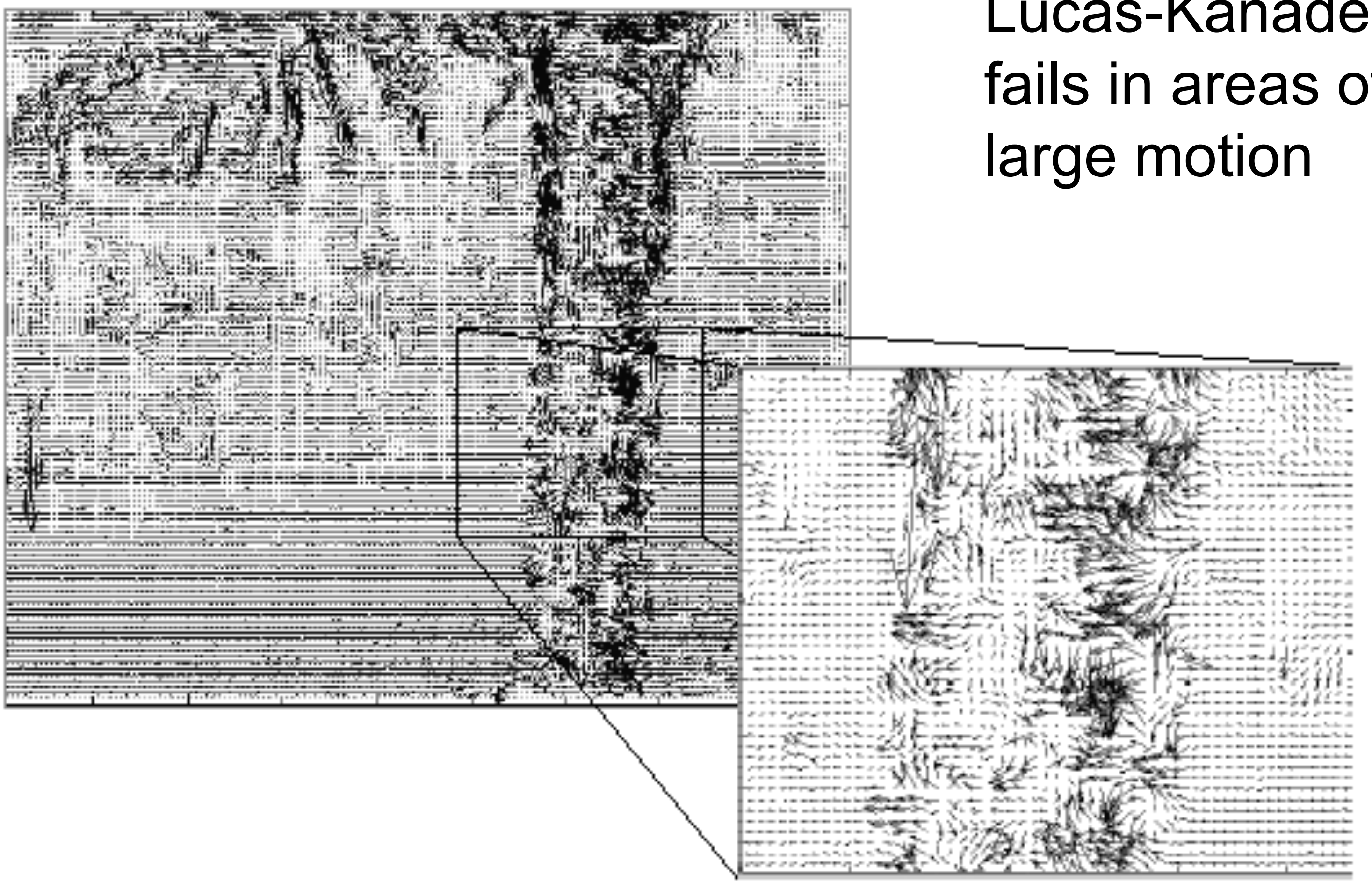
