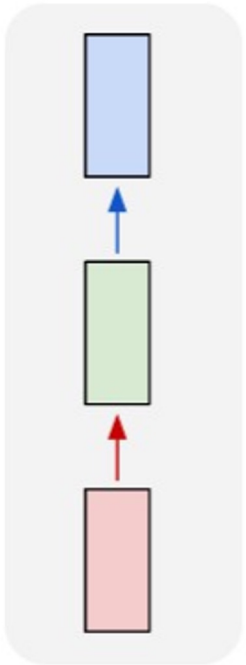


Sequence Modeling (RNNs)

So far: “Feedforward” Neural Networks

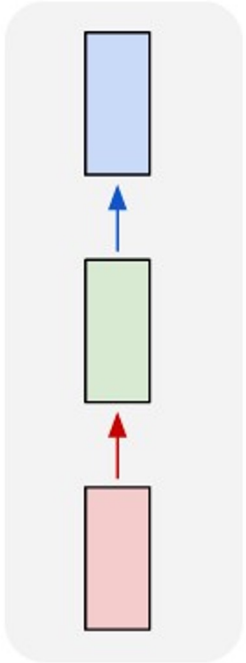
one to one



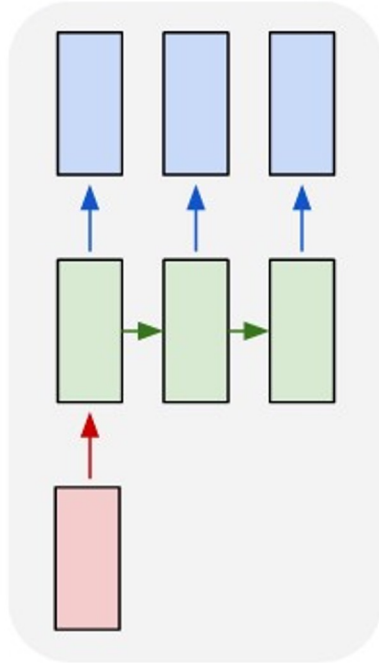
e.g. **Image classification**
Image -> Label

Recurrent Neural Networks: Process Sequences

one to one



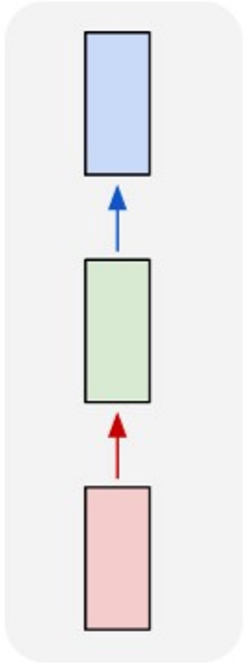
one to many



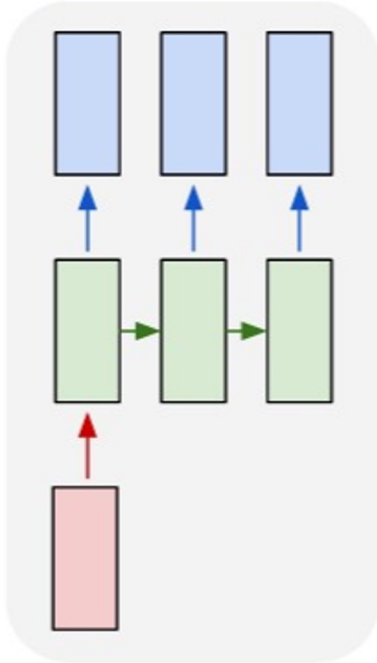
e.g. **Image Captioning:**
Image -> sequence of words

Recurrent Neural Networks: Process Sequences

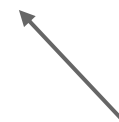
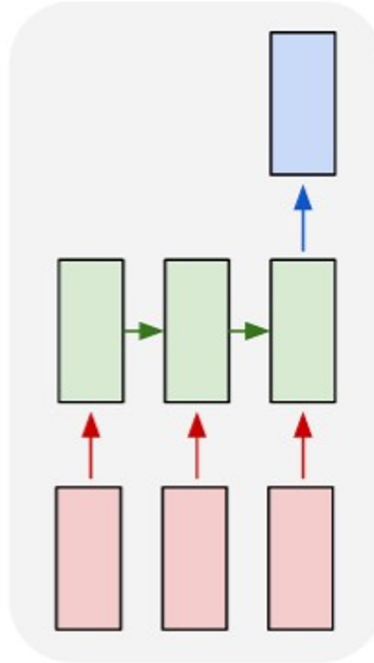
one to one



one to many



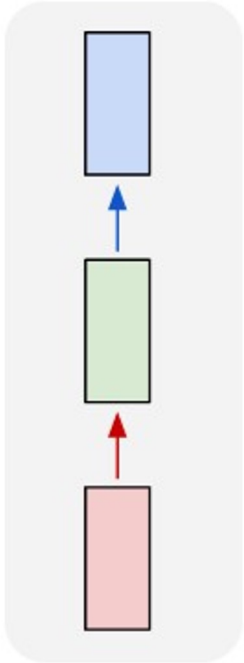
many to one



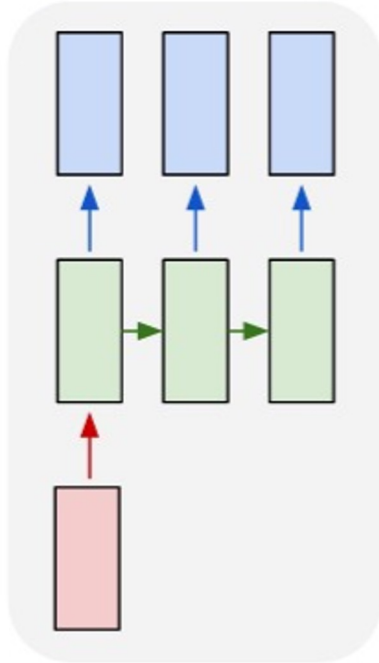
e.g. **Video classification:**
Sequence of images -> label

Recurrent Neural Networks: Process Sequences

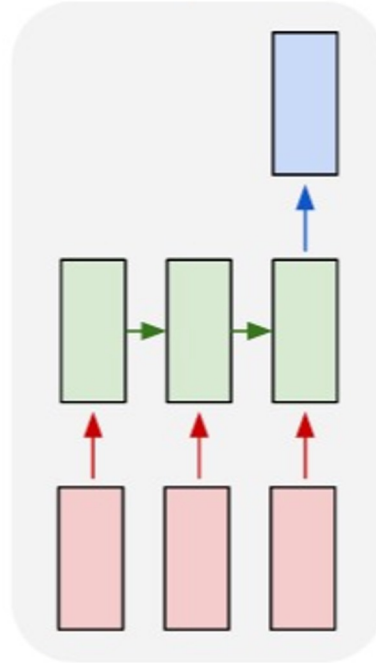
one to one



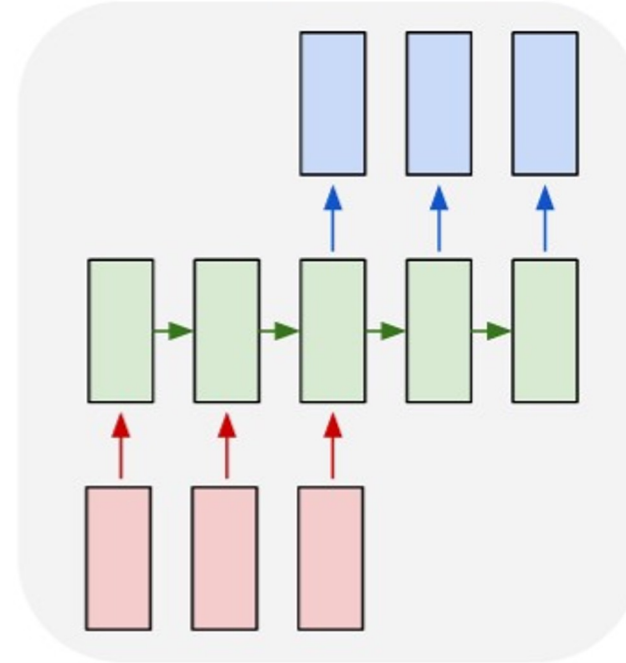
one to many



many to one



many to many

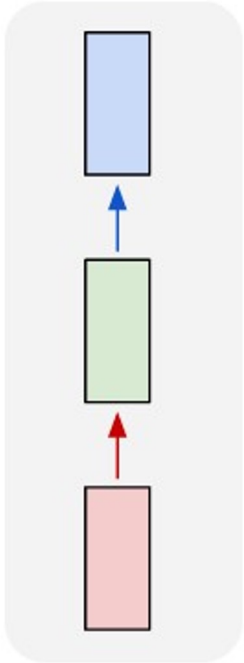


e.g. **Machine Translation:**

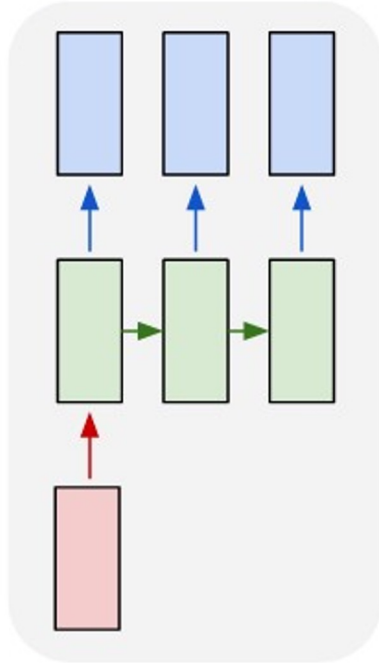
Sequence of words -> Sequence of words

Recurrent Neural Networks: Process Sequences

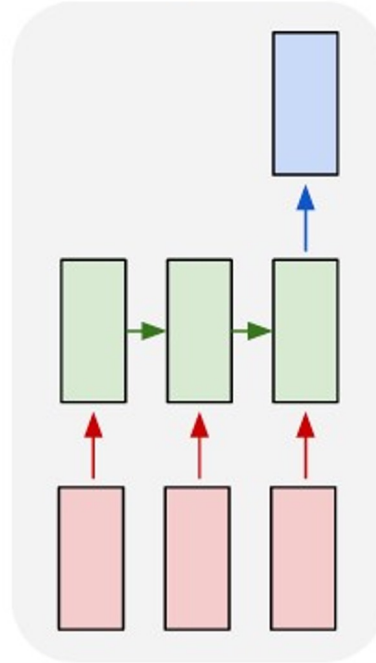
one to one



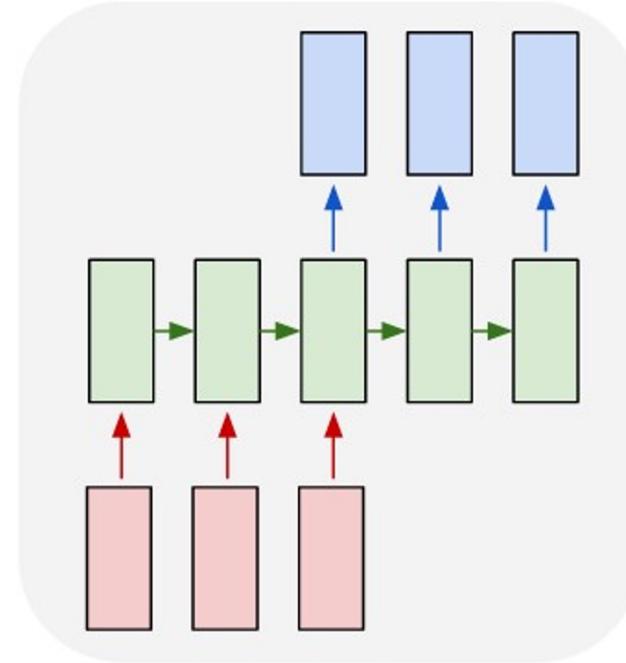
one to many



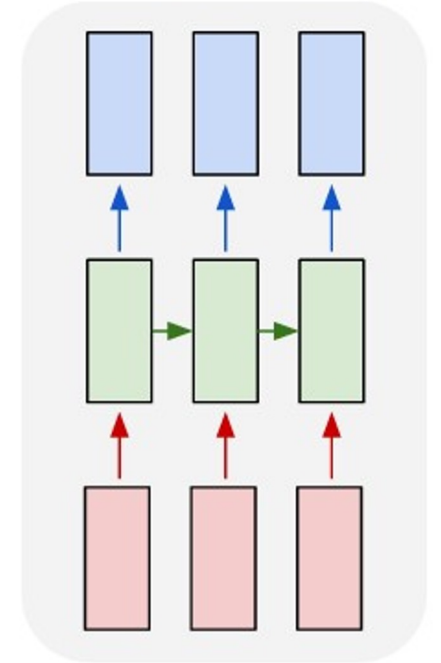
many to one



many to many



many to many



e.g. **Per-frame video classification:**
Sequence of images -> Sequence of labels

Sequential Processing of Non-Sequential Data

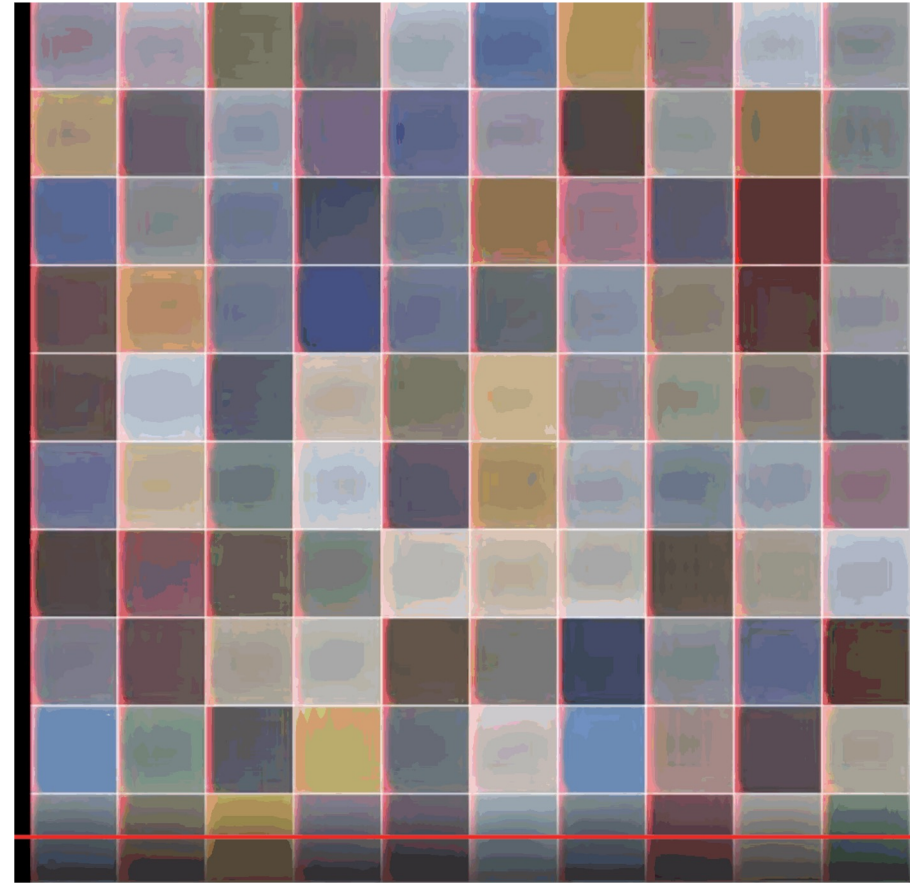
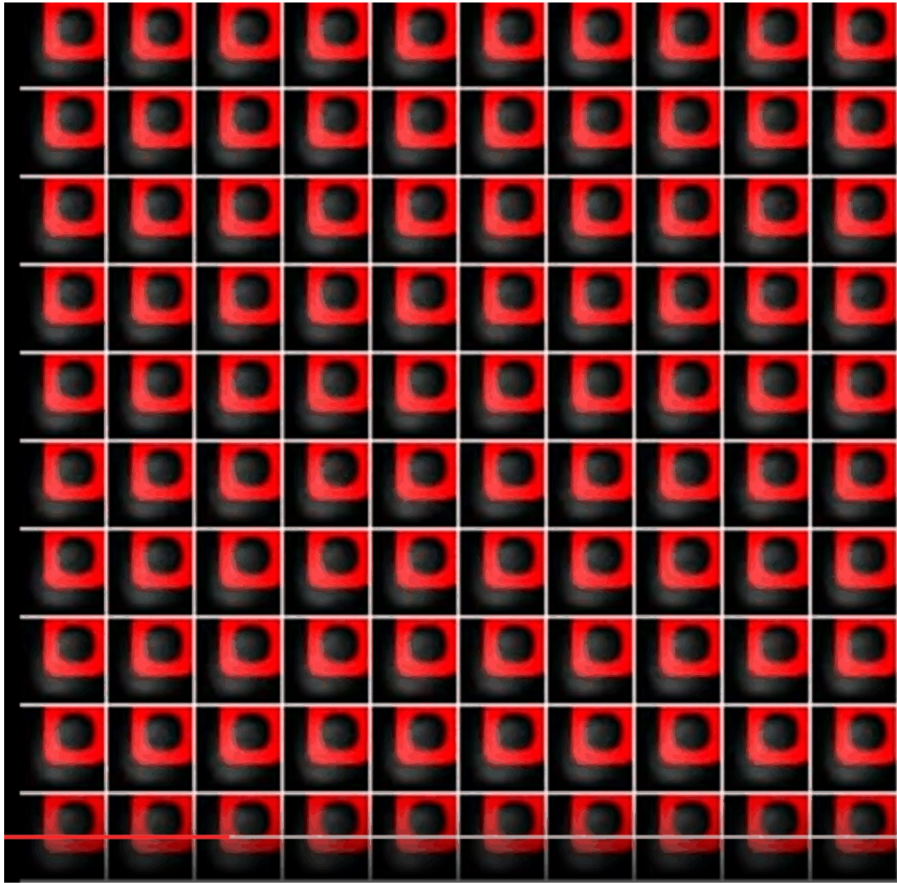
Classify images by taking a series of “glimpses”



Ba, Mnih, and Kavukcuoglu, “Multiple Object Recognition with Visual Attention”, ICLR 2015.
Gregor et al, “DRAW: A Recurrent Neural Network For Image Generation”, ICML 2015

Sequential Processing of Non-Sequential Data

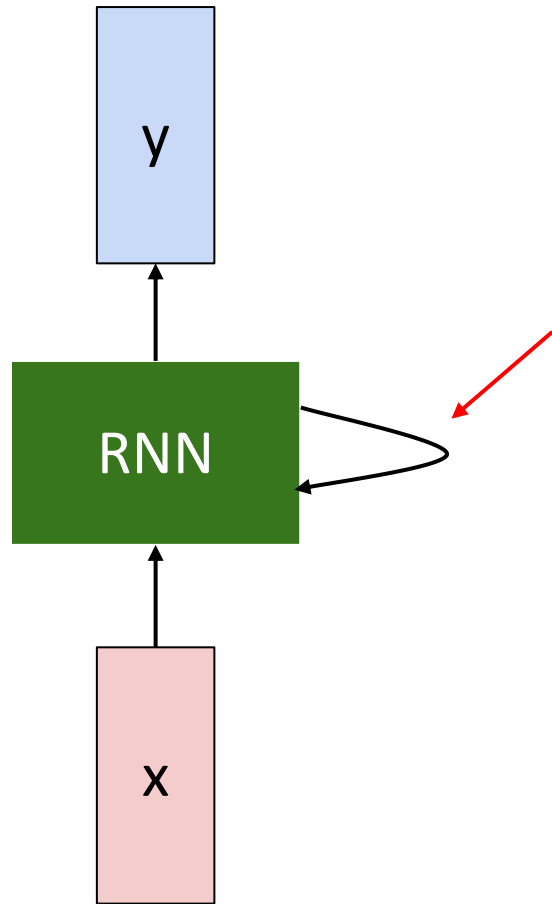
Generate images one piece at a time!



Gregor et al, "DRAW: A Recurrent Neural Network For Image Generation", ICML 2015

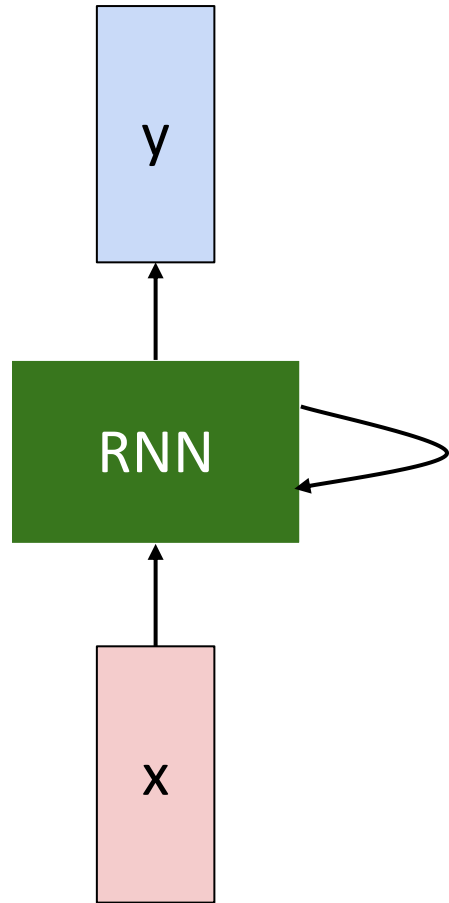
Slide from Justin Johnson

Recurrent Neural Networks



Key idea: RNNs have an “internal state” that is updated as a sequence is processed

Recurrent Neural Networks



We can process a sequence of vectors \mathbf{x} by applying a **recurrence formula** at every time step:

$$h_t = f_W(h_{t-1}, x_t)$$

new state

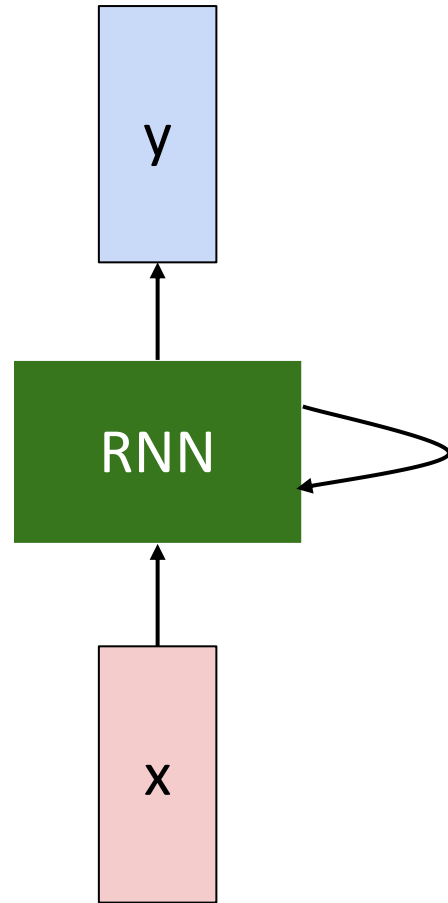
old state

input vector at
some time step

some function
with parameters W

Recurrent Neural Networks

Notice: the same function and the same set of parameters are used at every time step.