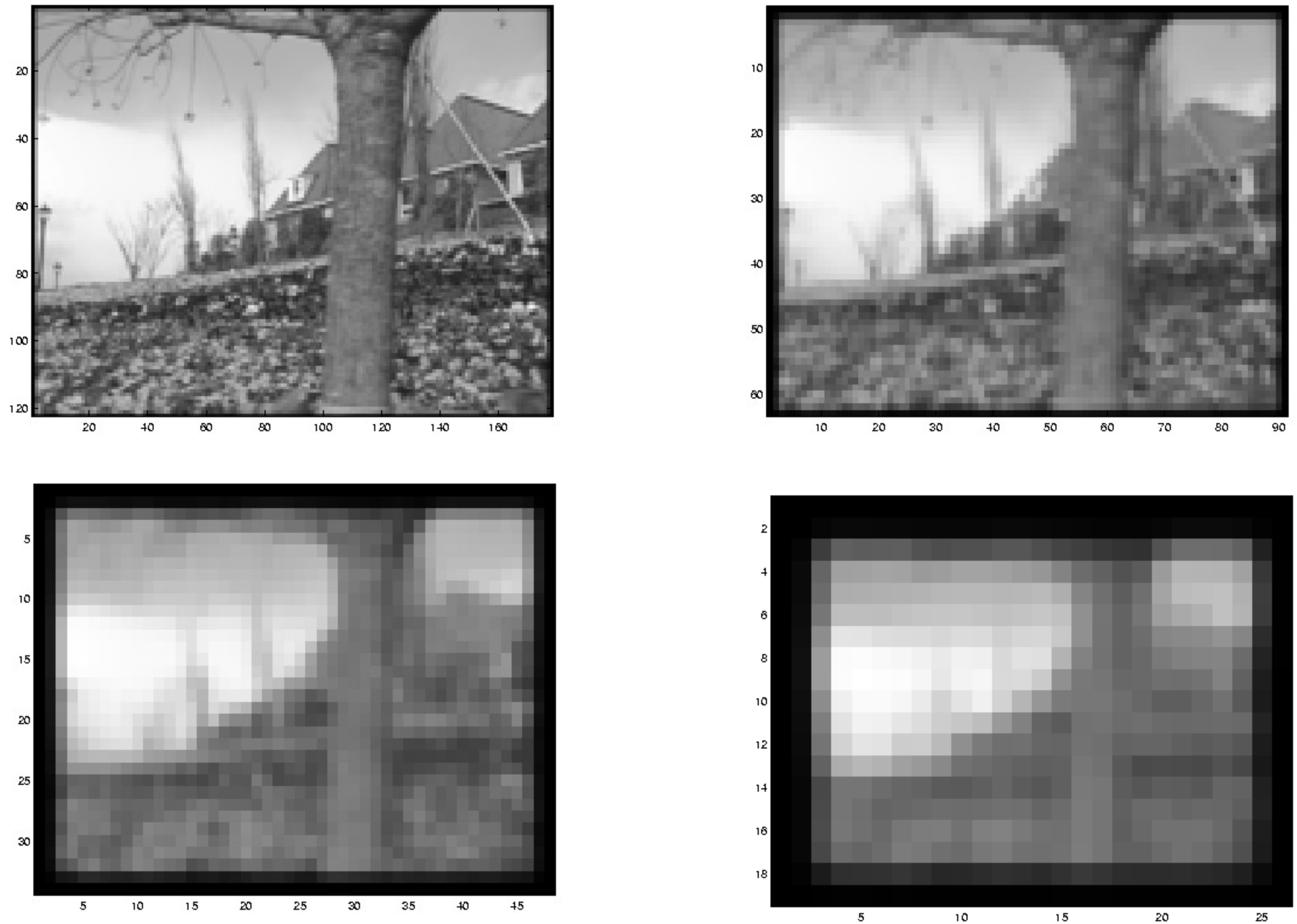