We can process a sequence of vectors \mathbf{x} by applying a **recurrence formula** at every time step:

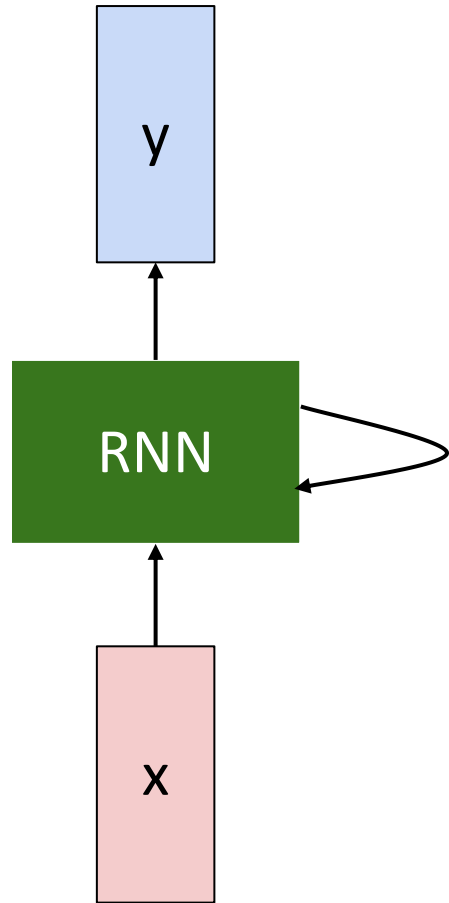
$$h_t = f_W(h_{t-1}, x_t)$$

new state some function with parameters W old state input vector at some time step

(Vanilla) Recurrent Neural Networks

The state consists of a single “hidden” vector \mathbf{h} :

$$h_t = f_W(h_{t-1}, x_t)$$



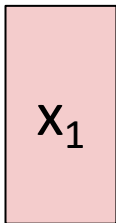
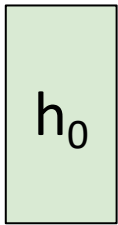
$$h_t = \tanh(W_{hh}h_{t-1} + W_{xh}x_t + b_h)$$

$$y_t = W_{hy}h_t + b_y$$

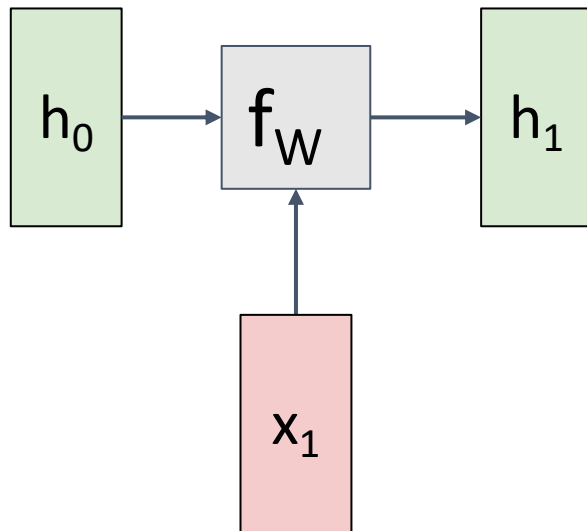
Sometimes called a “Vanilla RNN” or an “Elman RNN” after Prof. Jeffrey Elman

RNN Computational Graph

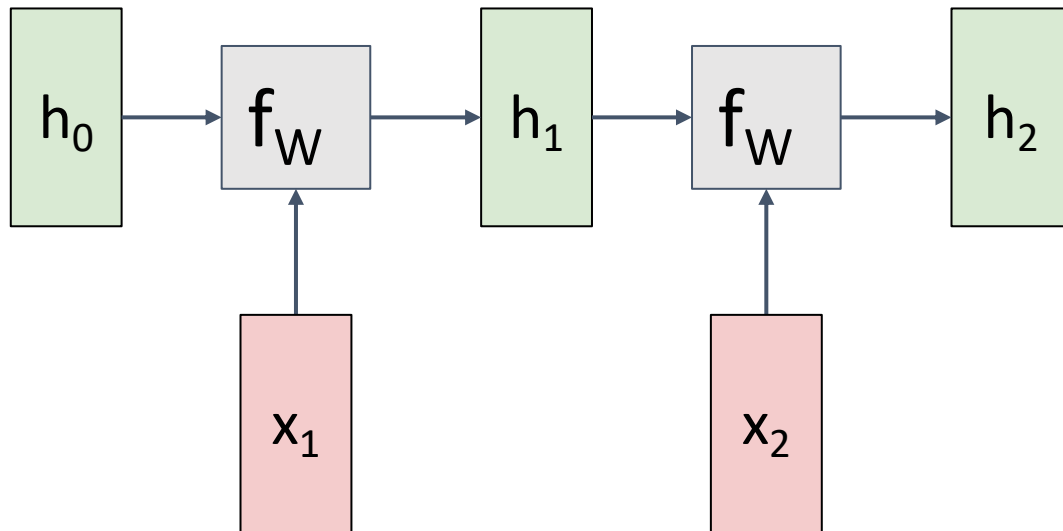
Initial hidden state
Either set to all 0,
Or learn it



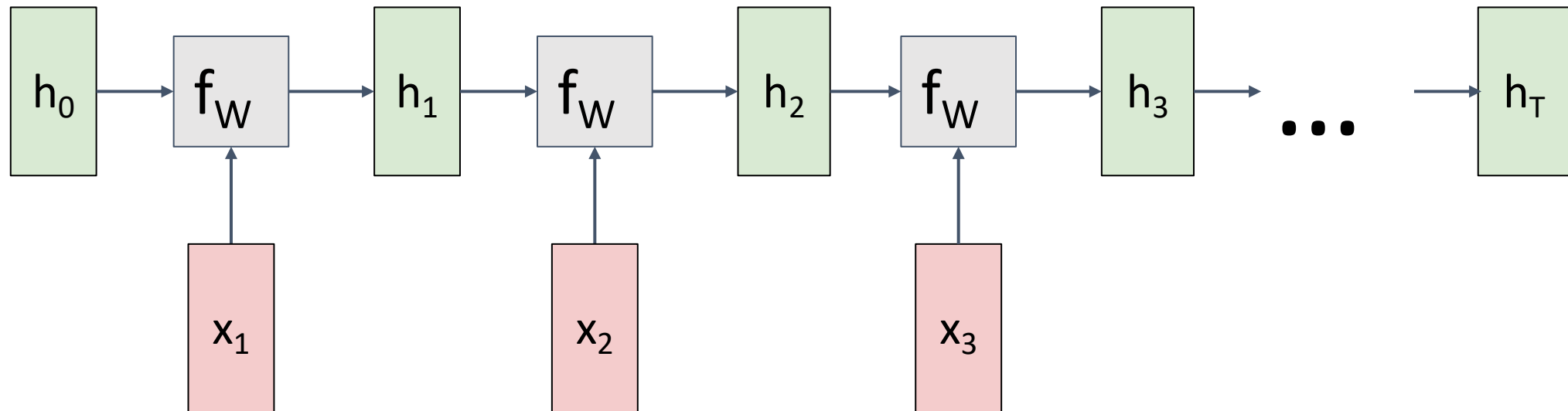
RNN Computational Graph



RNN Computational Graph

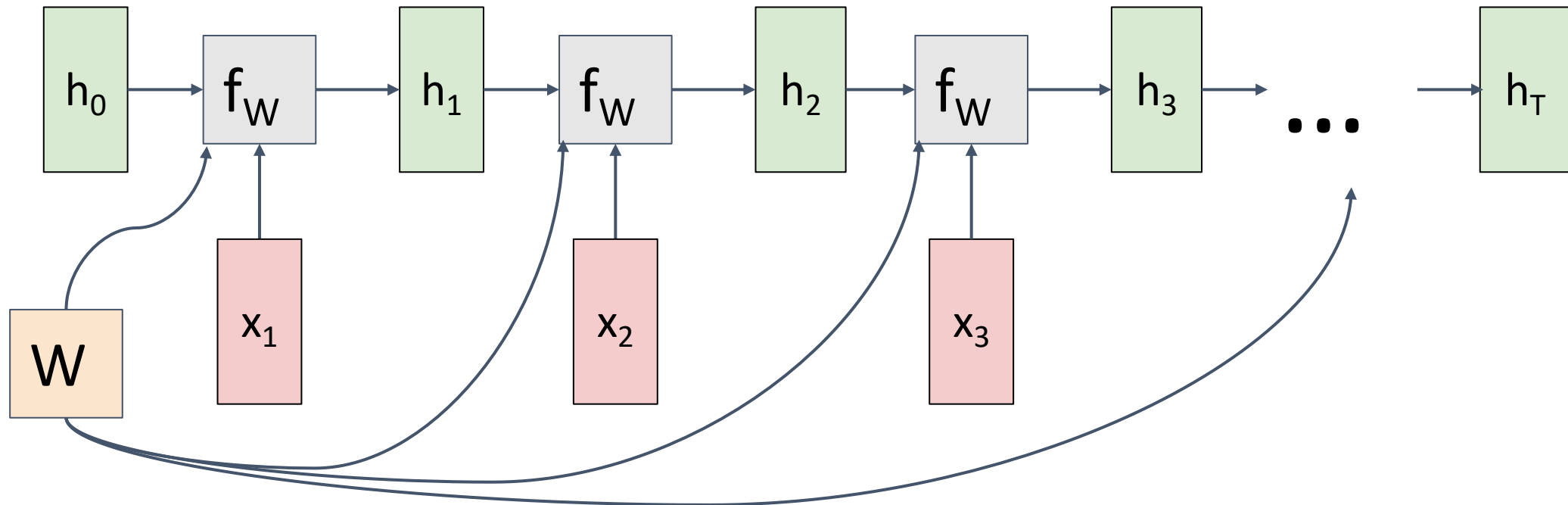


RNN Computational Graph

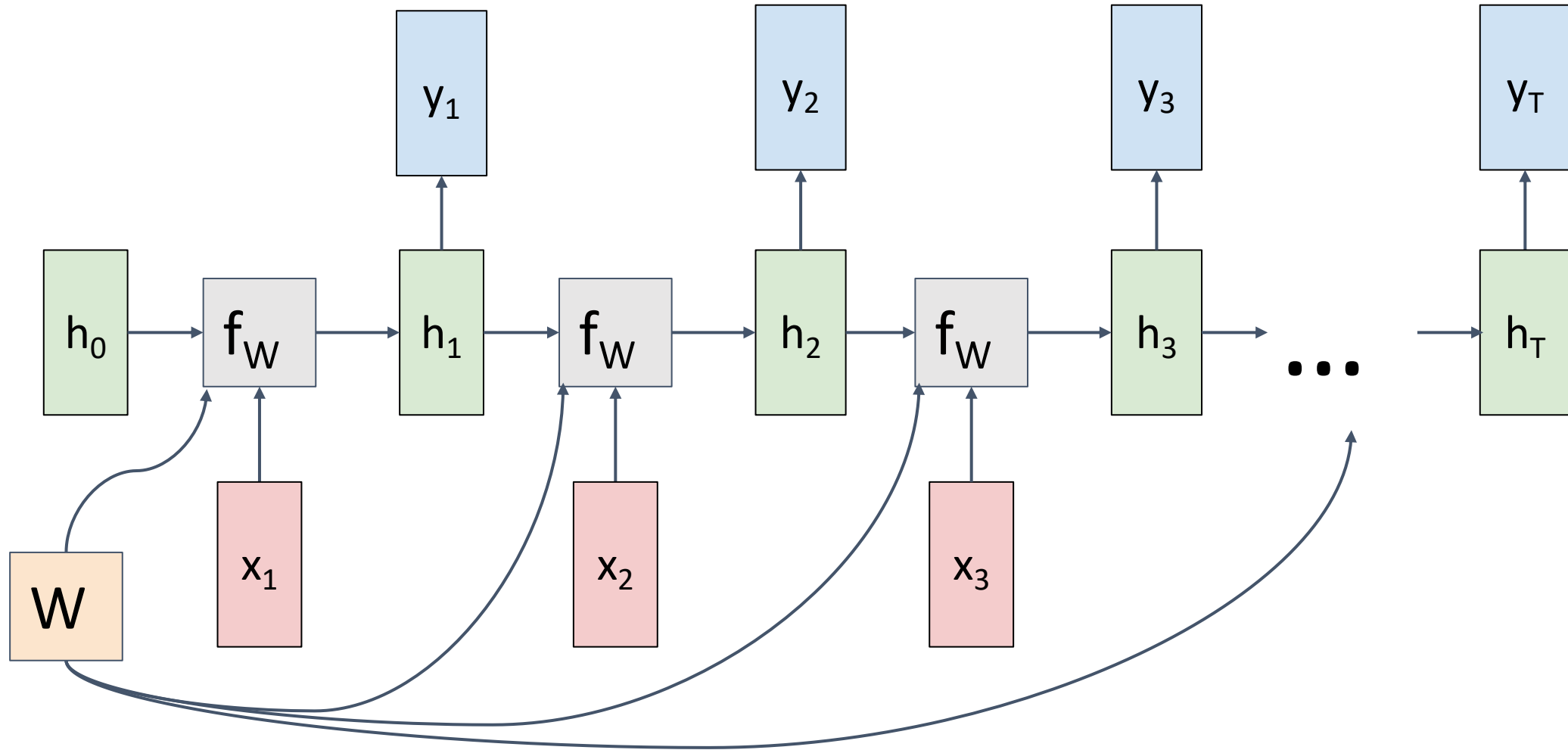


RNN Computational Graph

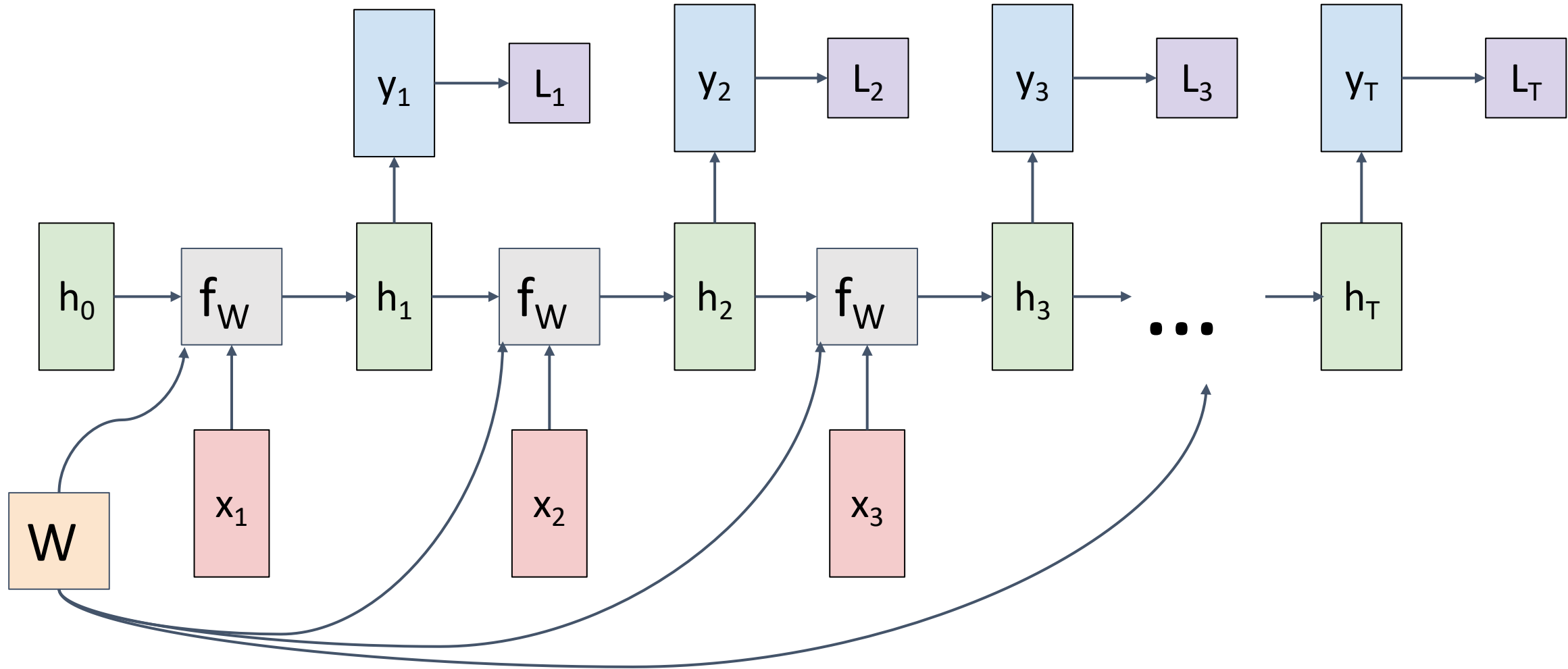
Re-use the same weight matrix at every time-step



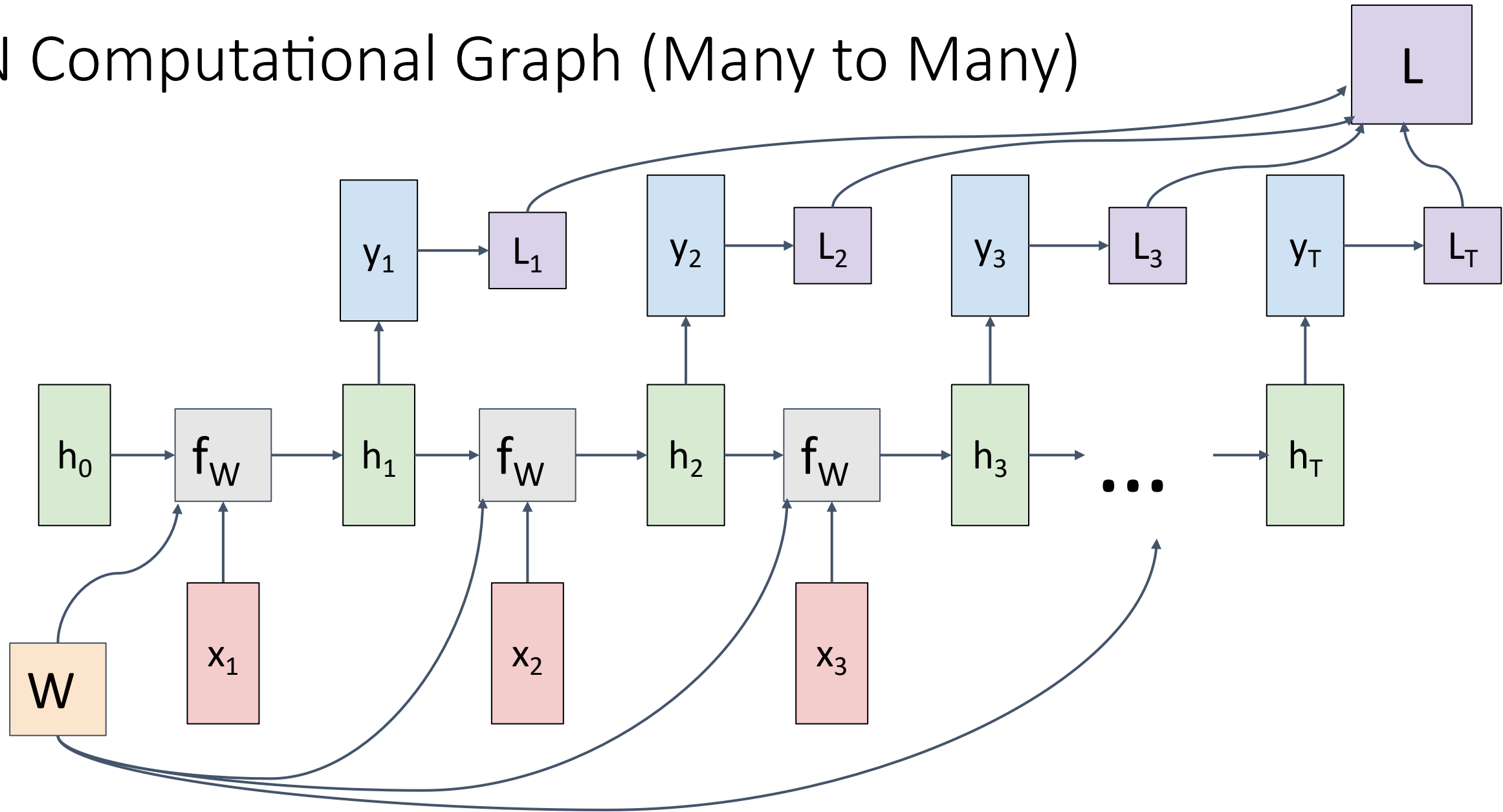
RNN Computational Graph (Many to Many)



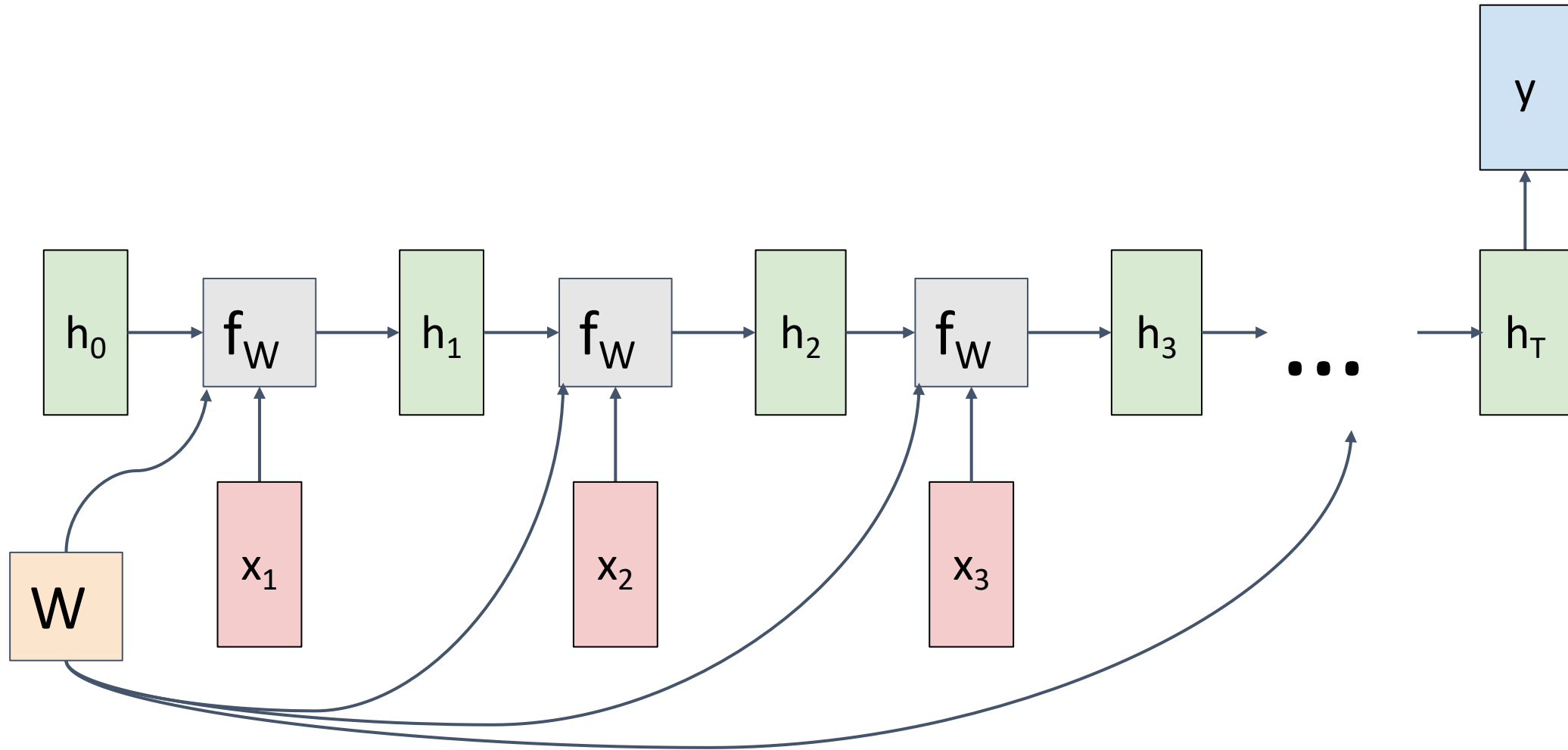
RNN Computational Graph (Many to Many)



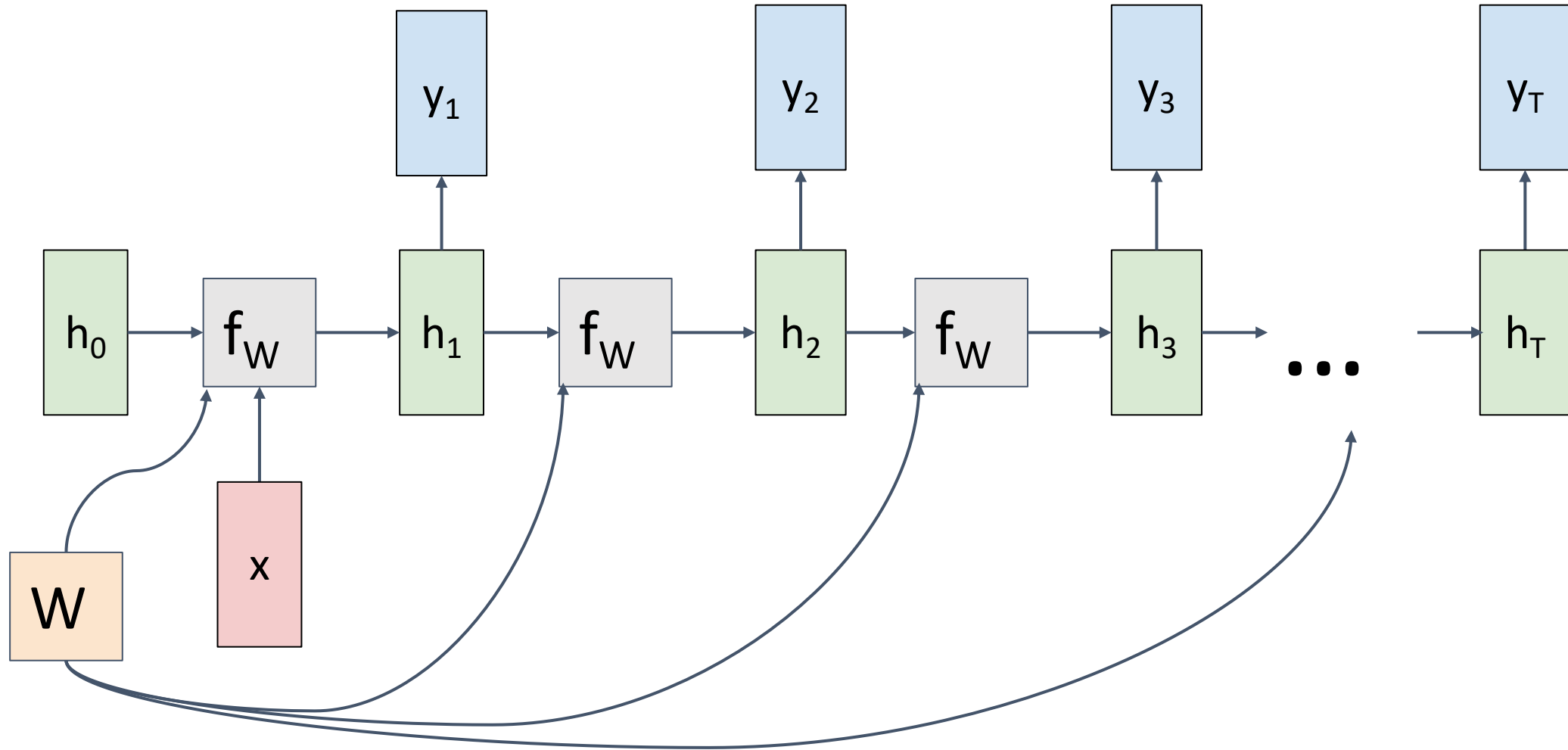
RNN Computational Graph (Many to Many)



RNN Computational Graph (Many to One)

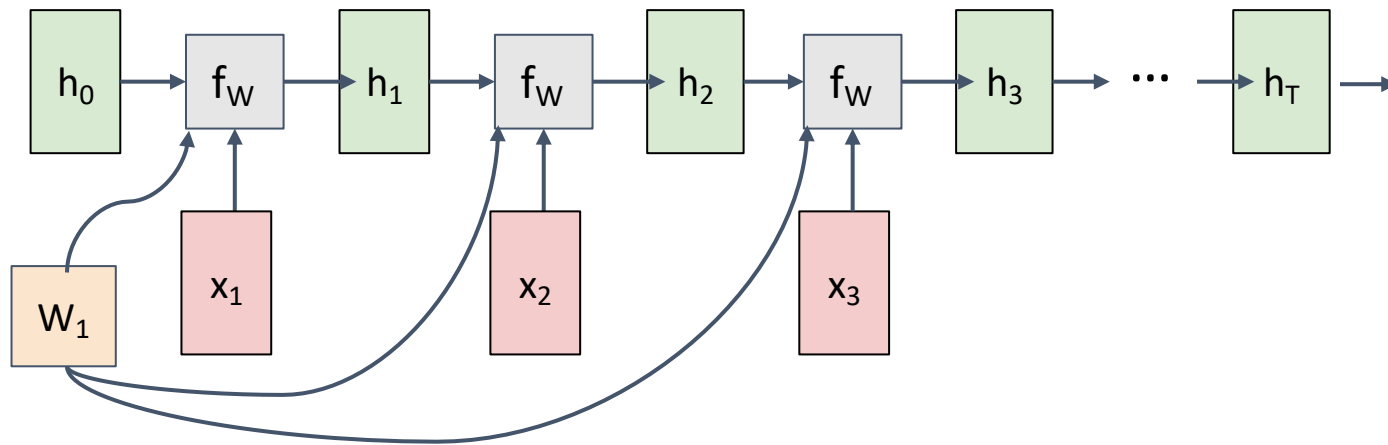


RNN Computational Graph (One to Many)



Sequence to Sequence (seq2seq) (Many to one) + (One to many)

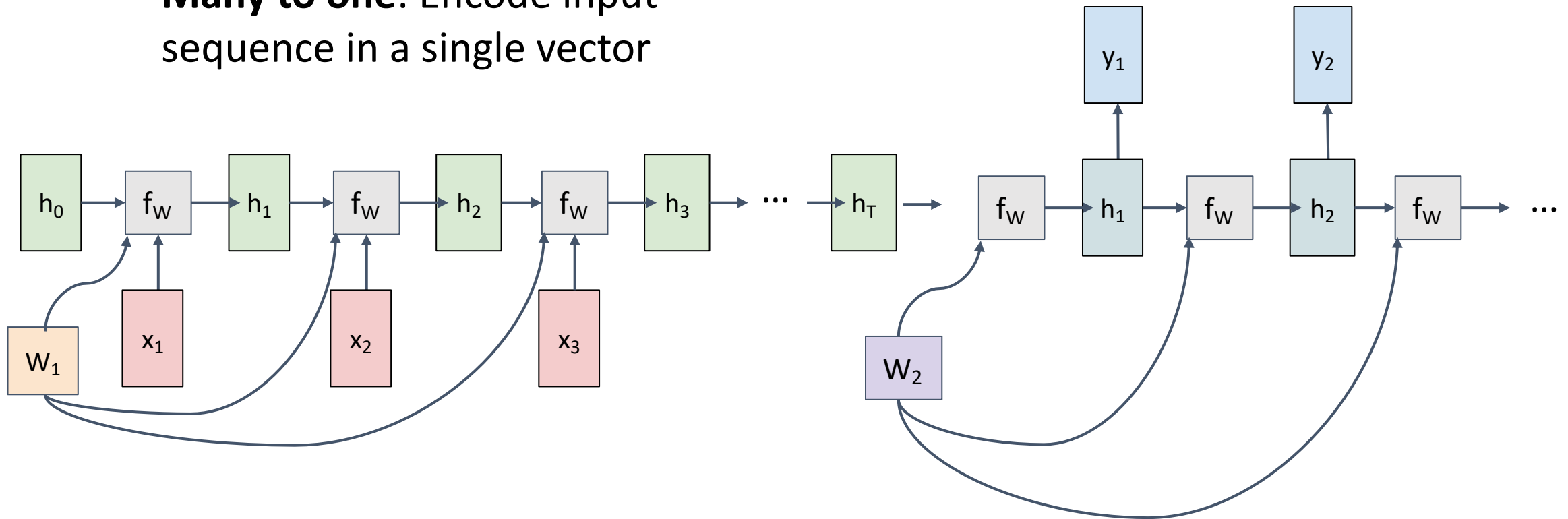
Many to one: Encode input sequence in a single vector



Sequence to Sequence (seq2seq) (Many to one) + (One to many)

One to many: Produce output sequence from single input vector

Many to one: Encode input sequence in a single vector

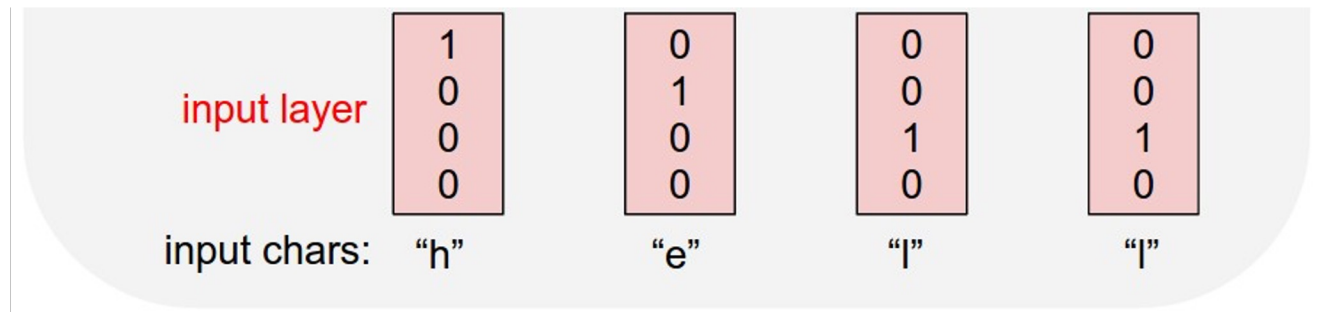


Example: Language Modeling

Given characters 1, 2, ..., t-1,
model predicts character t

Training sequence: "hello"

Vocabulary: [h, e, l, o]



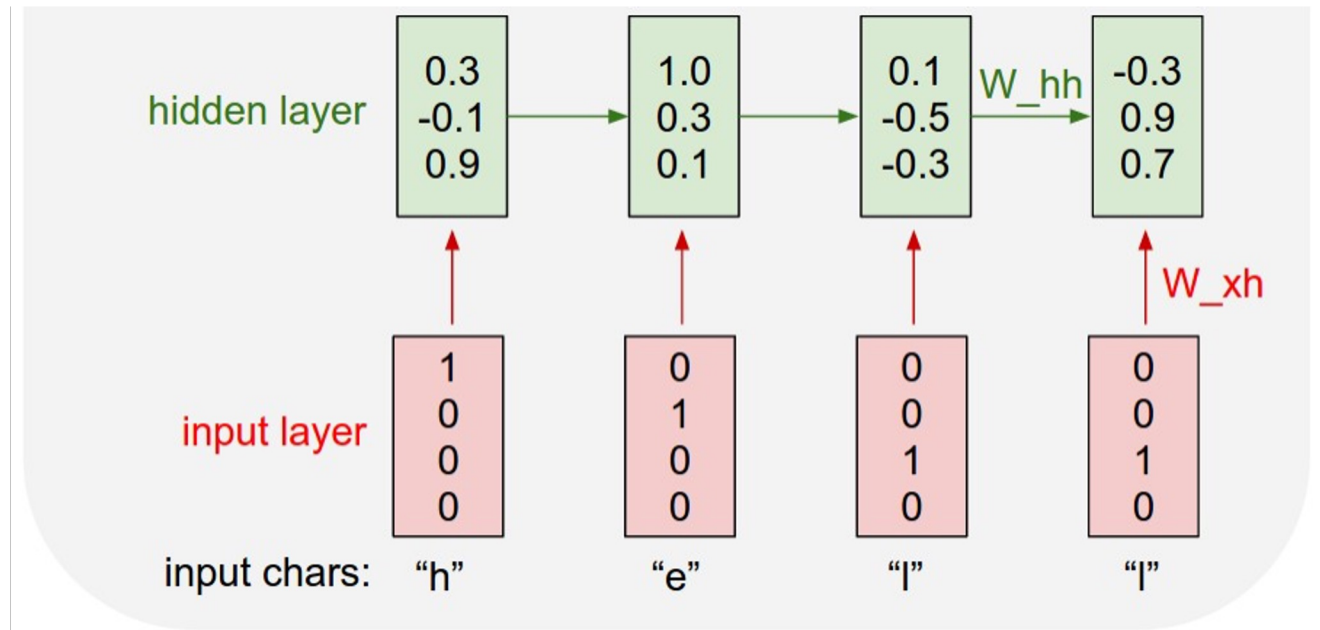
Example: Language Modeling

Given characters 1, 2, ..., t-1,
model predicts character t

$$h_t = \tanh(W_{hh}h_{t-1} + W_{xh}x_t)$$

Training sequence: "hello"

Vocabulary: [h, e, l, o]



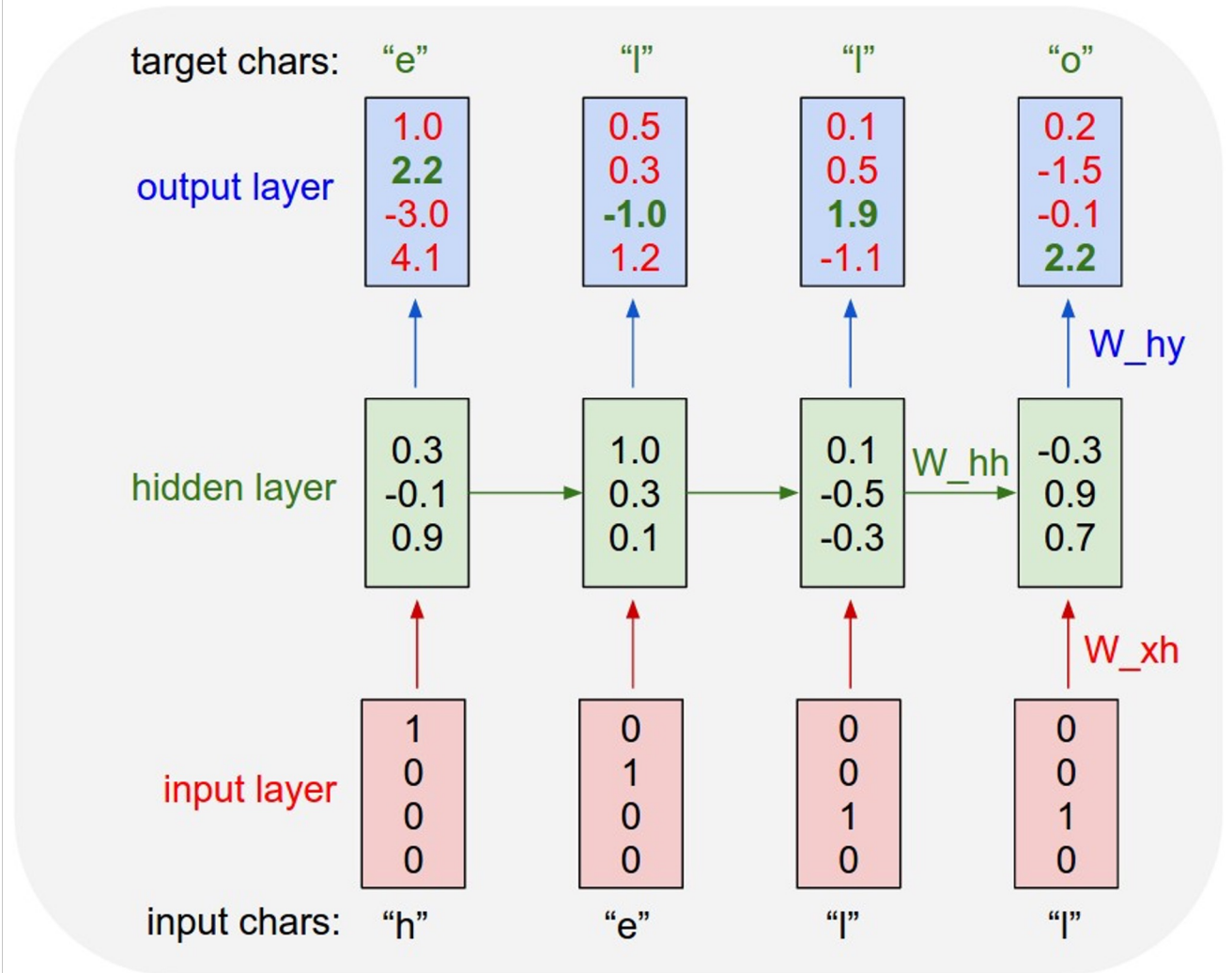
Example: Language Modeling

Given characters 1, 2, ..., t-1,
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Training sequence: "hello"

Vocabulary: [h, e, l, o]



Example: Language Modeling

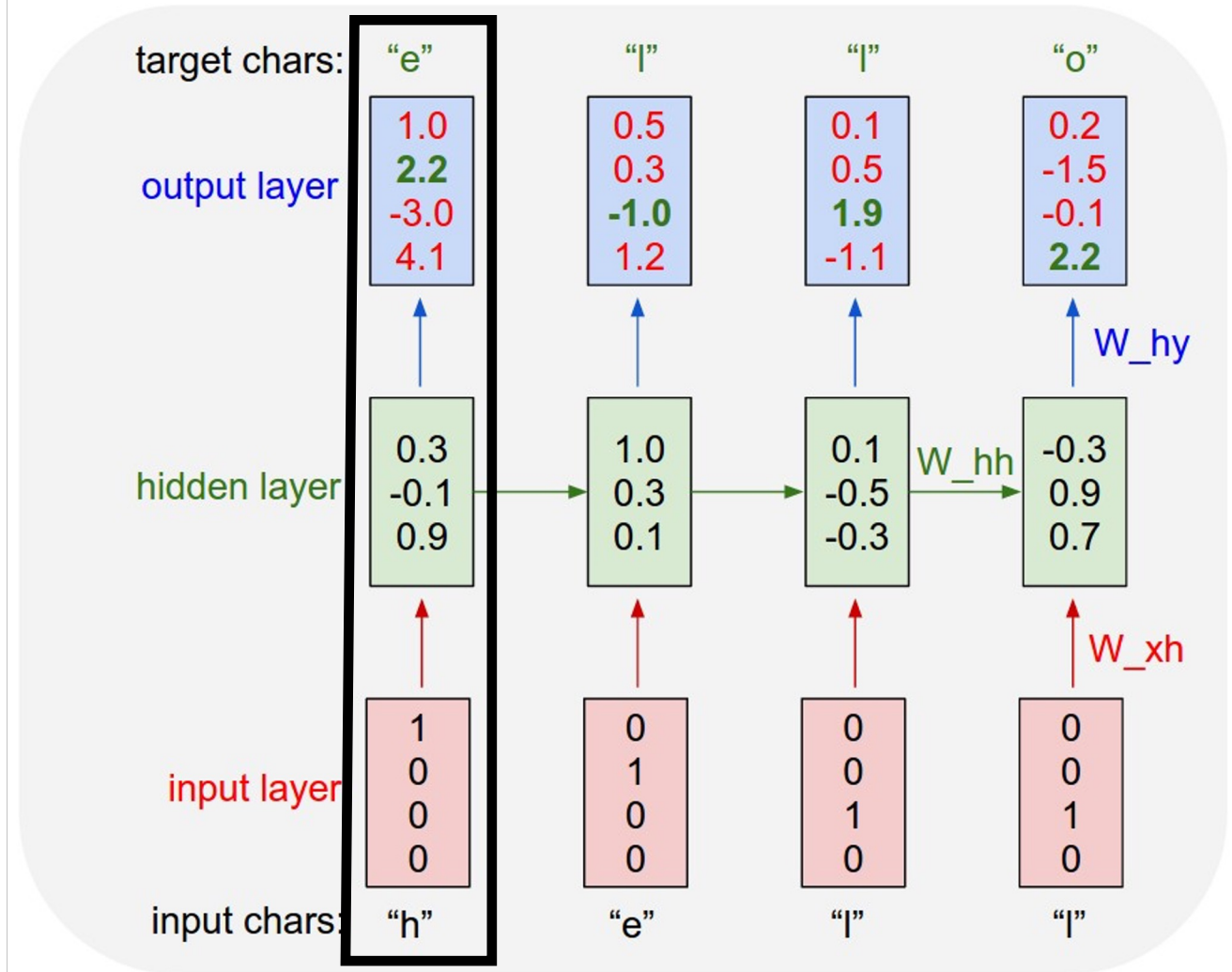
Given characters 1, 2, ..., t-1,
model predicts character t

$$h_t = \tanh(W_{hh}h_{t-1} + W_{xh}x_t)$$

Training sequence: "hello"

Vocabulary: [h, e, l, o]

Given "h", predict "e"



Example: Language Modeling

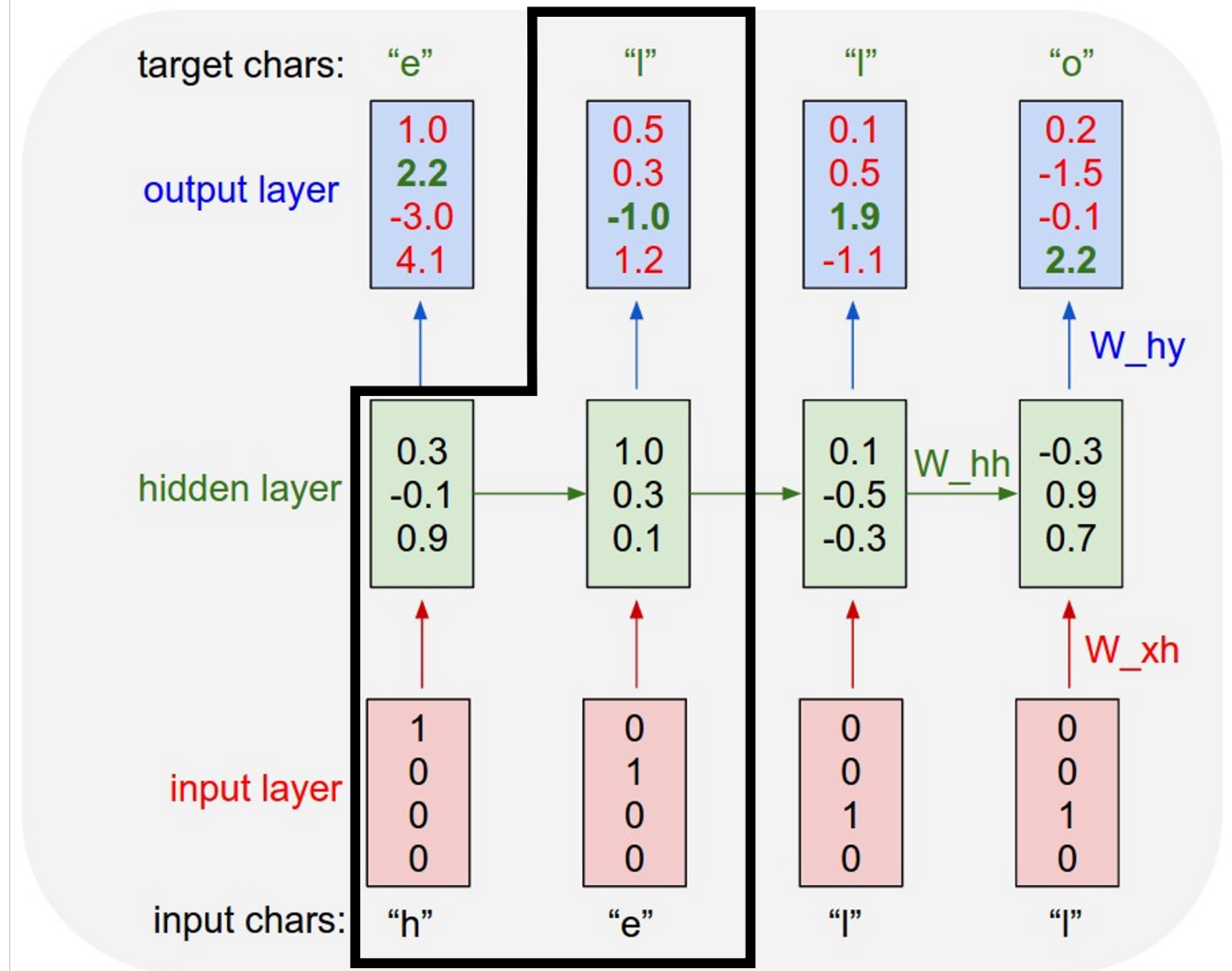
Given characters 1, 2, ..., t-1,
model predicts character t

$$h_t = \tanh(W_{hh}h_{t-1} + W_{xh}x_t)$$

Training sequence: "hello"

Vocabulary: [h, e, l, o]

Given "he", predict "l"



Example: Language Modeling

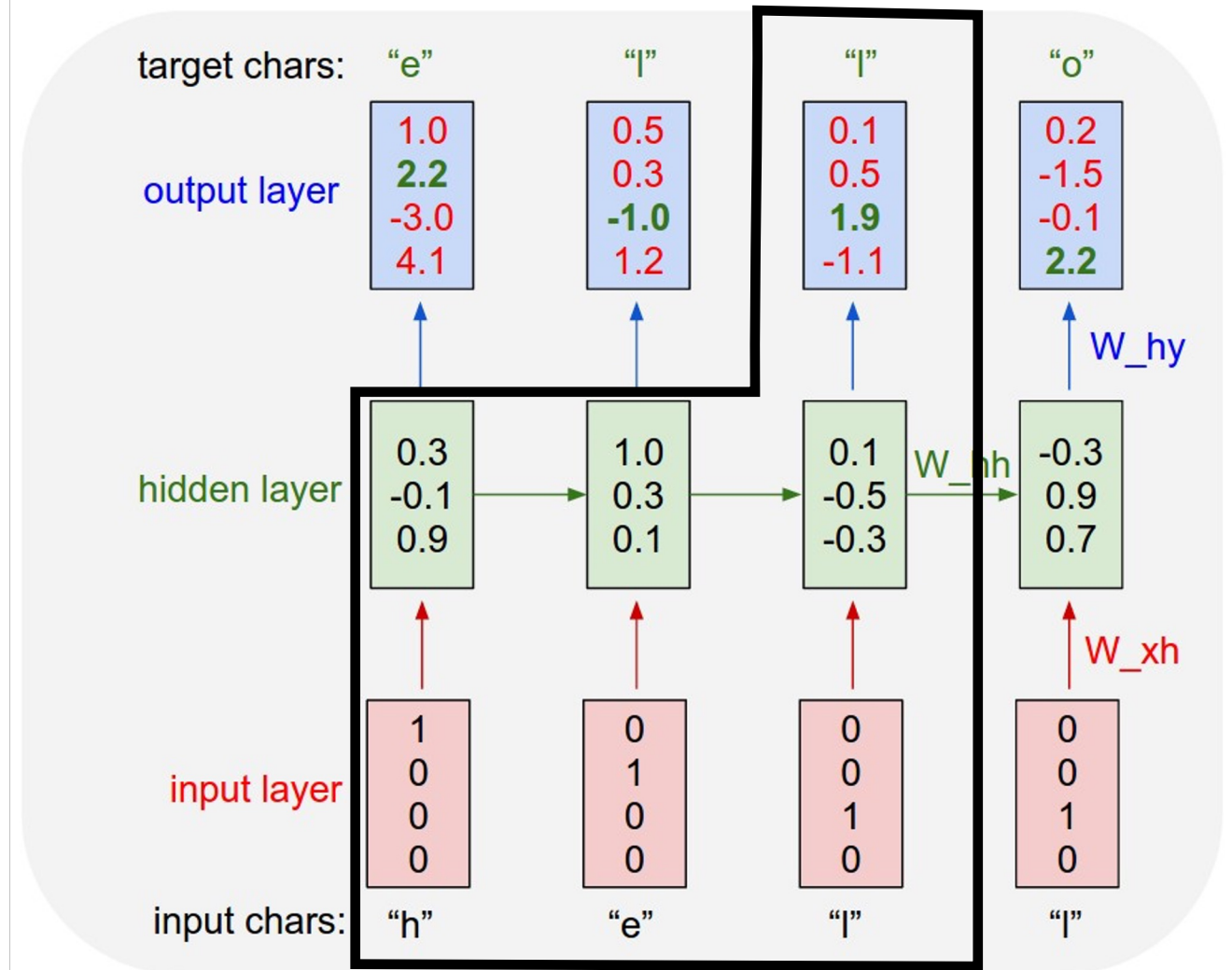
Given characters 1, 2, ..., t-1,
model predicts character t

$$h_t = \tanh(W_{hh}h_{t-1} + W_{xh}x_t)$$

Training sequence: "hello"