Lucas-Kanade fails in areas of large motion

Source: L. Lazebnik

*** From Khurram Hassan-Shafique CAP5415 Computer Vision 2003**

Multi-resolution estimation

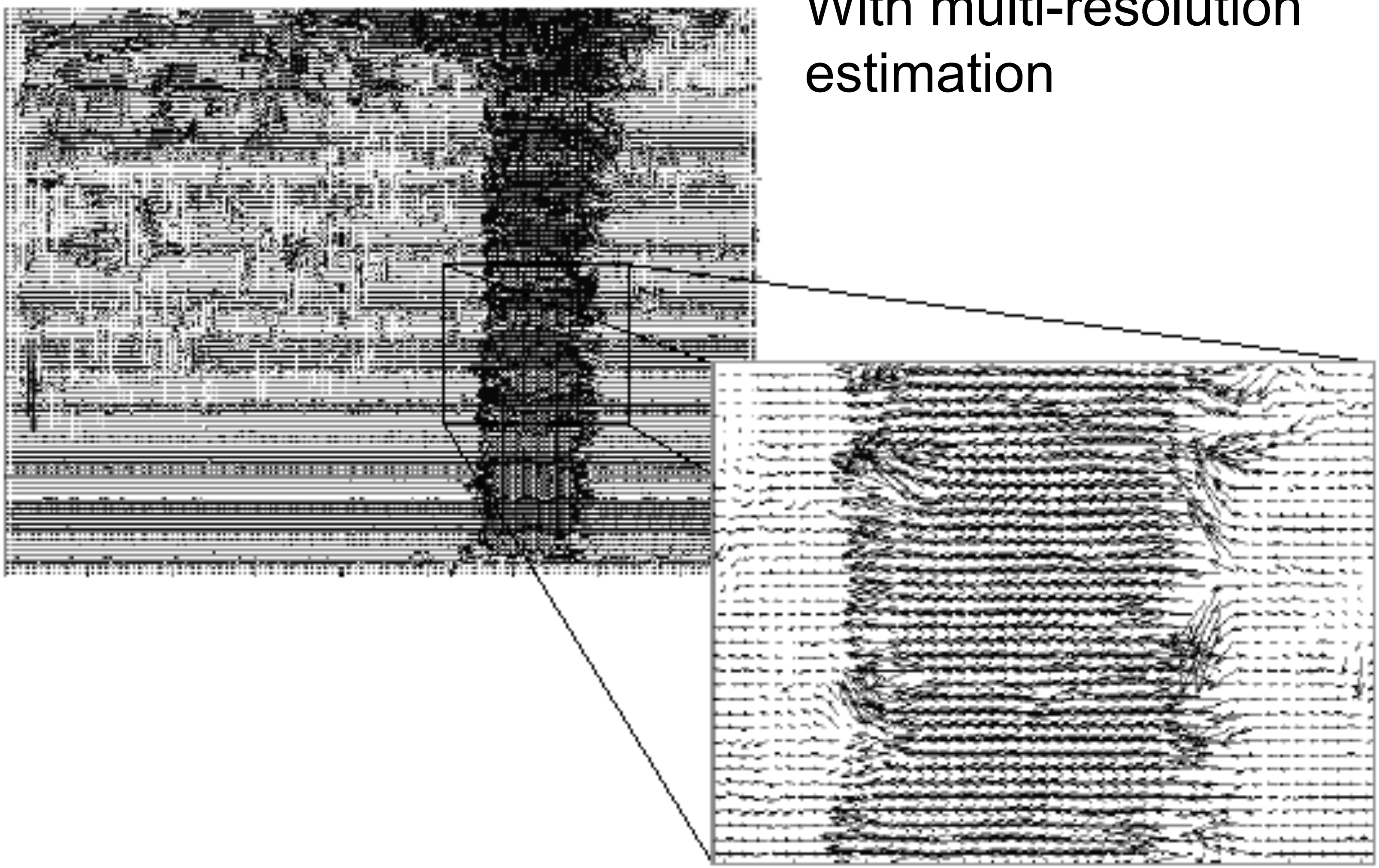


Source: L. Lazebnik

*** From Khurram Hassan-Shafique CAP5415 Computer Vision 2003**

Multi-resolution estimation

With multi-resolution estimation

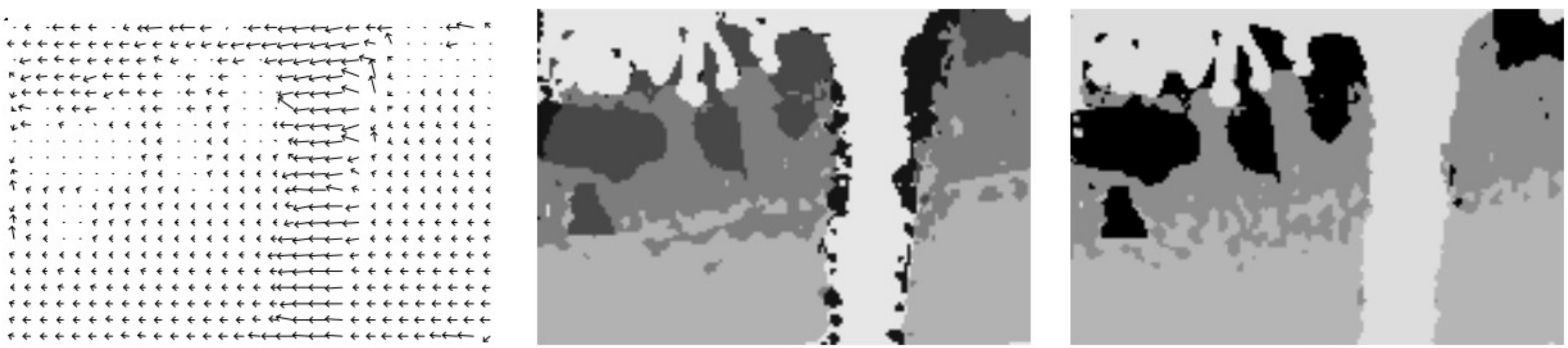


Source: L. Lazebnik

* From Khurram Hassan-Shafique CAP5415 Computer Vision 2003

Fixing the errors in Lucas-Kanade

- The motion is large (larger than a pixel)
  - Multi-resolution estimation, iterative refinement
  - Feature matching
- A point does not move like its neighbors
  - Motion segmentation



(a) (b) (c)

Figure 11: (a) The optic flow from multi-scale gradient method. (b) Segmentation obtained by clustering optic flow into affine motion regions. (c) Segmentation from consistency checking by image warping. Representing moving images with layers.

J. Wang and E. Adelson, Representing Moving Images with Layers, IEEE Transactions on Image Processing, 1994

Source: L. Lazebnik

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