Vocabulary: [h, e, l, o]

Given "hel", predict "l"



Example: Language Modeling

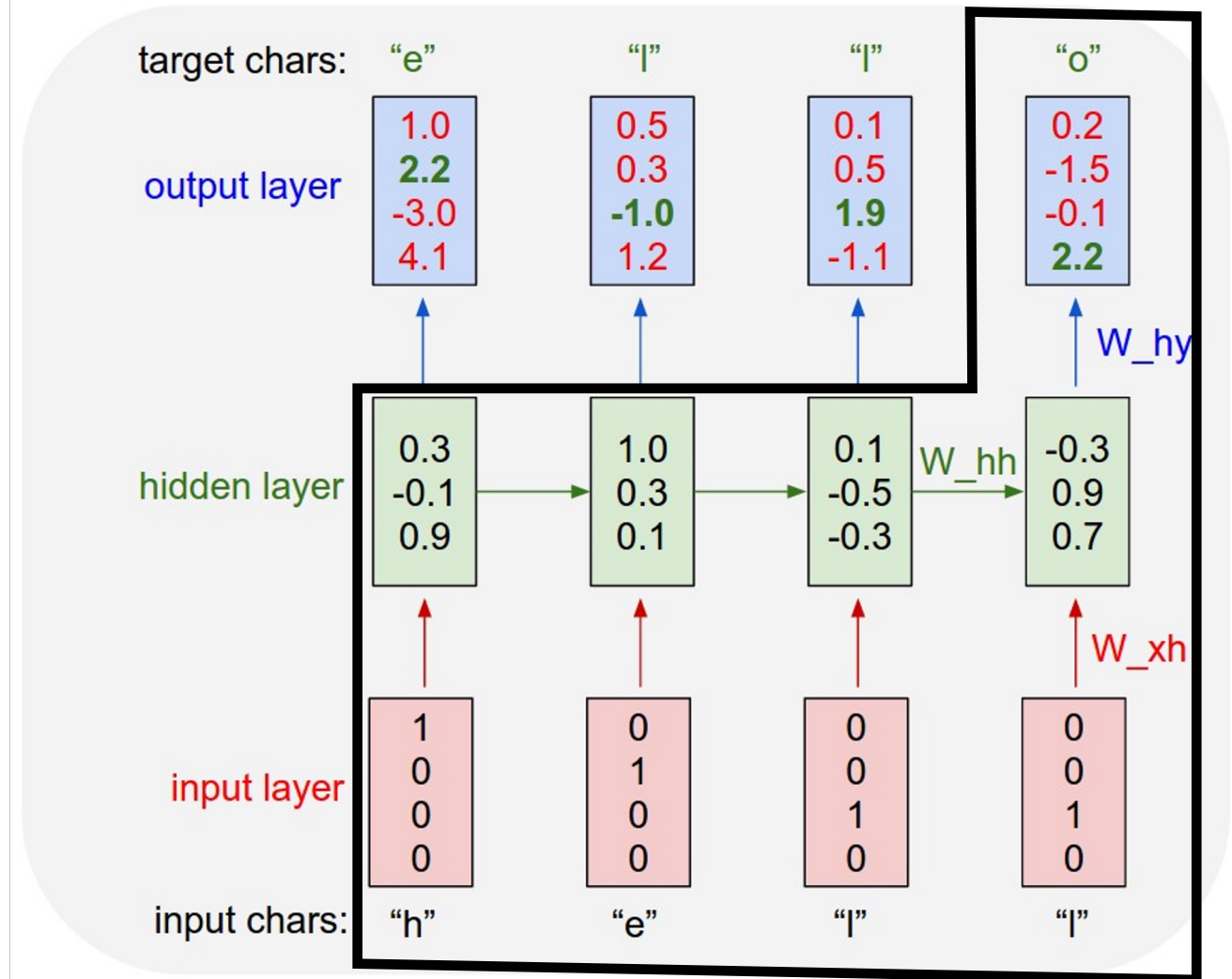
Given characters 1, 2, ..., t-1,
model predicts character t

$$h_t = \tanh(W_{hh}h_{t-1} + W_{xh}x_t)$$

Training sequence: "hello"

Vocabulary: [h, e, l, o]

Given "hell", predict "o"

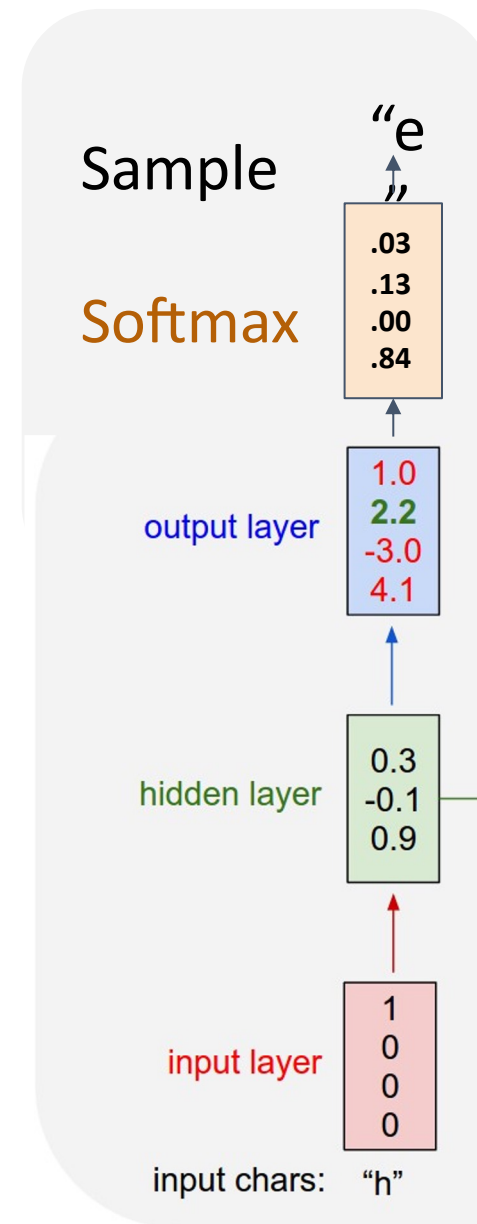


Example: Language Modeling

At test-time, **generate** new text: sample characters one at a time, feed back to model

Training sequence: "hello"

Vocabulary: [h, e, l, o]

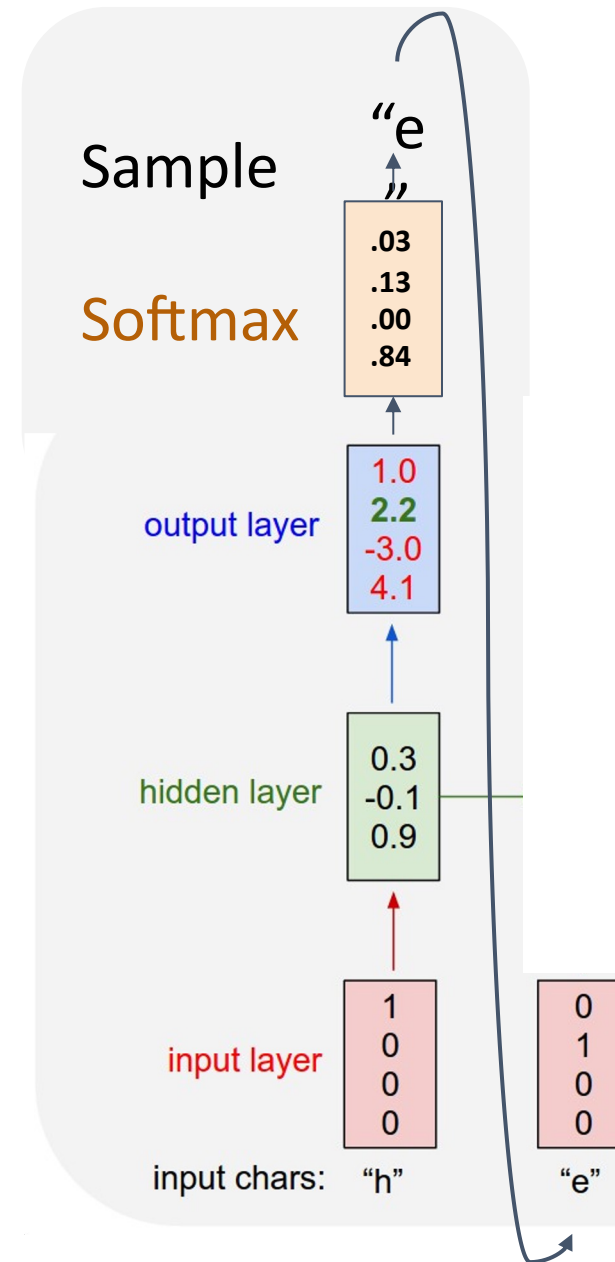


Example: Language Modeling

At test-time, **generate** new text: sample characters one at a time, feed back to model

Training sequence: "hello"

Vocabulary: [h, e, l, o]

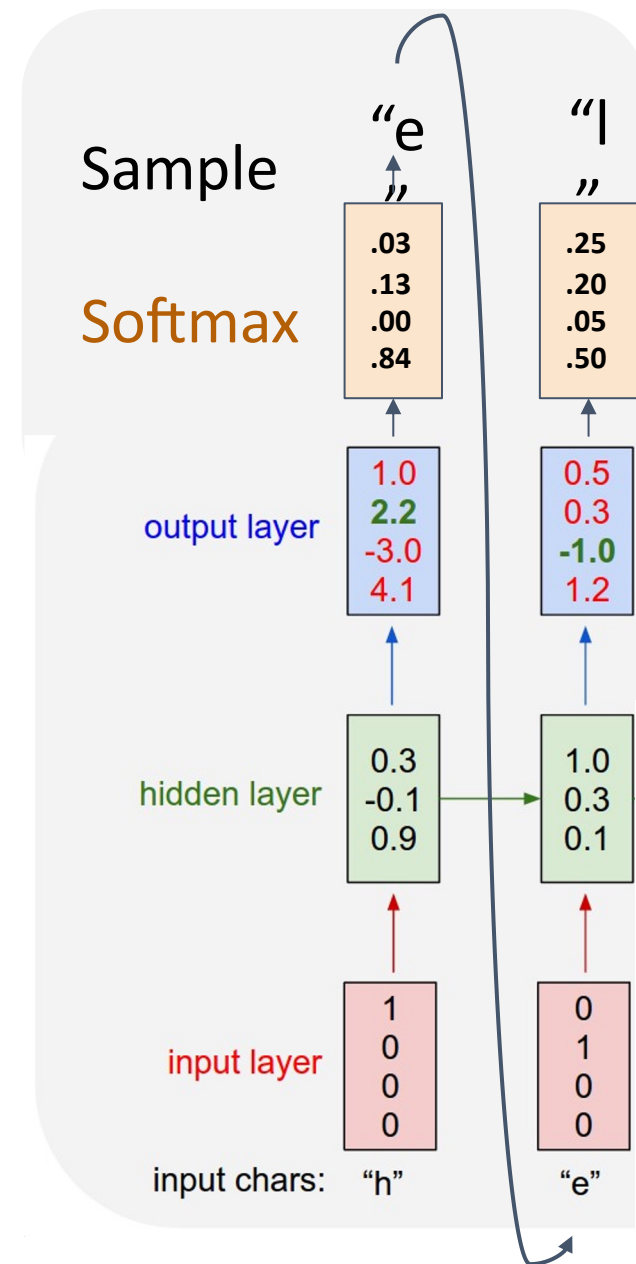


Example: Language Modeling

At test-time, **generate** new text: sample characters one at a time, feed back to model

Training sequence: "hello"

Vocabulary: [h, e, l, o]

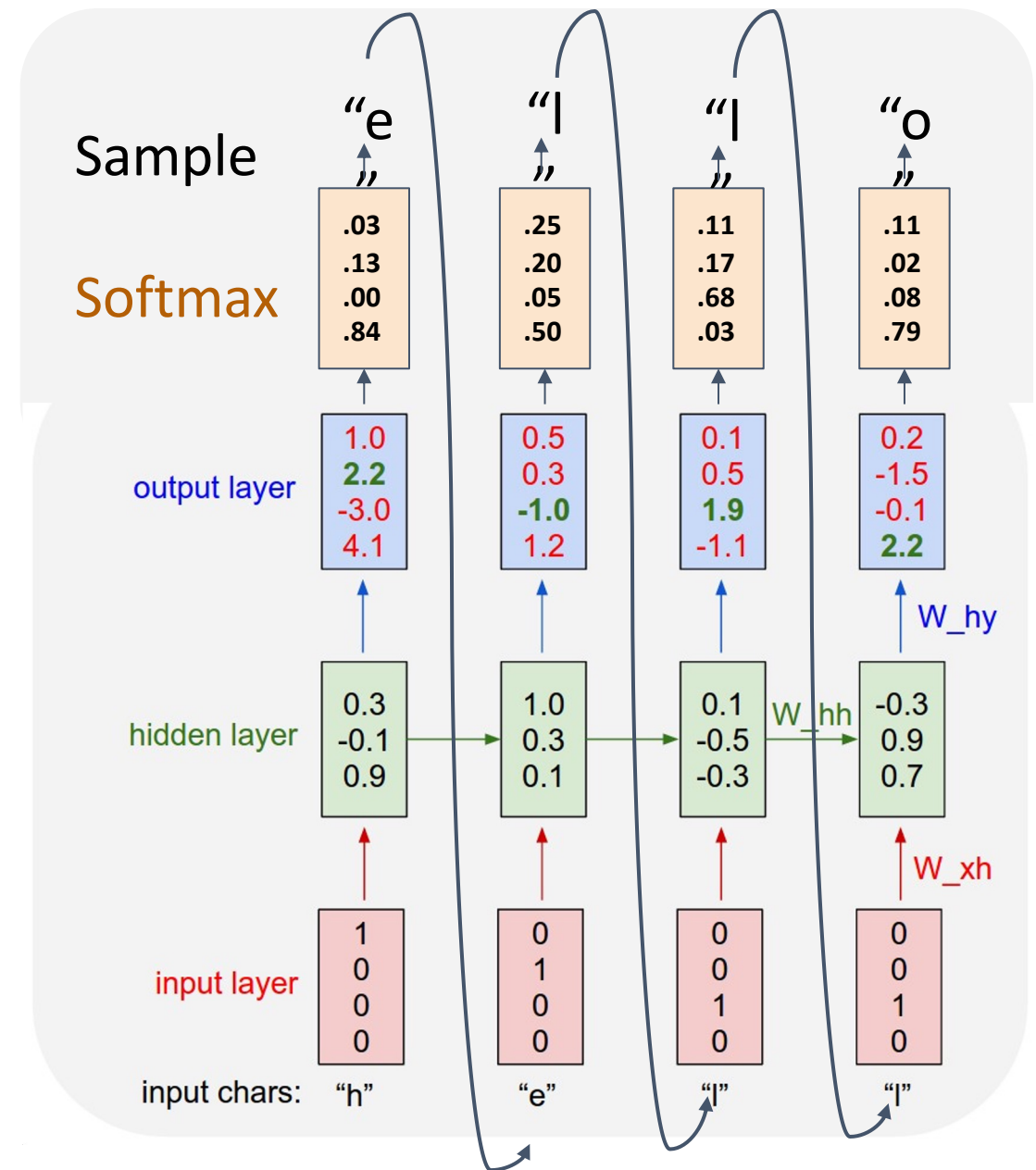


Example: Language Modeling

At test-time, **generate** new text: sample characters one at a time, feed back to model

Training sequence: "hello"

Vocabulary: [h, e, l, o]

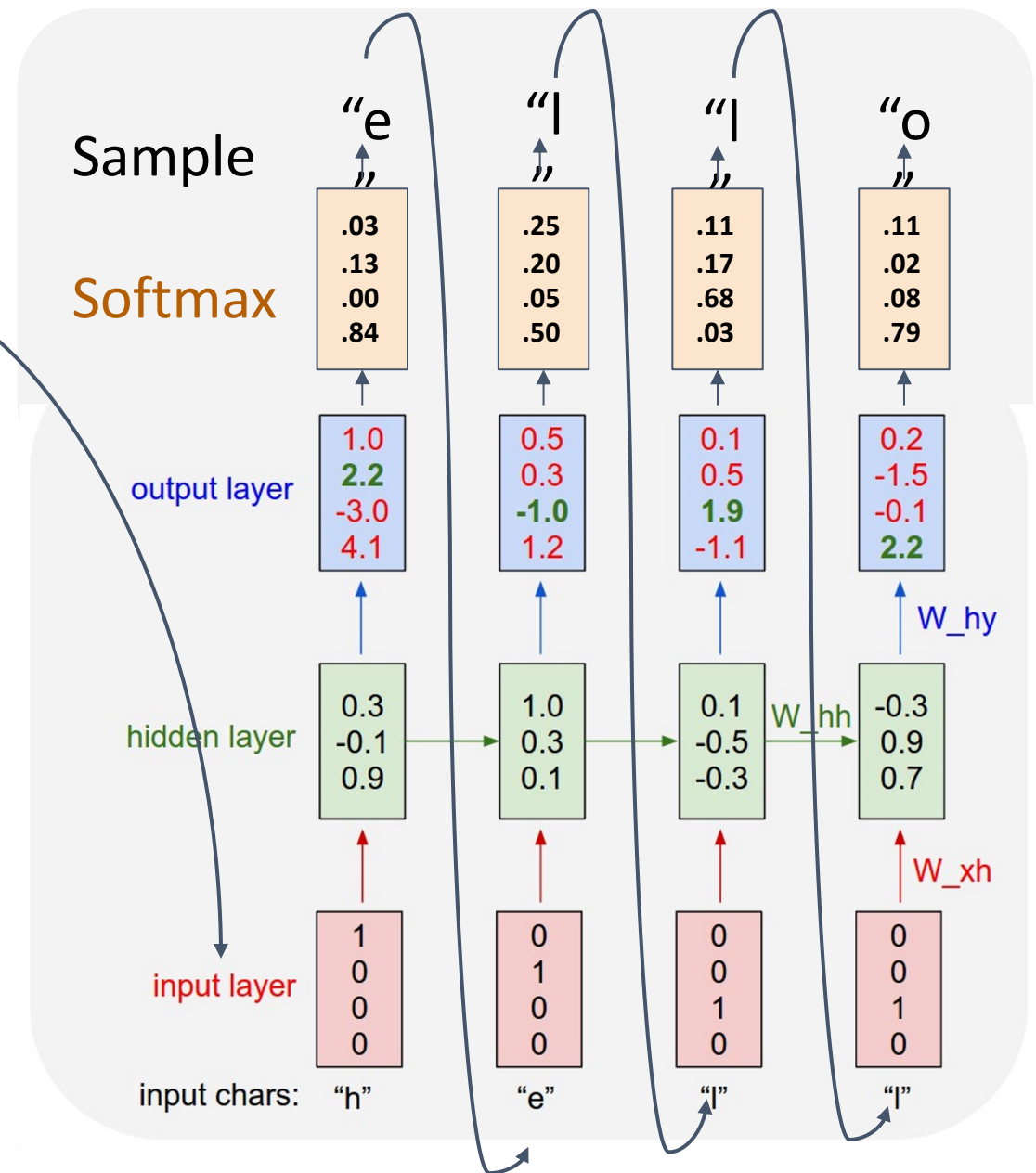


Example: Language Modeling

So far: encode inputs as **one-hot-vector**

$$\begin{bmatrix} w_{11} & w_{12} & w_{13} & w_{14} \\ w_{21} & w_{22} & w_{23} & w_{14} \\ w_{31} & w_{32} & w_{33} & w_{14} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} w_{11} \\ w_{21} \\ w_{31} \end{bmatrix}$$

Matrix multiply with a one-hot vector just extracts a column from the weight matrix. Often extract this into a separate **embedding** layer

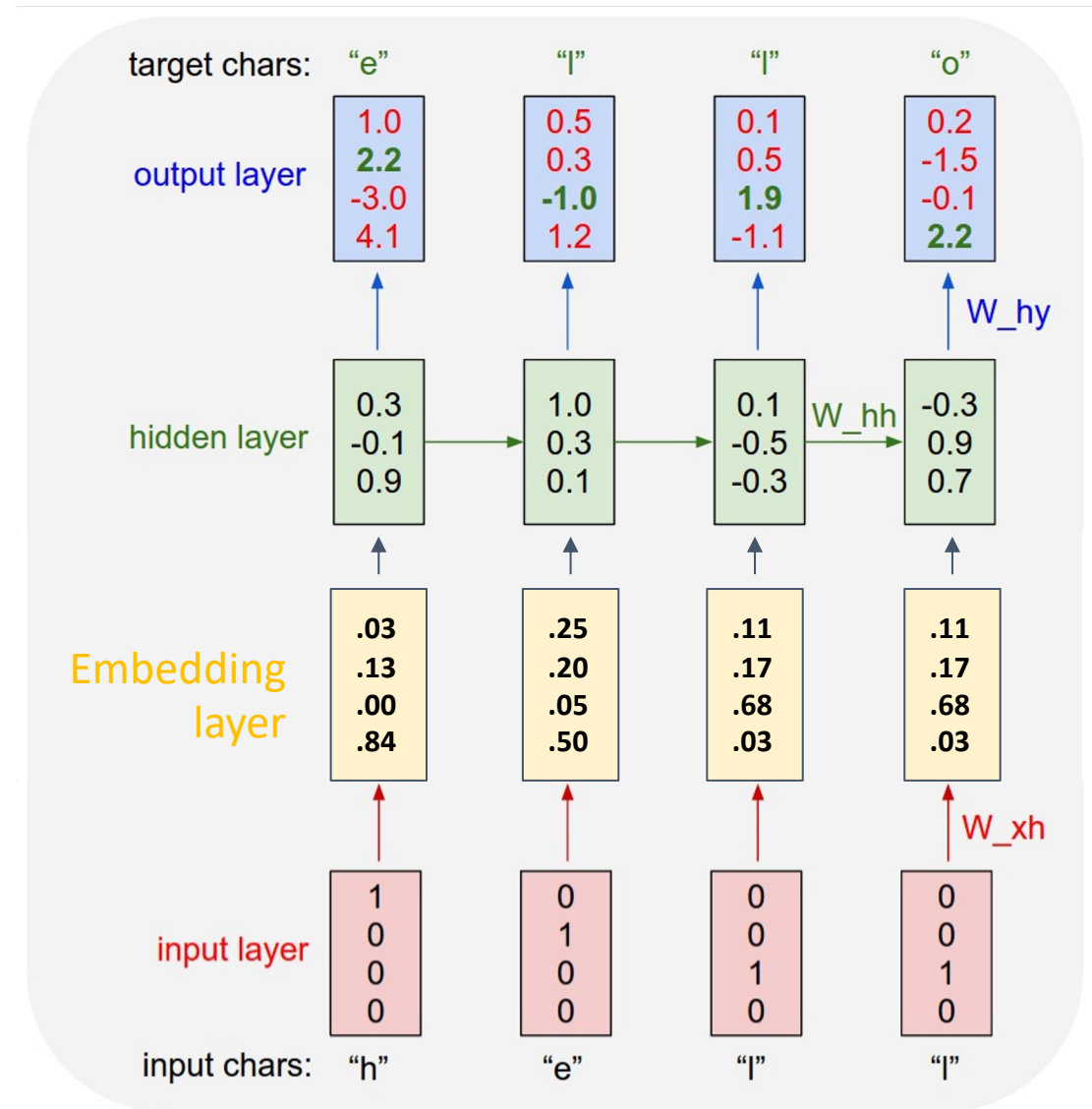


Example: Language Modeling

So far: encode inputs as **one-hot-vector**

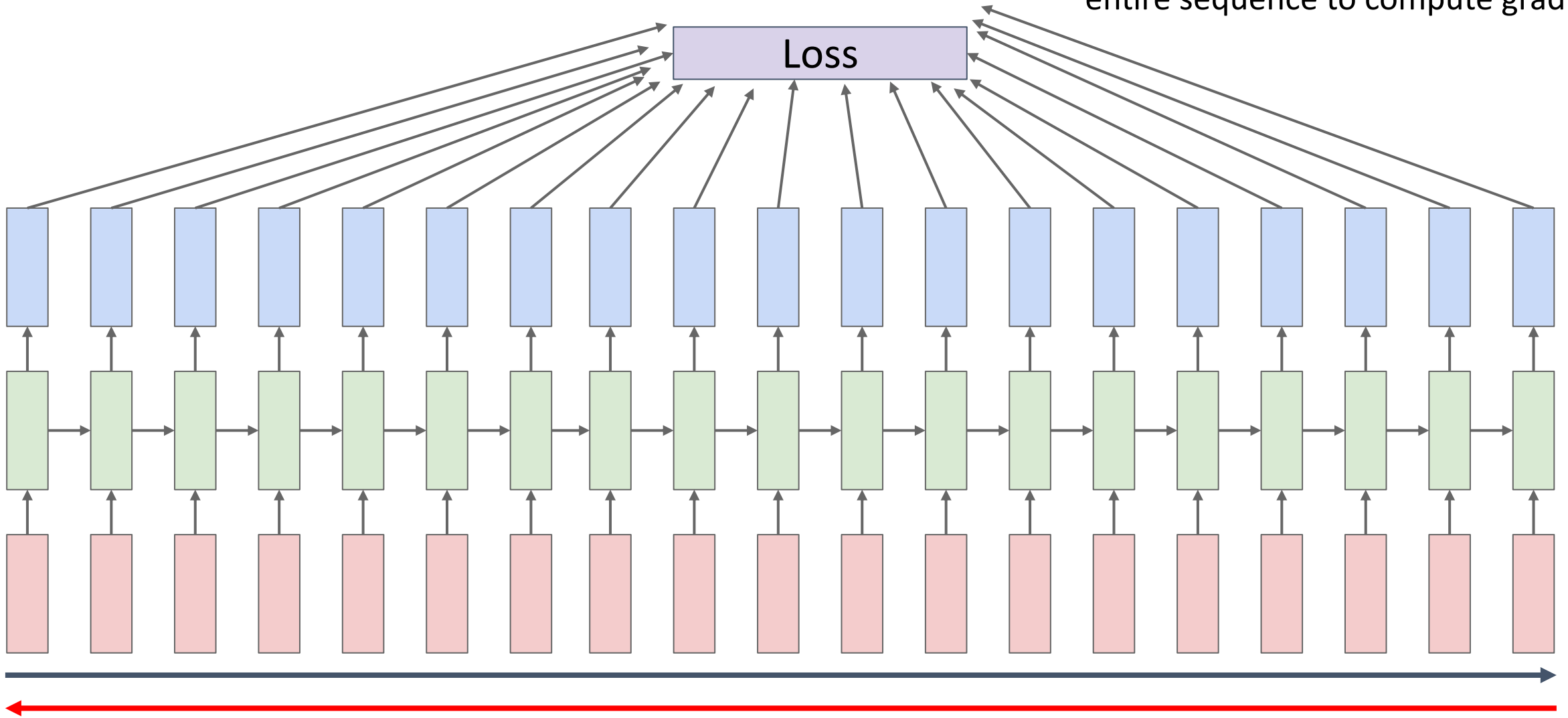
$$\begin{bmatrix} w_{11} & w_{12} & w_{13} & w_{14} \\ w_{21} & w_{22} & w_{23} & w_{14} \\ w_{31} & w_{32} & w_{33} & w_{14} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} w_{11} \\ w_{21} \\ w_{31} \end{bmatrix}$$

Matrix multiply with a one-hot vector just extracts a column from the weight matrix. Often extract this into a separate **embedding** layer



Backpropagation Through Time

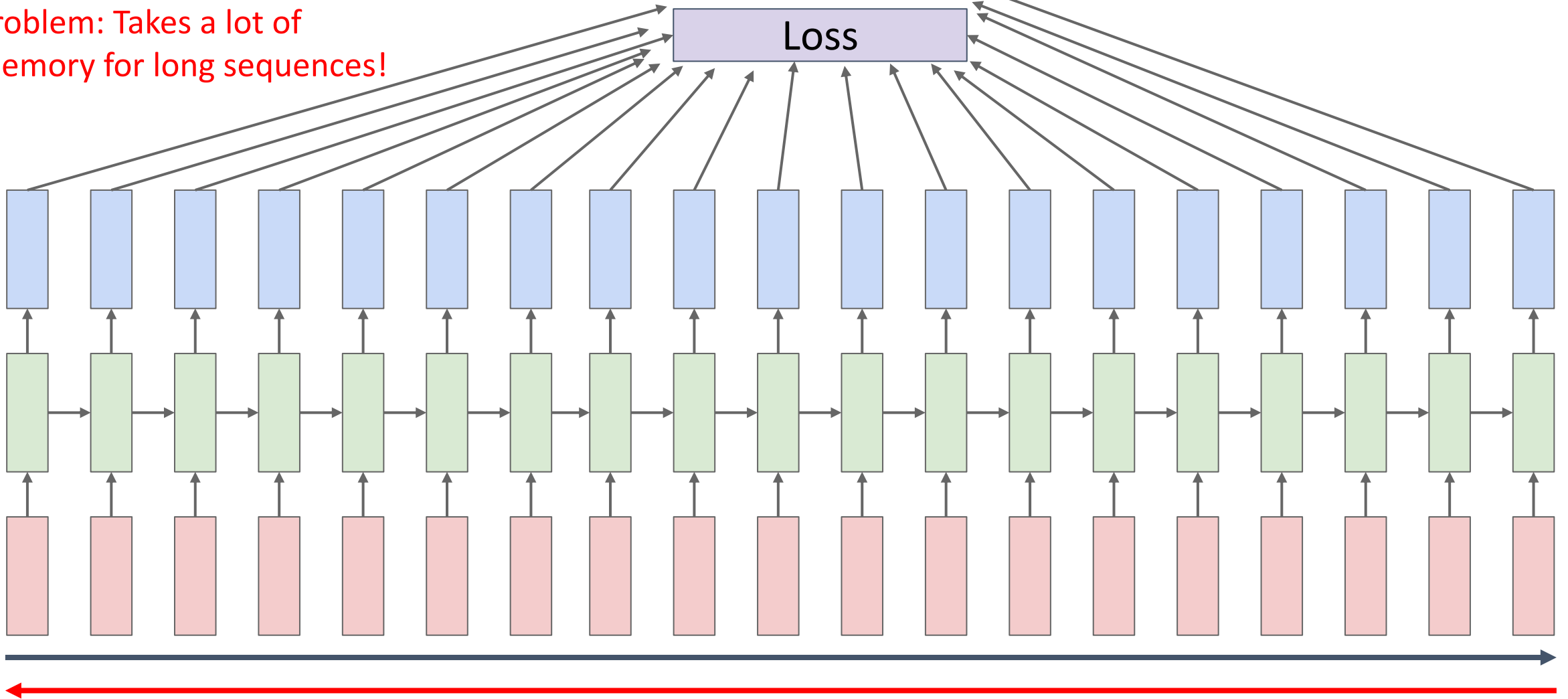
Forward through entire sequence to compute loss, then backward through entire sequence to compute gradient



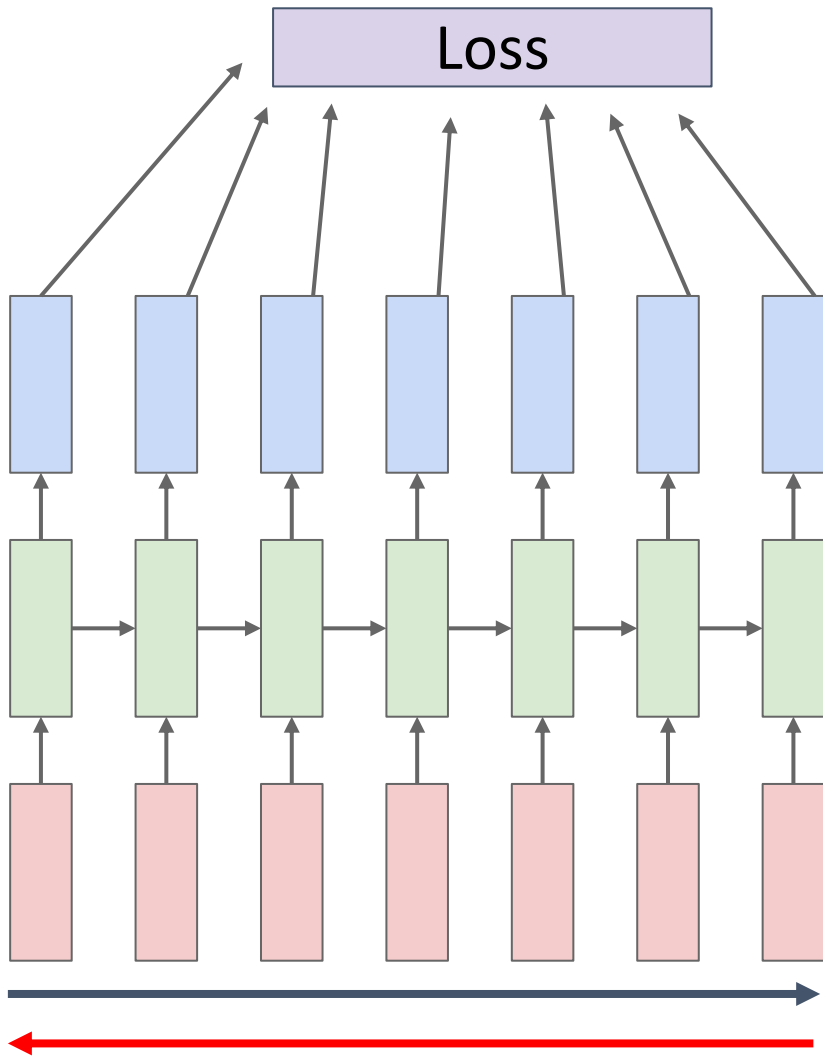
Backpropagation Through Time

Forward through entire sequence to compute loss, then backward through entire sequence to compute gradient

Problem: Takes a lot of memory for long sequences!

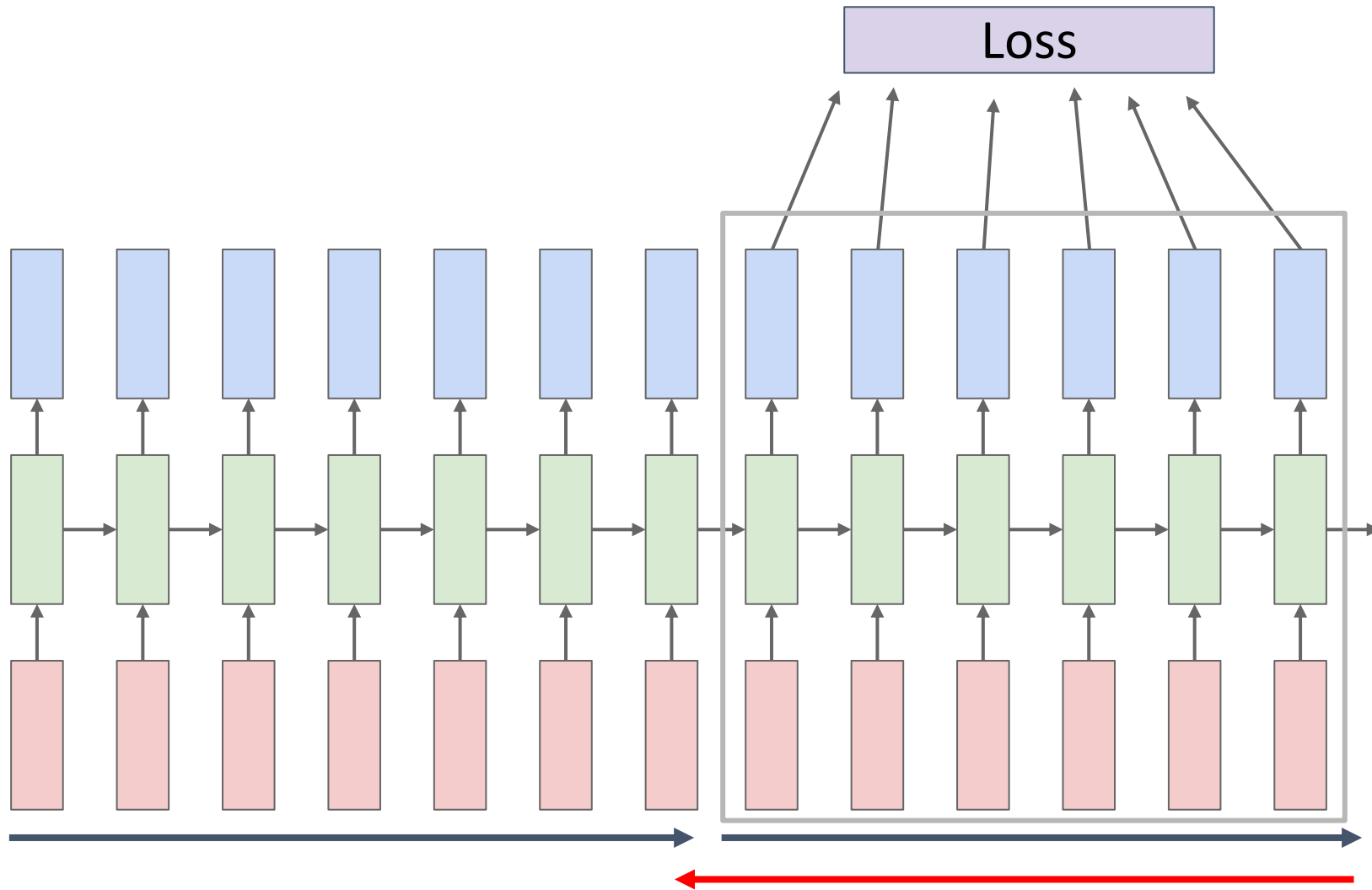


Truncated Backpropagation Through Time



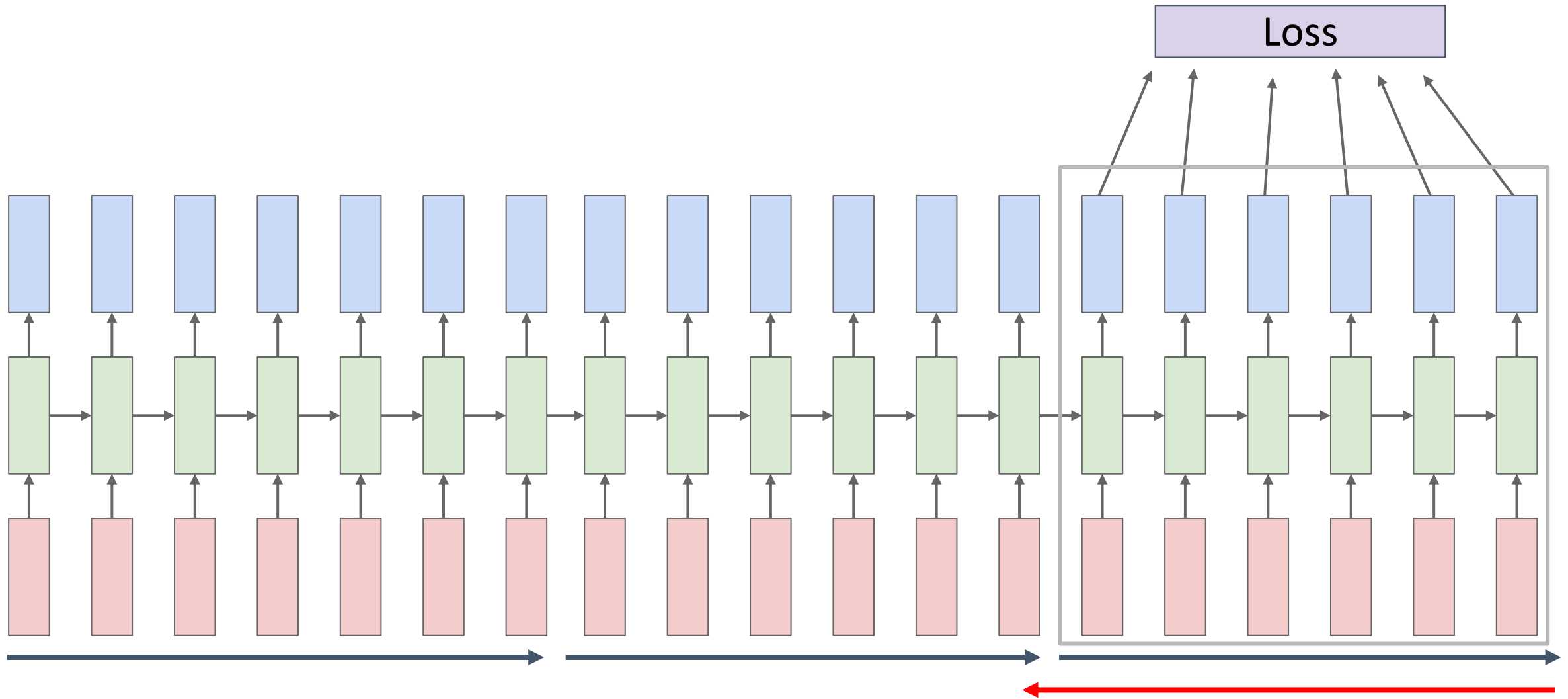
Run forward and backward through chunks of the sequence instead of whole sequence

Truncated Backpropagation Through Time

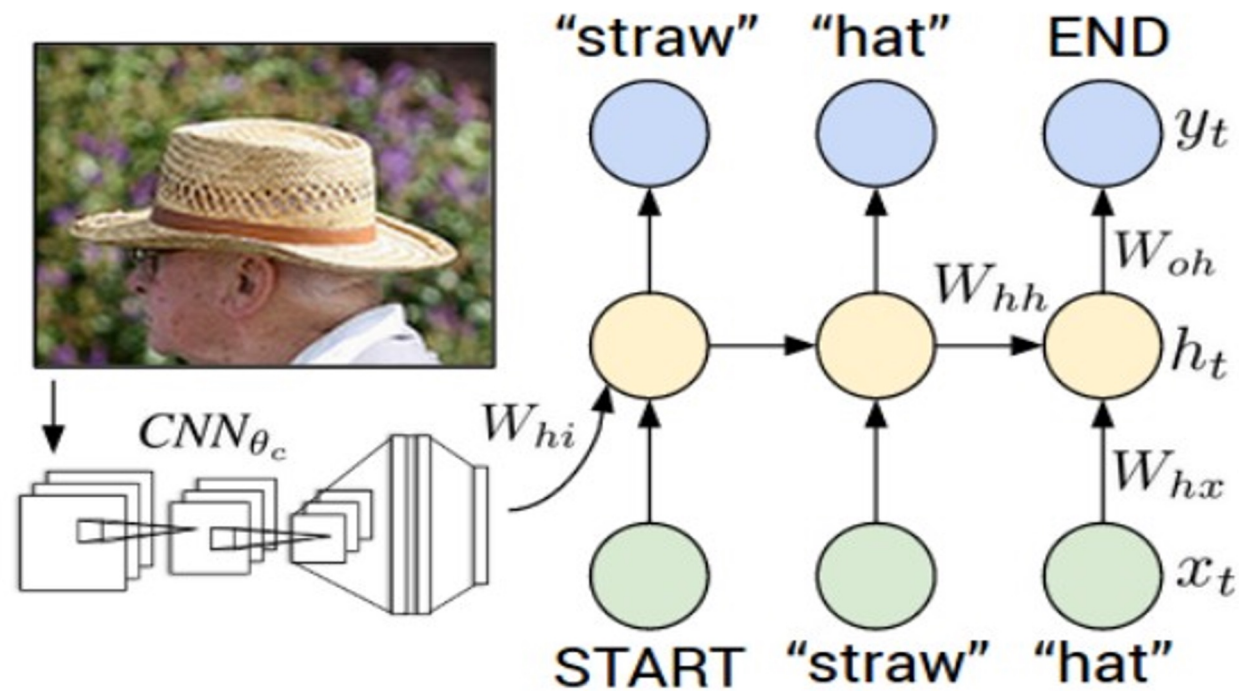


Carry hidden states forward in time forever, but only backpropagate for some smaller number of steps

Truncated Backpropagation Through Time



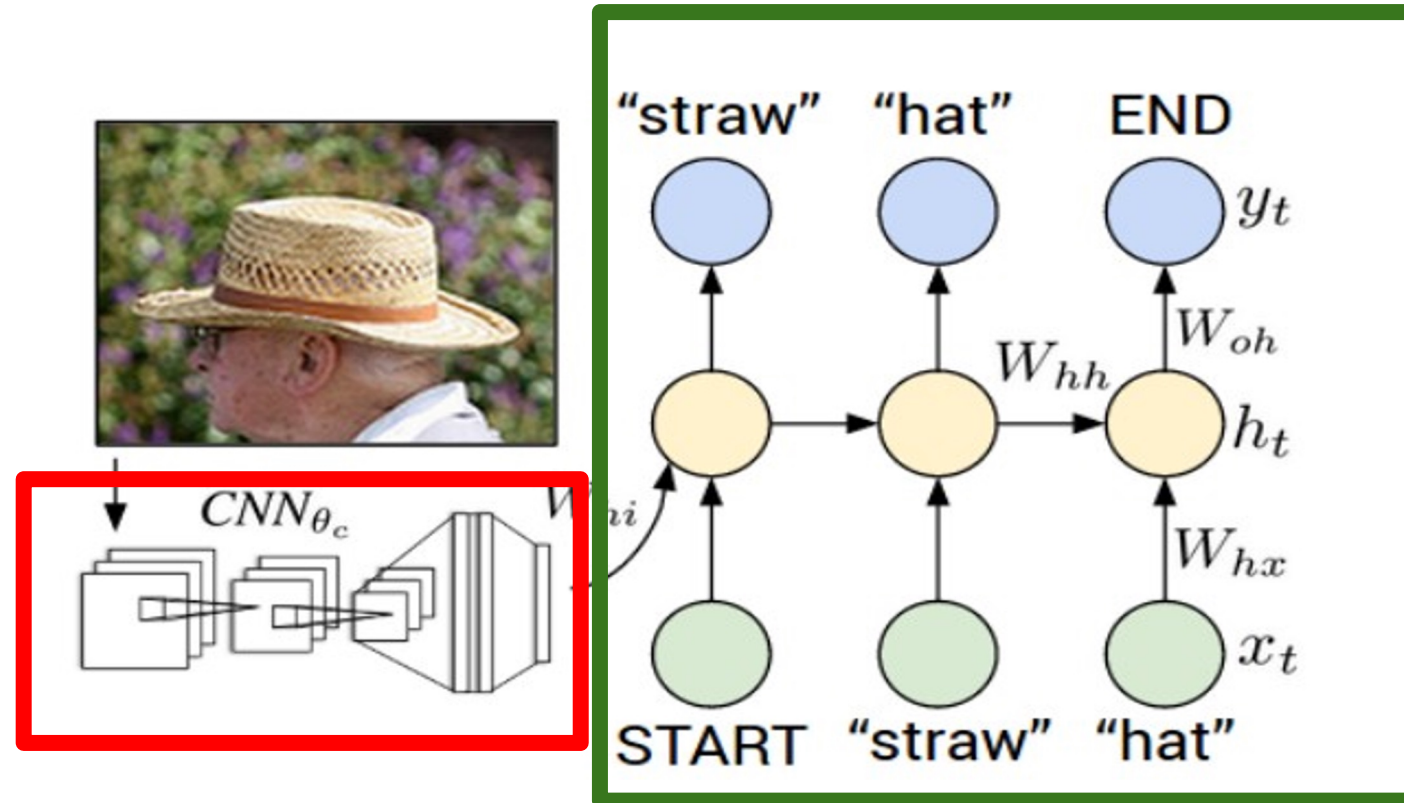
Example: Image Captioning



- Mao et al, "Explain Images with Multimodal Recurrent Neural Networks", NeurIPS 2014 Deep Learning and Representation Workshop
- Karpathy and Fei-Fei, "Deep Visual-Semantic Alignments for Generating Image Descriptions", CVPR 2015
- Vinyals et al, "Show and Tell: A Neural Image Caption Generator", CVPR 2015
- Donahue et al, "Long-term Recurrent Convolutional Networks for Visual Recognition and Description", CVPR 2015
- Chen and Zitnick, "Learning a Recurrent Visual Representation for Image Caption Generation", CVPR 2015

Figure from Karpathy et al, "Deep Visual-Semantic Alignments for Generating Image Descriptions", CVPR 2015

Example: Image Captioning



**Recurrent
Neural
Network**

Convolutional Neural Network

Figure from Karpathy et al, "Deep Visual-Semantic Alignments for Generating Image Descriptions", CVPR 2015

[This image is CC0 public domain](#)



image

conv-64

conv-64

maxpool

conv-128

conv-128

maxpool

conv-256

conv-256

maxpool

conv-512

conv-512

maxpool

conv-512

conv-512

maxpool

FC-4096

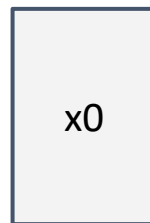
FC-4096

FC-1000

softmax

Transfer learning: Take
CNN trained on ImageNet,
chop off last layer

This image is [CC0 public domain](#)



<START>

- image
- conv-64
- conv-64
- maxpool
- conv-128
- conv-128
- maxpool
- conv-256
- conv-256
- maxpool
- conv-512
- conv-512
- maxpool
- conv-512
- conv-512
- maxpool
- FC-4096
- FC-4096

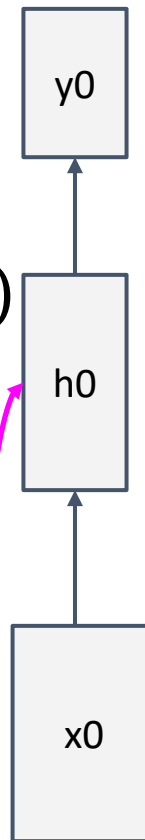


Before:

$$h_t = \tanh(W_{hh}h_{t-1} + W_{xh}x_t + b_h)$$

Now:

$$\tanh(W_{hh}h_{t-1} + W_{xh}x_t + W_{ih}v + b_h)$$



W_{ih}

- image
- conv-64
- conv-64
- maxpool
- conv-128
- conv-128
- maxpool
- conv-256
- conv-256
- maxpool
- conv-512
- conv-512
- maxpool
- conv-512
- conv-512
- maxpool
- FC-4096
- FC-4096

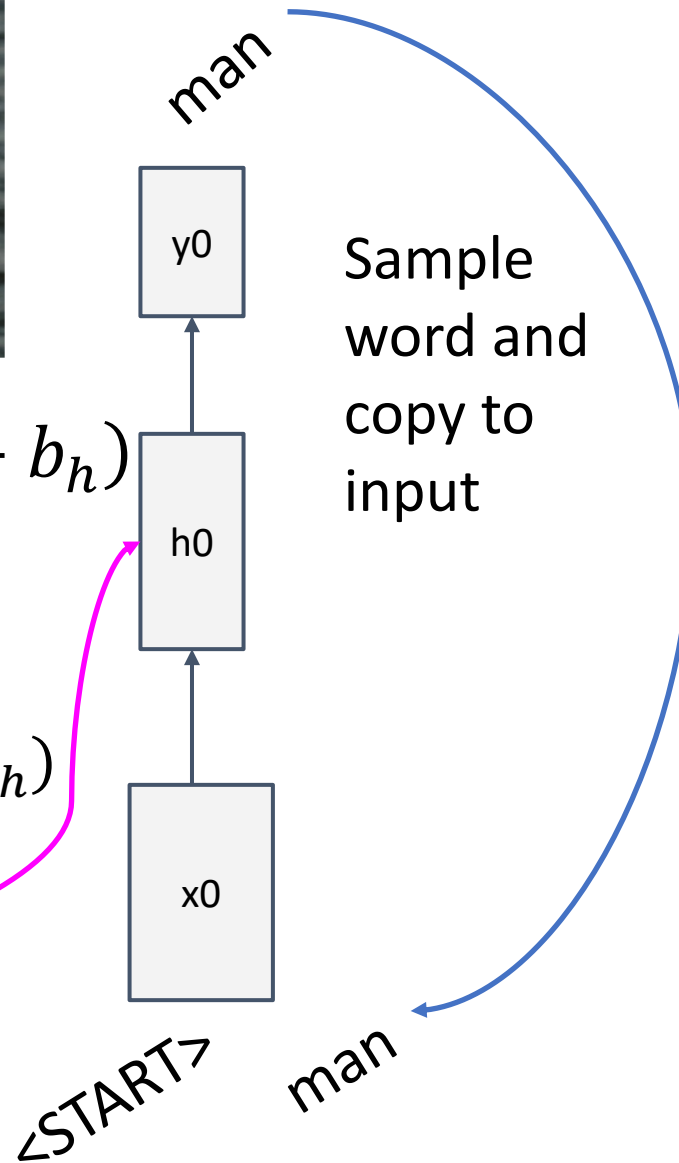


Before:

$$h_t = \tanh(W_{hh}h_{t-1} + W_{xh}x_t + b_h)$$

Now:

$$\tanh(W_{hh}h_{t-1} + W_{xh}x_t + W_{ih}v + b_h)$$



- image
- conv-64
- conv-64
- maxpool
- conv-128
- conv-128
- maxpool
- conv-256
- conv-256
- maxpool
- conv-512
- conv-512
- maxpool
- conv-512
- conv-512
- maxpool
- FC-4096
- FC-4096

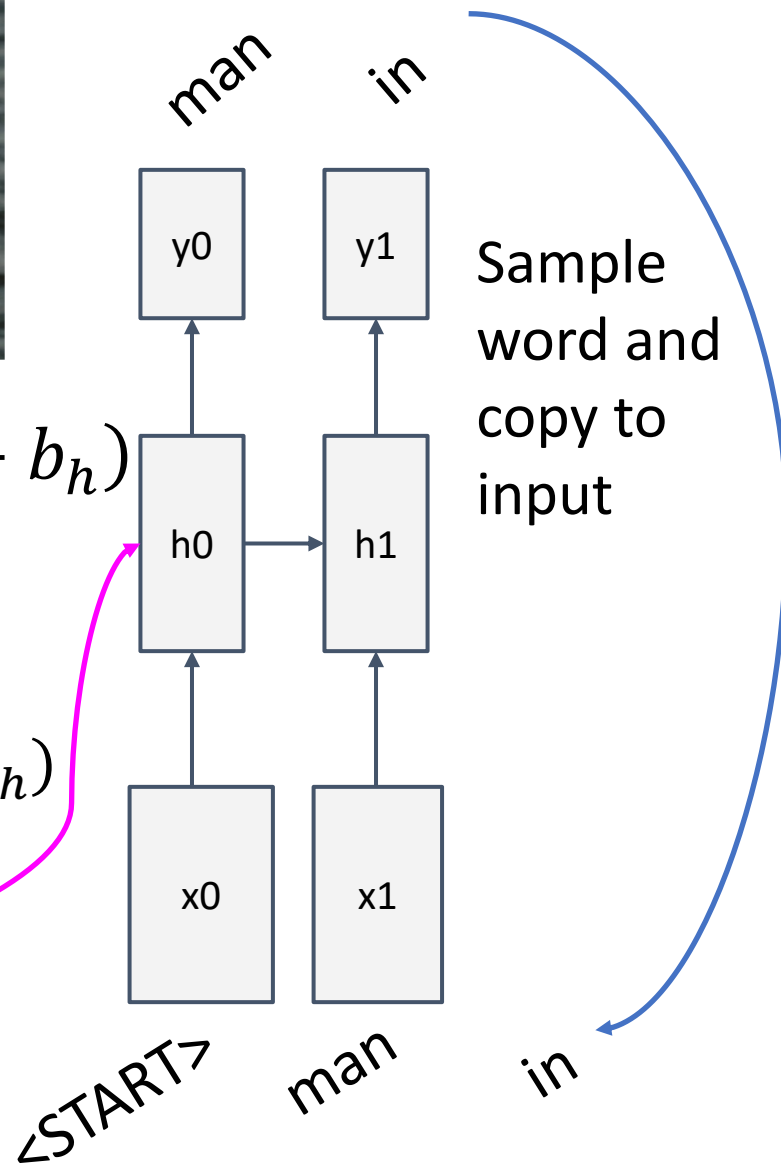


Before:

$$h_t = \tanh(W_{hh}h_{t-1} + W_{xh}x_t + b_h)$$

Now:

$$\tanh(W_{hh}h_{t-1} + W_{xh}x_t + W_{ih}v + b_h)$$



- image
- conv-64
- conv-64
- maxpool
- conv-128
- conv-128
- maxpool
- conv-256
- conv-256
- maxpool
- conv-512
- conv-512
- maxpool
- conv-512
- conv-512
- maxpool
- FC-4096
- FC-4096



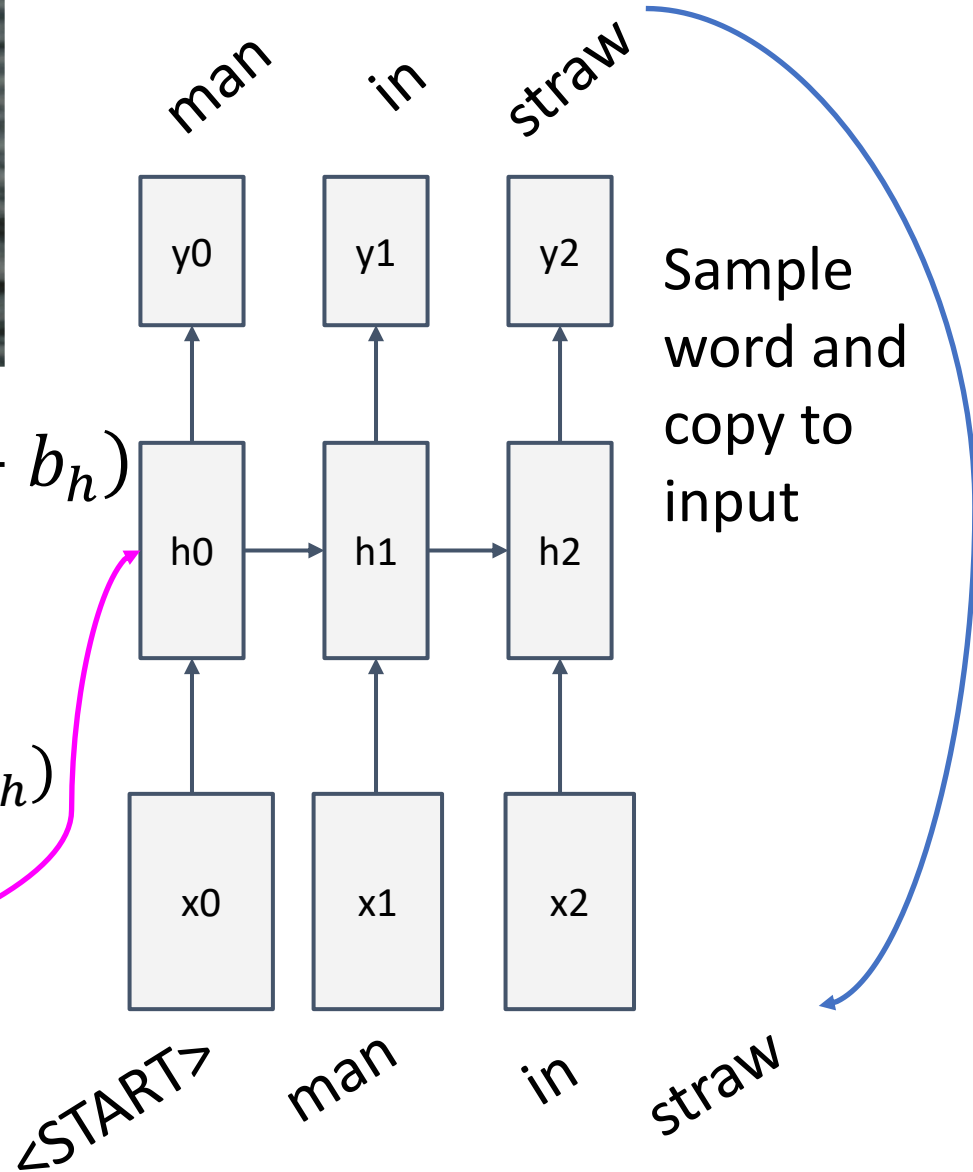
Before:

$$h_t = \tanh(W_{hh}h_{t-1} + W_{xh}x_t + b_h)$$

Now:

$$\tanh(W_{hh}h_{t-1} + W_{xh}x_t + W_{ih}v + b_h)$$

W_{ih}



- image
- conv-64
- conv-64
- maxpool
- conv-128
- conv-128
- maxpool
- conv-256
- conv-256
- maxpool
- conv-512
- conv-512
- maxpool
- conv-512
- conv-512
- maxpool
- FC-4096
- FC-4096

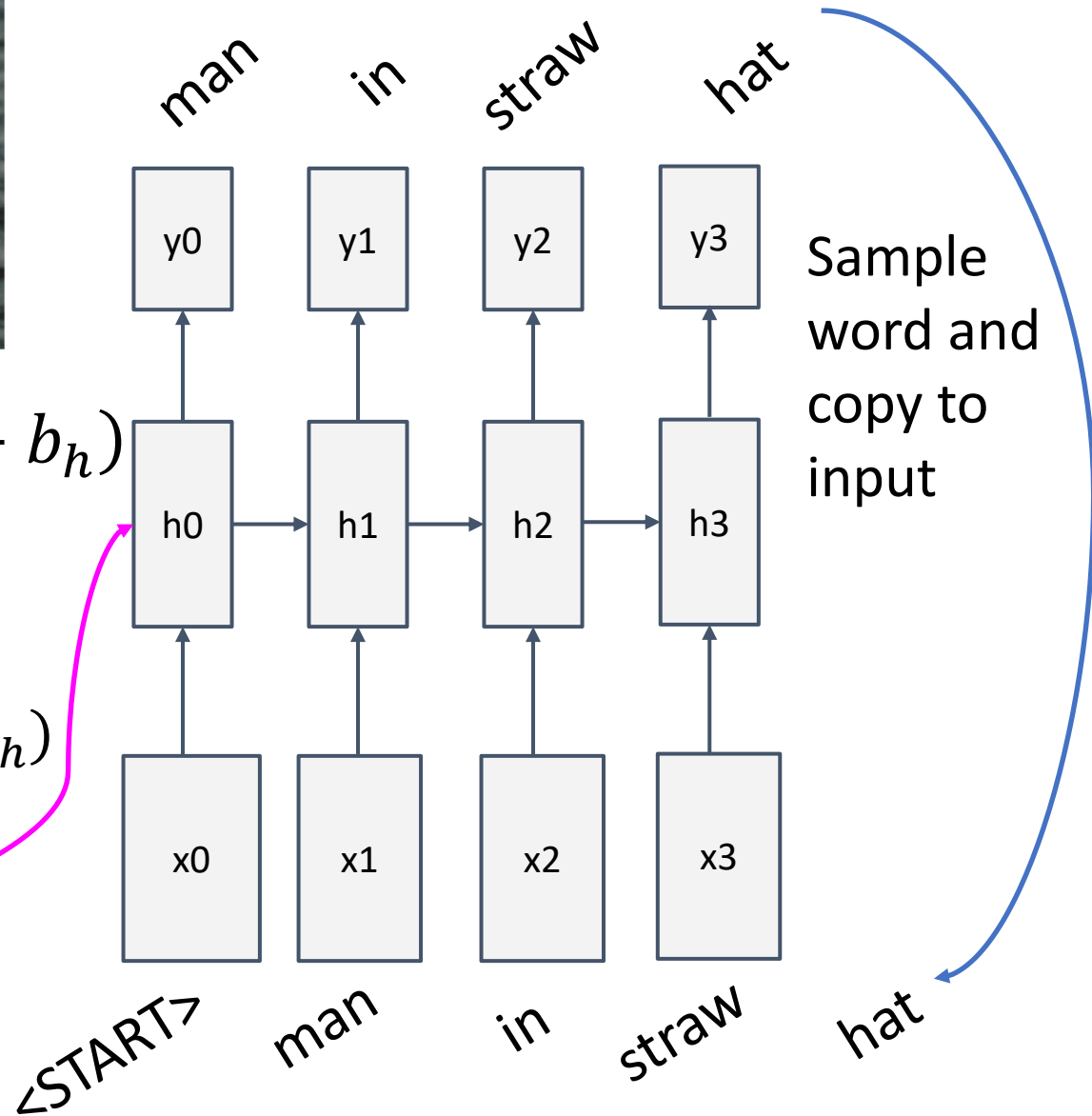


Before:

$$h_t = \tanh(W_{hh}h_{t-1} + W_{xh}x_t + b_h)$$

Now:

$$\tanh(W_{hh}h_{t-1} + W_{xh}x_t + W_{ih}v + b_h)$$



- image
- conv-64
- conv-64
- maxpool
- conv-128
- conv-128
- maxpool
- conv-256
- conv-256
- maxpool
- conv-512
- conv-512
- maxpool
- conv-512
- conv-512
- maxpool
- FC-4096
- FC-4096



Before:

$$h_t = \tanh(W_{hh}h_{t-1} + W_{xh}x_t + b_h)$$

Now:

$$\tanh(W_{hh}h_{t-1} + W_{xh}x_t + W_{ih}v + b_h)$$

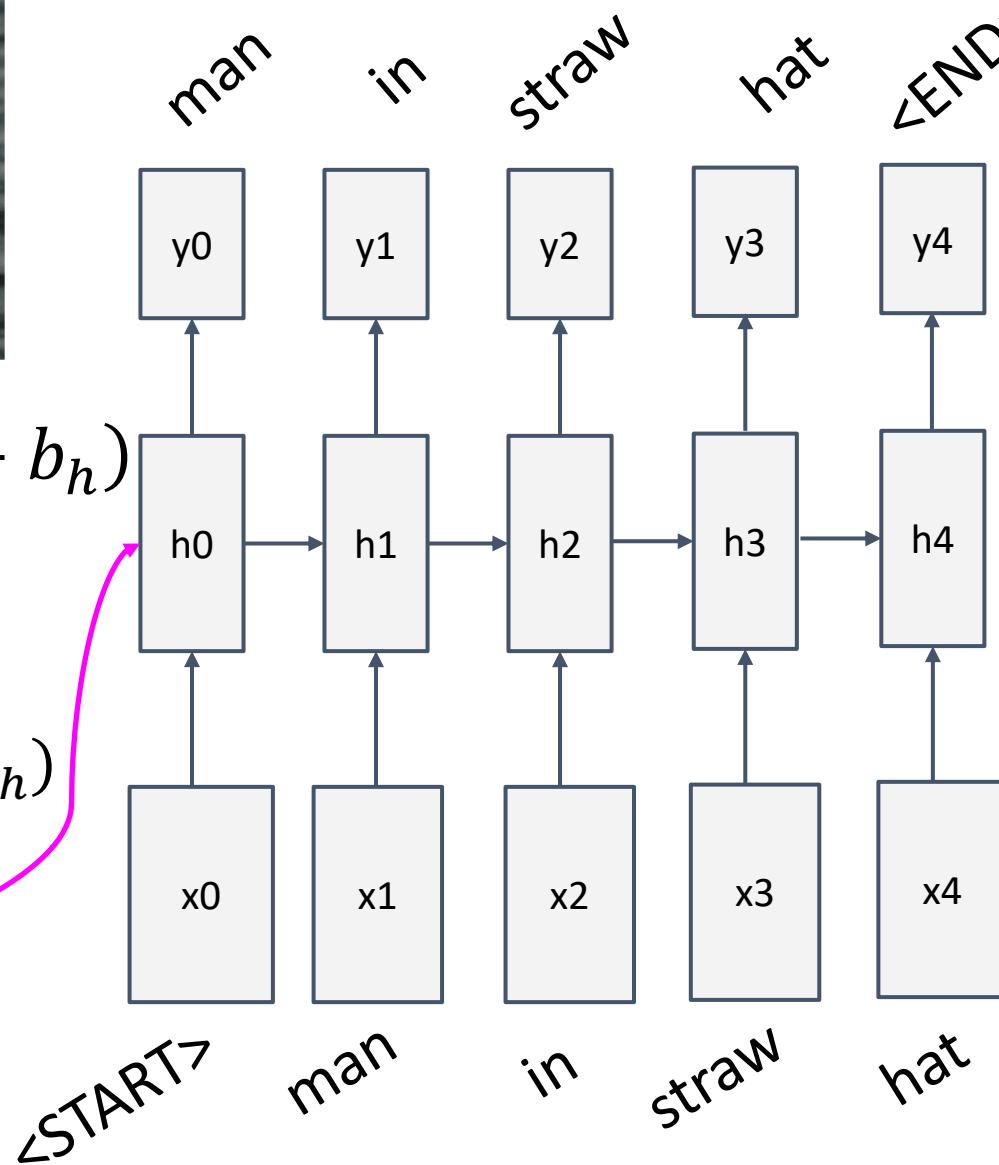


Image Captioning: Example Results



A cat sitting on a suitcase on the floor



A cat is sitting on a tree branch



A dog is running in the grass with a frisbee



A white teddy bear sitting in the grass



Two people walking on the beach with surfboards



A tennis player in action on the court



Two giraffes standing in a grassy field



A man riding a dirt bike on a dirt track

Image Captioning: Failure Cases



*A woman is holding a cat
in her hand*



*A woman standing on a beach
holding a surfboard*



*A bird is perched on a
tree branch*

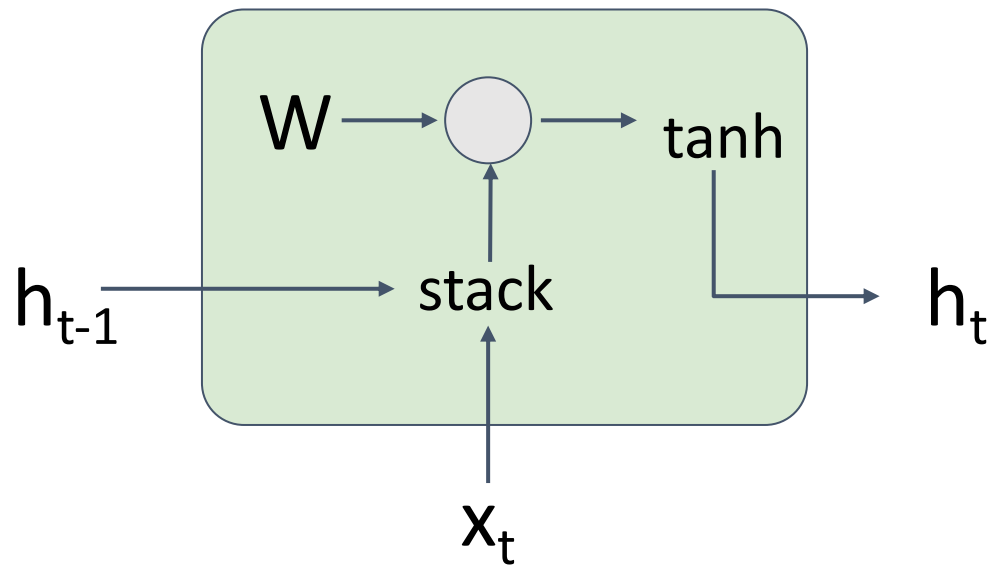


*A person holding a computer
mouse on a desk*



*A man in a
baseball uniform
throwing a ball*

Vanilla RNN Gradient Flow

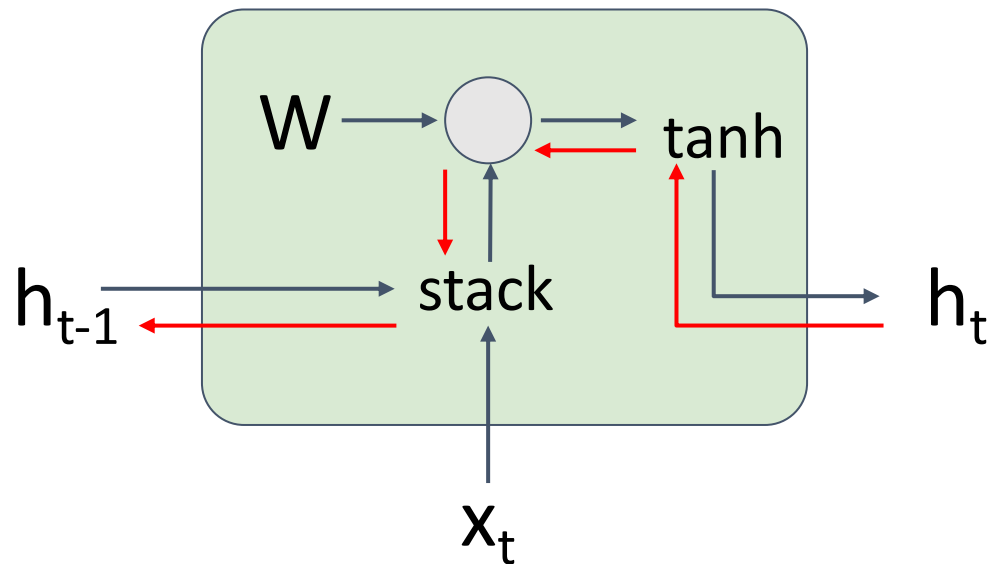


$$\begin{aligned} h_t &= \tanh(W_{hh}h_{t-1} + W_{xh}x_t + b_h) \\ &= \tanh\left((W_{hh} \quad W_{hx}) \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix} + b_h\right) \\ &= \tanh\left(W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix} + b_h\right) \end{aligned}$$

Bengio et al, "Learning long-term dependencies with gradient descent is difficult", IEEE Transactions on Neural Networks, 1994
Pascanu et al, "On the difficulty of training recurrent neural networks", ICML 2013

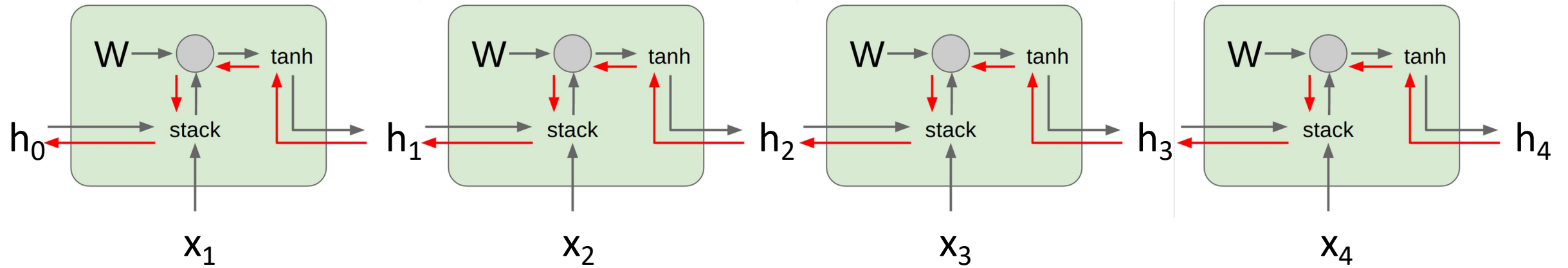
Vanilla RNN Gradient Flow

Backpropagation from h_t to h_{t-1} multiplies by W (actually W_{hh}^T)



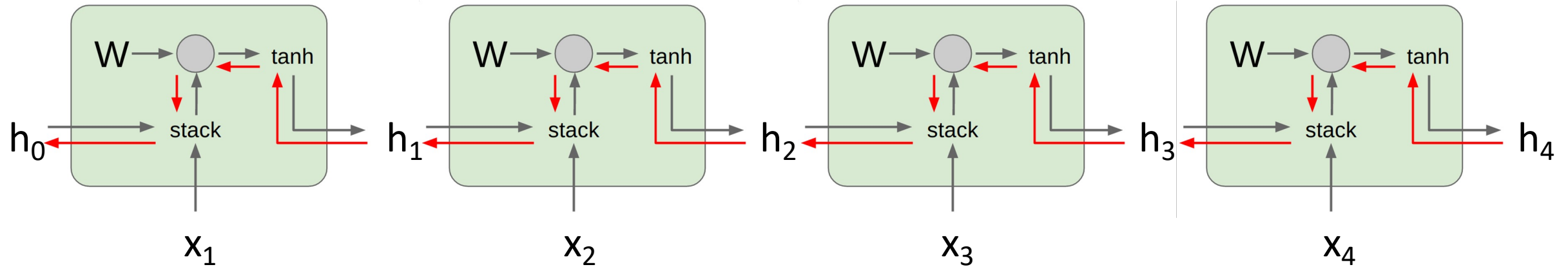
$$\begin{aligned} h_t &= \tanh(W_{hh}h_{t-1} + W_{xh}x_t + b_h) \\ &= \tanh\left((W_{hh} \quad W_{hx}) \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix} + b_h\right) \\ &= \tanh\left(W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix} + b_h\right) \end{aligned}$$

Vanilla RNN Gradient Flow



Computing gradient of h_0 involves many factors of W (and repeated \tanh)

Vanilla RNN Gradient Flow

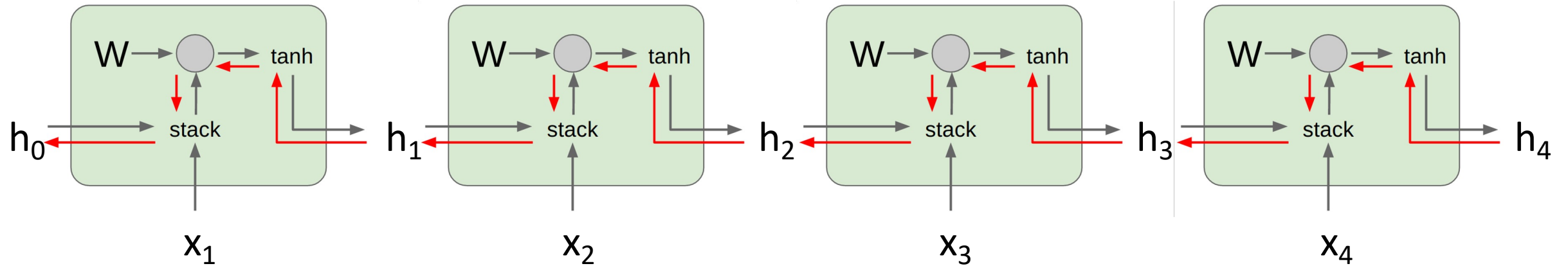


Computing gradient of h_0 involves many factors of W (and repeated tanh)

Largest singular value > 1 :
Exploding gradients

Largest singular value < 1 :
Vanishing gradients

Vanilla RNN Gradient Flow



Computing gradient of h_0 involves many factors of W (and repeated \tanh)

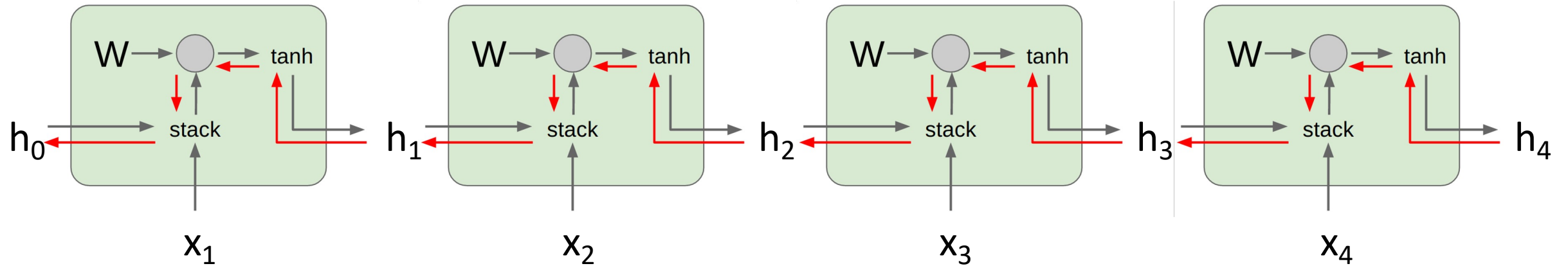
Largest singular value > 1 :
Exploding gradients

Largest singular value < 1 :
Vanishing gradients

Gradient clipping: Scale gradient if its norm is too big

```
grad_norm = np.sum(grad * grad)
if grad_norm > threshold:
    grad *= (threshold / grad_norm)
```

Vanilla RNN Gradient Flow



Computing gradient of h_0 involves many factors of W (and repeated \tanh)

Largest singular value > 1 :
Exploding gradients

Largest singular value < 1 :
Vanishing gradients

→ **Change RNN architecture!**

Long Short Term Memory (LSTM)

Vanilla RNN

$$h_t = \tanh \left(W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix} + b_h \right)$$

Long Short Term Memory (LSTM)

Vanilla RNN

$$h_t = \tanh \left(W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix} + b_h \right)$$

LSTM

$$\begin{pmatrix} i_t \\ f_t \\ o_t \\ g_t \end{pmatrix} = \begin{pmatrix} \sigma \\ \sigma \\ \sigma \\ \tanh \end{pmatrix} \left(W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix} + b_h \right)$$
$$c_t = f_t \odot c_{t-1} + i_t \odot g_t$$
$$h_t = o_t \odot \tanh(c_t)$$

Long Short Term Memory (LSTM)

Vanilla RNN

$$h_t = \tanh \left(W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix} + b_h \right)$$

Two vectors at each timestep:

Cell state: $c_t \in \mathbb{R}^H$

Hidden state: $h_t \in \mathbb{R}^H$

LSTM

$$\begin{pmatrix} i_t \\ f_t \\ o_t \\ g_t \end{pmatrix} = \begin{pmatrix} \sigma \\ \sigma \\ \sigma \\ \tanh \end{pmatrix} \left(W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix} + b_h \right)$$
$$c_t = f_t \odot c_{t-1} + i_t \odot g_t$$
$$h_t = o_t \odot \tanh(c_t)$$



Long Short Term Memory (LSTM)

Vanilla RNN

$$h_t = \tanh \left(W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix} + b_h \right)$$

Compute four “gates” per timestep:

Input gate: $i_t \in \mathbb{R}^H$

Forget gate: $f_t \in \mathbb{R}^H$

Output gate: $o_t \in \mathbb{R}^H$

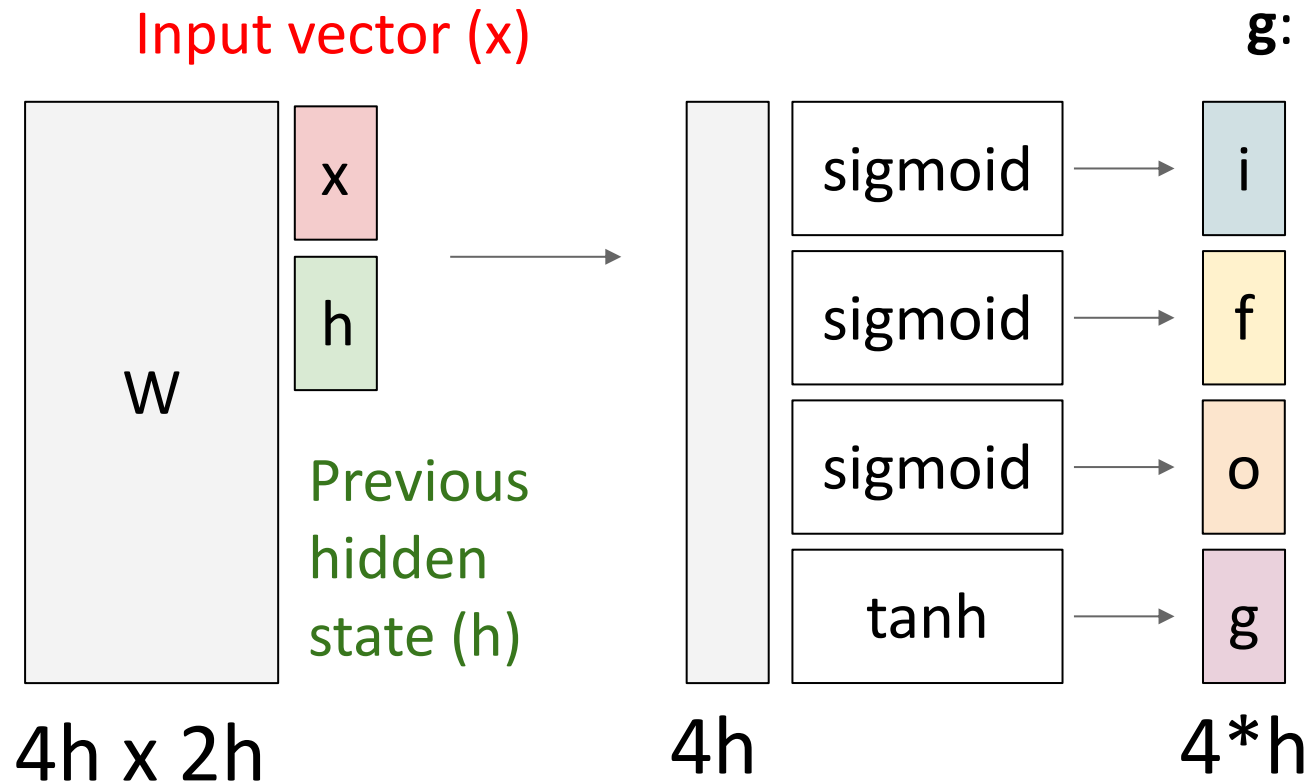
“Gate?” gate: $g_t \in \mathbb{R}^H$

LSTM

$$\begin{pmatrix} i_t \\ f_t \\ o_t \\ g_t \end{pmatrix} = \begin{pmatrix} \sigma \\ \sigma \\ \sigma \\ \tanh \end{pmatrix} \left(W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix} + b_h \right)$$
$$c_t = f_t \odot c_{t-1} + i_t \odot g_t$$
$$h_t = o_t \odot \tanh(c_t)$$

Long Short Term Memory (LSTM)

- i**: Input gate, whether to write to cell
- f**: Forget gate, Whether to erase cell
- o**: Output gate, How much to reveal cell
- g**: Gate gate (?), How much to write to cell

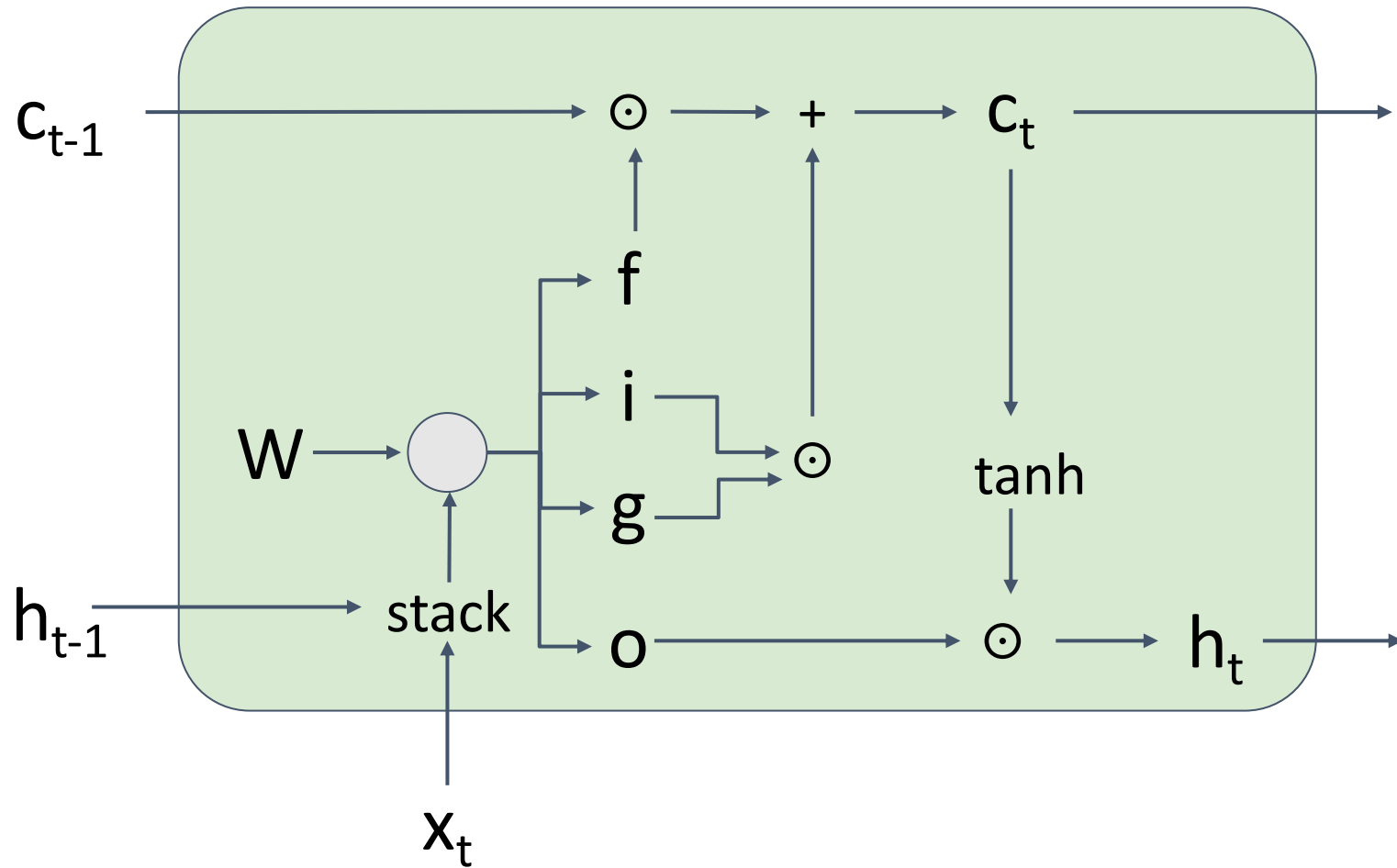


$$\begin{pmatrix} i_t \\ f_t \\ o_t \\ g_t \end{pmatrix} = \begin{pmatrix} \sigma \\ \sigma \\ \sigma \\ \tanh \end{pmatrix} \left(W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix} + b_h \right)$$

$$c_t = f_t \odot c_{t-1} + i_t \odot g_t$$

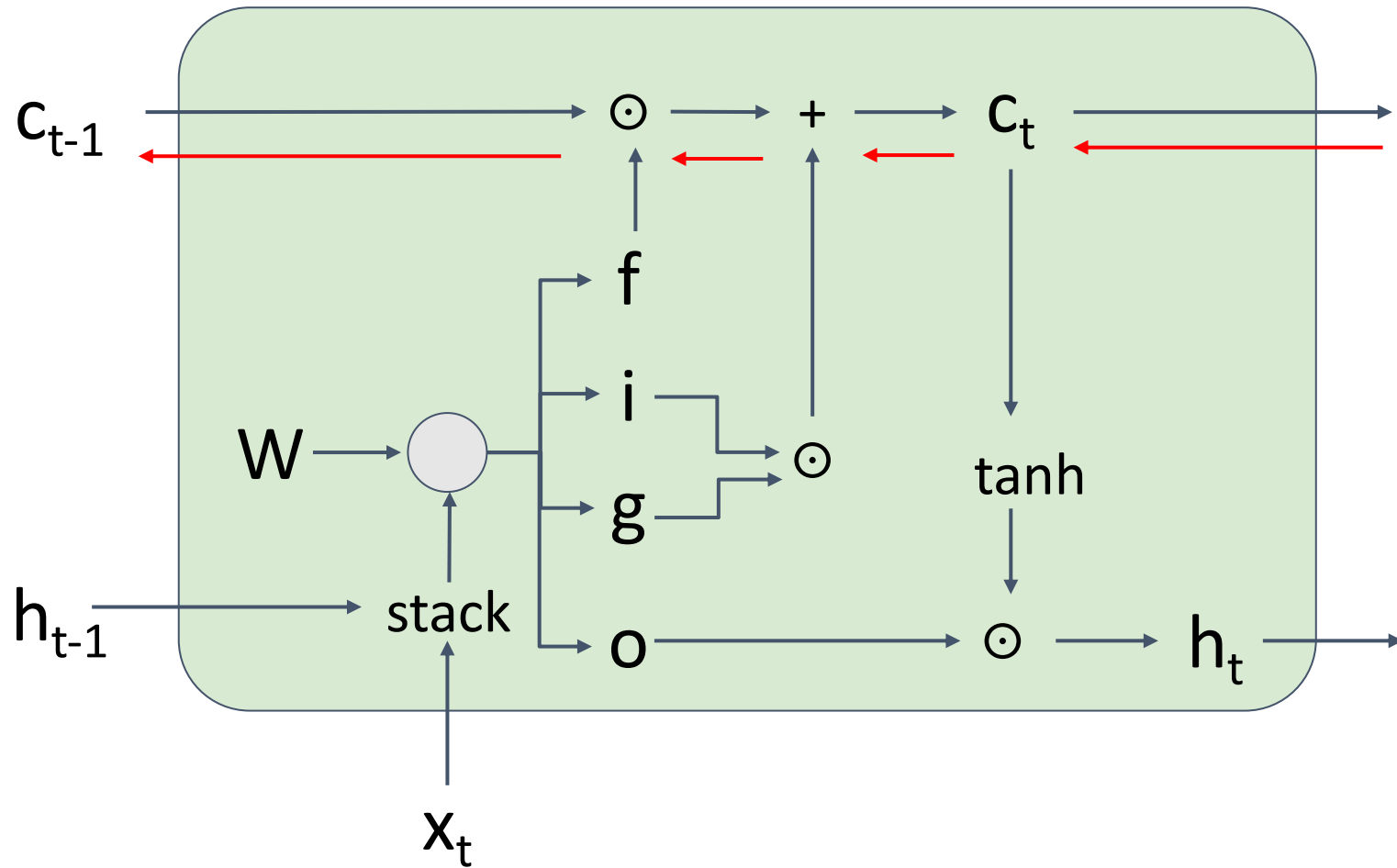
$$h_t = o_t \odot \tanh(c_t)$$

Long Short Term Memory (LSTM)



$$\begin{pmatrix} i_t \\ f_t \\ o_t \\ g_t \end{pmatrix} = \begin{pmatrix} \sigma \\ \sigma \\ \sigma \\ \tanh \end{pmatrix} \left(W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix} + b_h \right)$$
$$c_t = f_t \odot c_{t-1} + i_t \odot g_t$$
$$h_t = o_t \odot \tanh(c_t)$$

Long Short Term Memory (LSTM): Gradient Flow



Backpropagation from c_t to c_{t-1} only elementwise multiplication by f , no matrix multiply by W

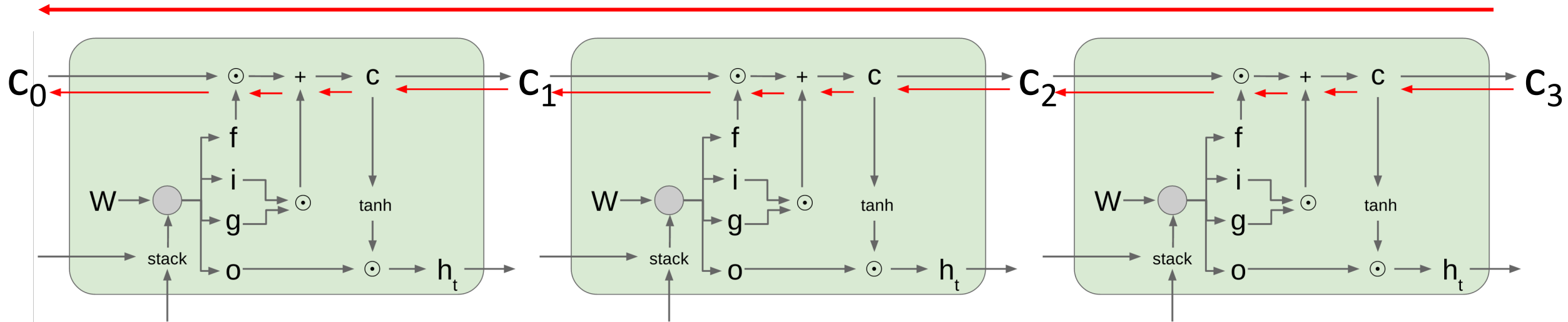
$$\begin{pmatrix} i_t \\ f_t \\ o_t \\ g_t \end{pmatrix} = \begin{pmatrix} \sigma \\ \sigma \\ \sigma \\ \tanh \end{pmatrix} \left(W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix} + b_h \right)$$

$$c_t = f_t \odot c_{t-1} + i_t \odot g_t$$

$$h_t = o_t \odot \tanh(c_t)$$

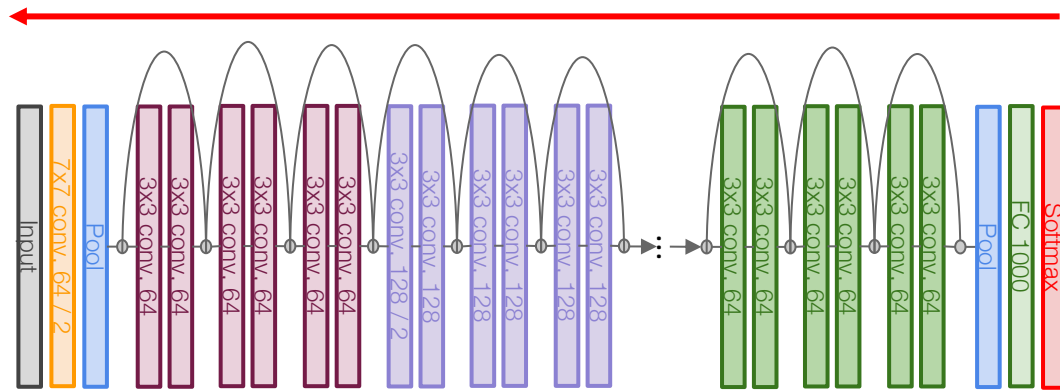
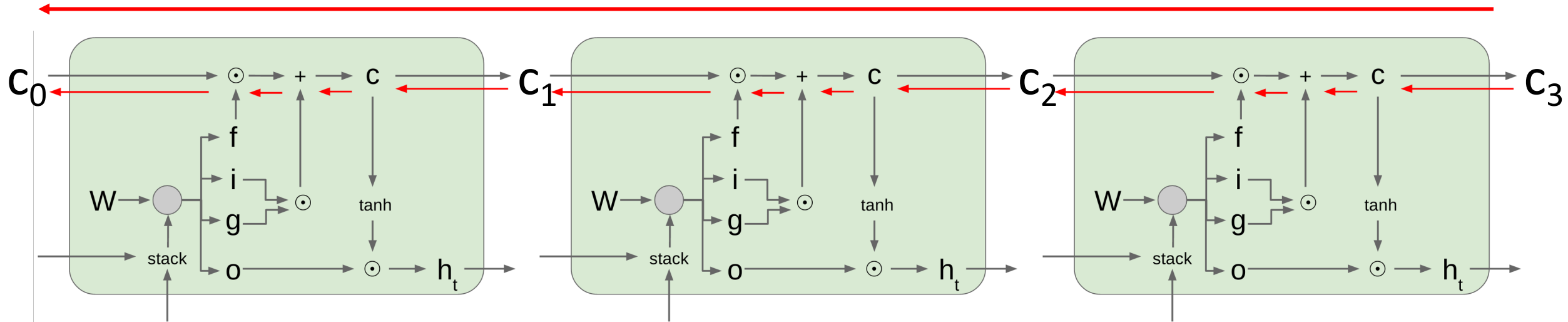
Long Short Term Memory (LSTM): Gradient Flow

Uninterrupted gradient flow!



Long Short Term Memory (LSTM): Gradient Flow

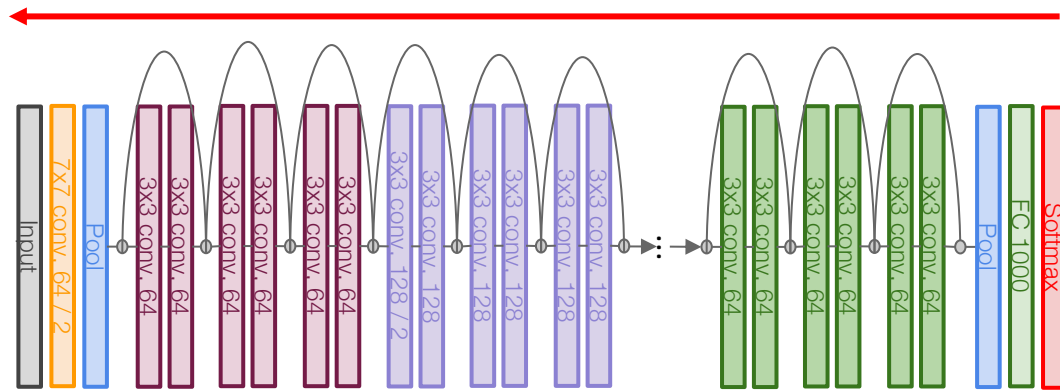
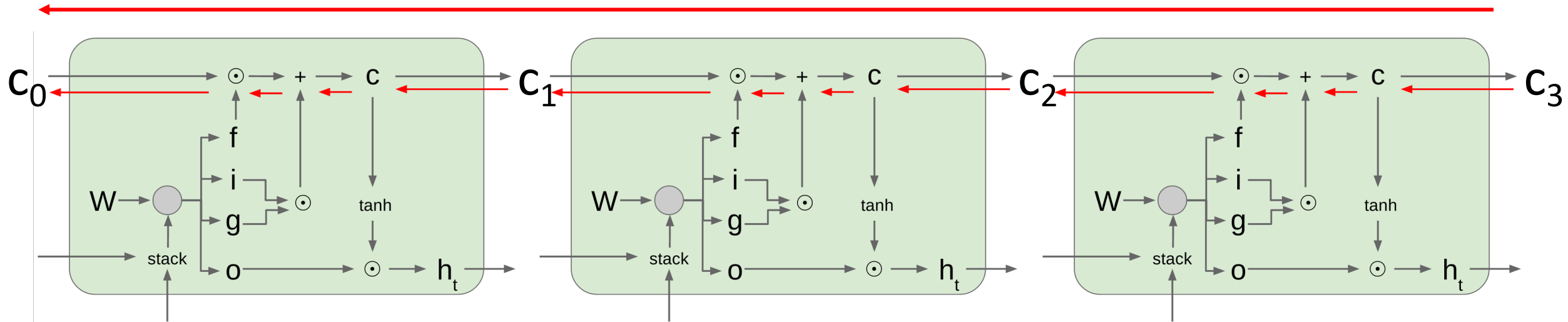
Uninterrupted gradient flow!



Similar to ResNet!

Long Short Term Memory (LSTM): Gradient Flow

Uninterrupted gradient flow!



Similar to ResNet!

In between: **Highway Networks**

$$g_t = F(x, W_t)$$

$$y_t = g_t \odot H(x, W_h) + (1 - g_t) \odot x_t$$

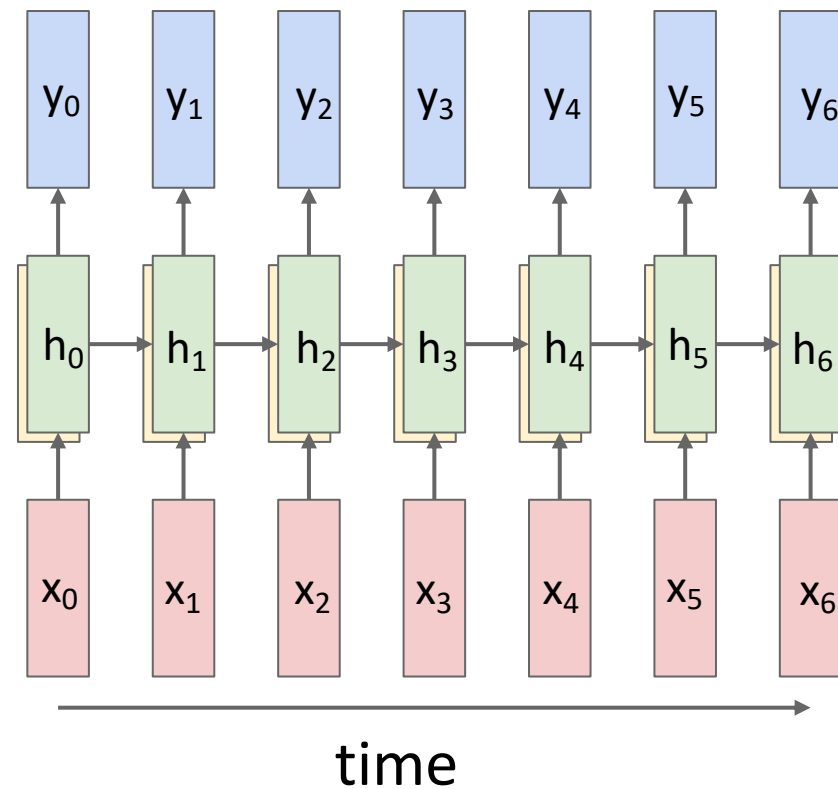
Srivastava et al, "Highway Networks", ICML DL Workshop 2015

Single-Layer RNNs

$$h_t = \tanh \left(W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix} + b_h \right)$$

LSTM:

$$\begin{pmatrix} i_t \\ f_t \\ o_t \\ g_t \end{pmatrix} = \begin{pmatrix} \sigma \\ \sigma \\ \sigma \\ \tanh \end{pmatrix} \left(W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix} + b_h \right)$$
$$c_t = f_t \odot c_{t-1} + i_t \odot g_t$$
$$h_t = o_t \odot \tanh(c_t)$$



Multilayer RNNs

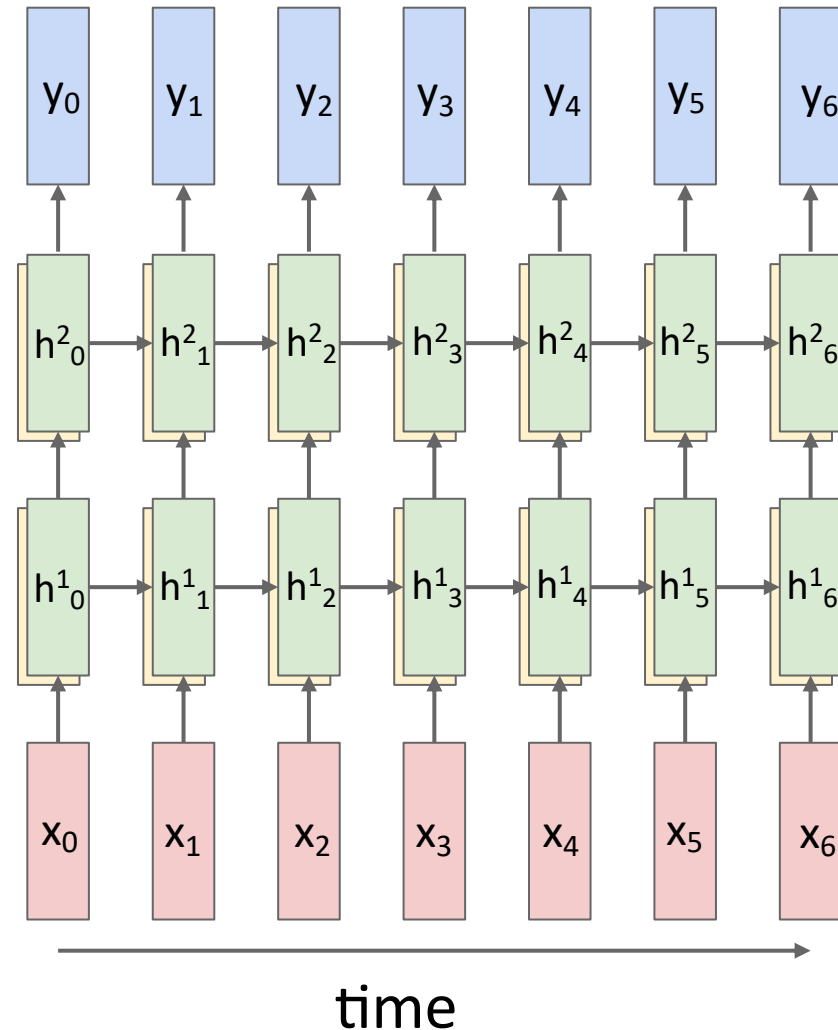
depth ↑

$$h_t^\ell = \tanh \left(W \begin{pmatrix} h_{t-1}^\ell \\ h_t^{\ell-1} \end{pmatrix} + b_h^\ell \right)$$

LSTM:

$$\begin{pmatrix} i_t^\ell \\ f_t^\ell \\ o_t^\ell \\ g_t^\ell \end{pmatrix} = \begin{pmatrix} \sigma \\ \sigma \\ \sigma \\ \tanh \end{pmatrix} \left(W \begin{pmatrix} h_{t-1}^\ell \\ h_t^{\ell-1} \end{pmatrix} + b_h^\ell \right)$$
$$c_t^\ell = f_t^\ell \odot c_{t-1}^\ell + i_t^\ell \odot g_t^\ell$$
$$h_t^\ell = o_t^\ell \odot \tanh(c_t^\ell)$$

Two-layer RNN: Pass hidden states from one RNN as inputs to another RNN

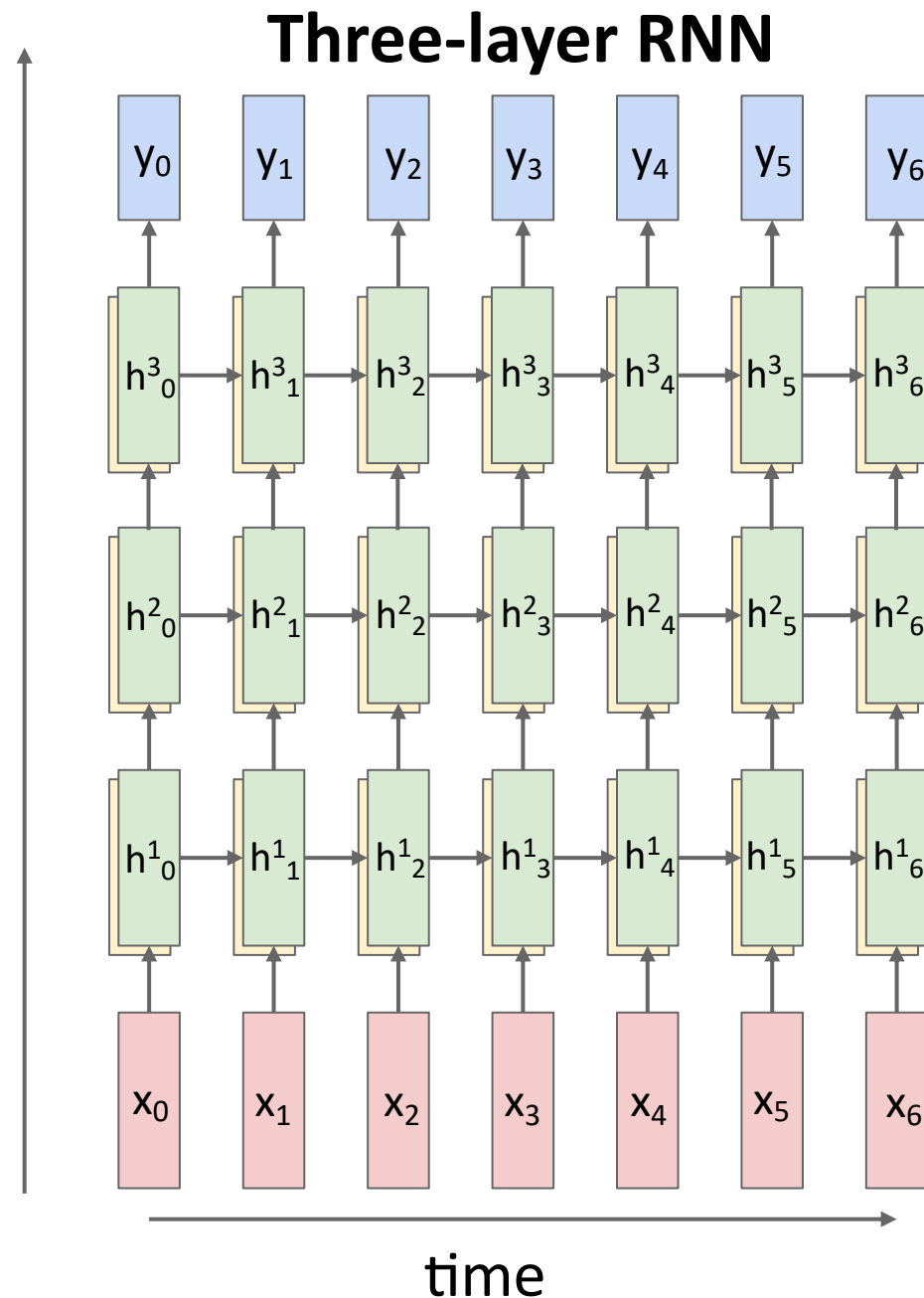


Multilayer RNNs

$$h_t^\ell = \tanh \left(W \begin{pmatrix} h_{t-1}^\ell \\ h_t^{\ell-1} \end{pmatrix} + b_h^\ell \right)$$

LSTM:

$$\begin{pmatrix} i_t^\ell \\ f_t^\ell \\ o_t^\ell \\ g_t^\ell \end{pmatrix} = \begin{pmatrix} \sigma \\ \sigma \\ \sigma \\ \tanh \end{pmatrix} \left(W \begin{pmatrix} h_{t-1}^\ell \\ h_t^{\ell-1} \end{pmatrix} + b_h^\ell \right)$$
$$c_t^\ell = f_t^\ell \odot c_{t-1}^\ell + i_t^\ell \odot g_t^\ell$$
$$h_t^\ell = o_t^\ell \odot \tanh(c_t^\ell)$$



Other RNN Variants

Gated Recurrent Unit (GRU)

Cho et al “Learning phrase representations using RNN encoder-decoder for statistical machine translation”, 2014

$$r_t = \sigma(W_{xr}x_t + W_{hr}h_{t-1} + b_r)$$

$$z_t = \sigma(W_{xz}x_t + W_{hz}h_{t-1} + b_z)$$

$$\tilde{h}_t = \tanh(W_{xh}x_t + W_{hh}(r_t \odot h_{t-1}) + b_h)$$

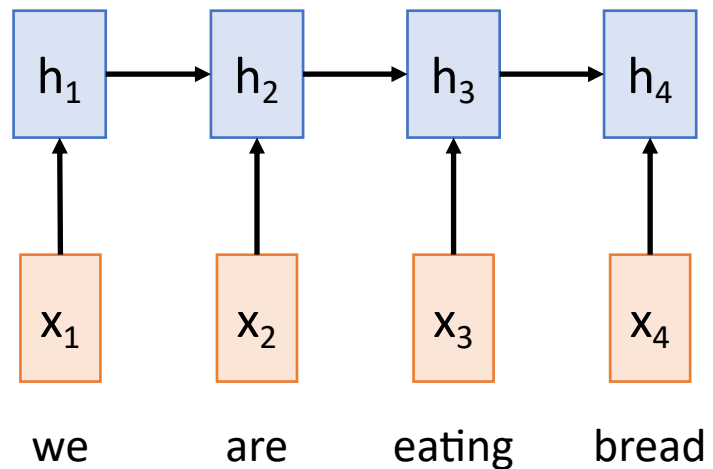
$$h_t = z_t \odot h_{t-1} + (1 - z_t) \odot \tilde{h}_t$$

Sequence-to-Sequence with RNNs

Input: Sequence x_1, \dots, x_T

Output: Sequence y_1, \dots, y_T

Encoder: $h_t = f_W(x_t, h_{t-1})$



Sequence-to-Sequence with RNNs

Input: Sequence x_1, \dots, x_T

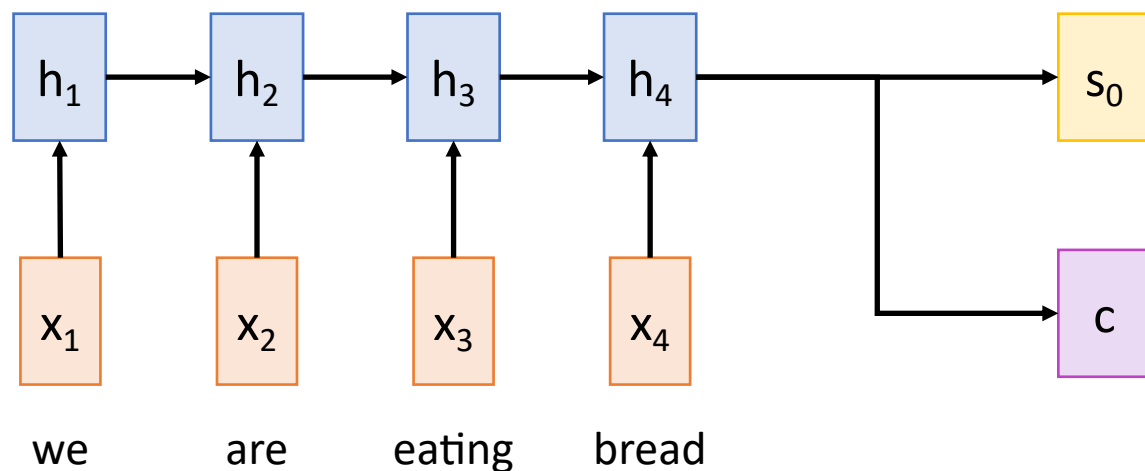
Output: Sequence y_1, \dots, y_T

Encoder: $h_t = f_W(x_t, h_{t-1})$

From final hidden state predict:

Initial decoder state s_0

Context vector c (often $c=h_T$)



Sequence-to-Sequence with RNNs

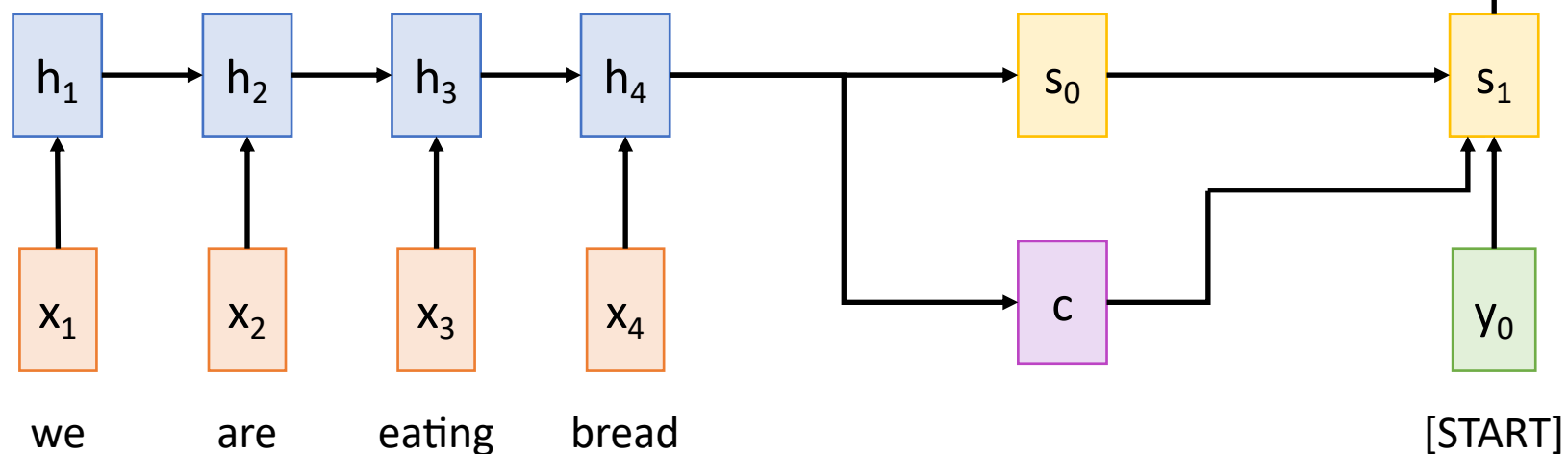
Input: Sequence x_1, \dots, x_T

Output: Sequence y_1, \dots, y_T

Decoder: $s_t = g_U(y_{t-1}, s_{t-1}, c)$

Encoder: $h_t = f_W(x_t, h_{t-1})$

From final hidden state predict:
Initial decoder state s_0
Context vector c (often $c=h_T$)



Sequence-to-Sequence with RNNs

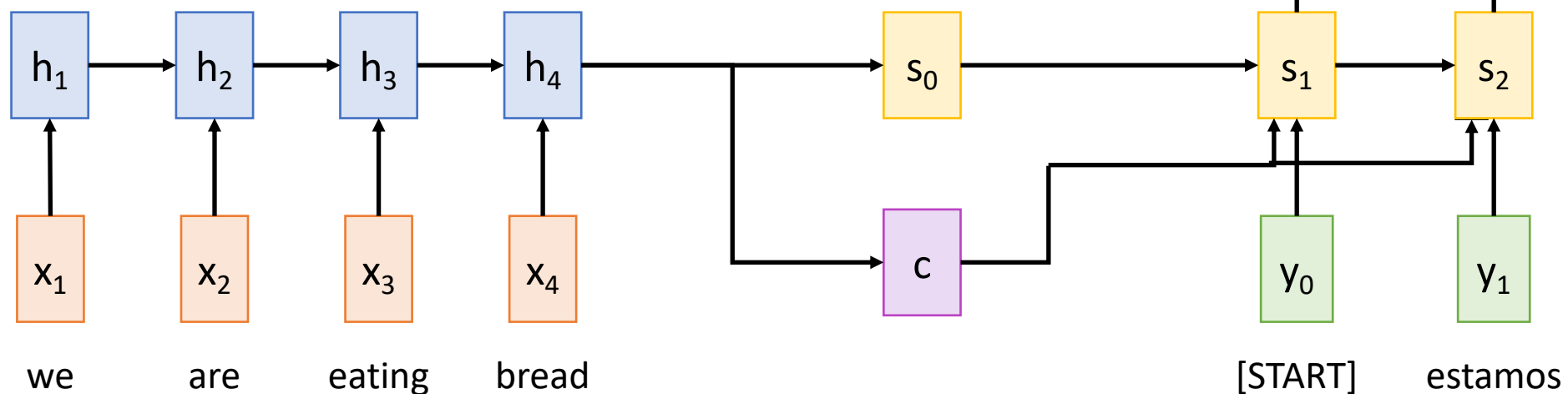
Input: Sequence x_1, \dots, x_T

Output: Sequence y_1, \dots, y_T

Decoder: $s_t = g_U(y_{t-1}, s_{t-1}, c)$

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Sequence-to-Sequence with RNNs

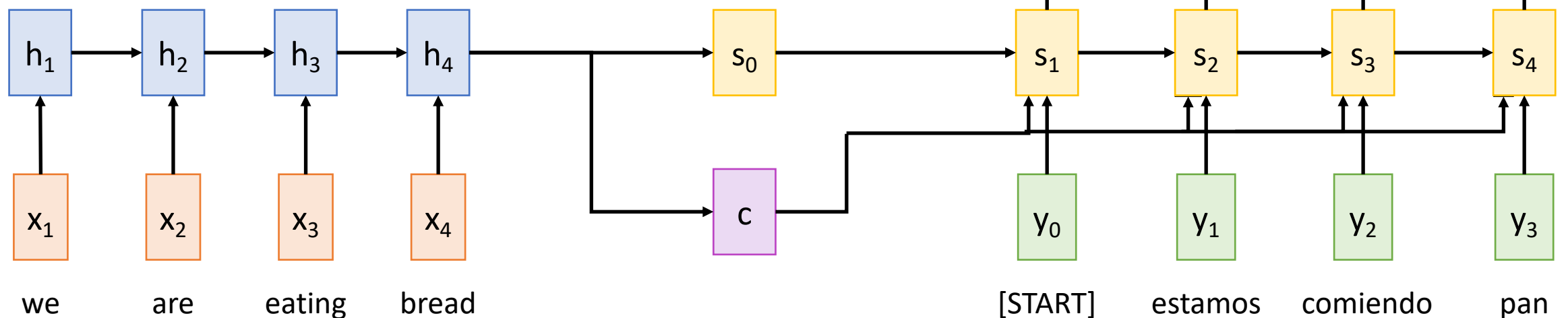
Input: Sequence x_1, \dots, x_T

Output: Sequence y_1, \dots, y_T

Decoder: $s_t = g_U(y_{t-1}, s_{t-1}, c)$

Encoder: $h_t = f_W(x_t, h_{t-1})$

From final hidden state predict:
Initial decoder state s_0
Context vector c (often $c=h_T$)

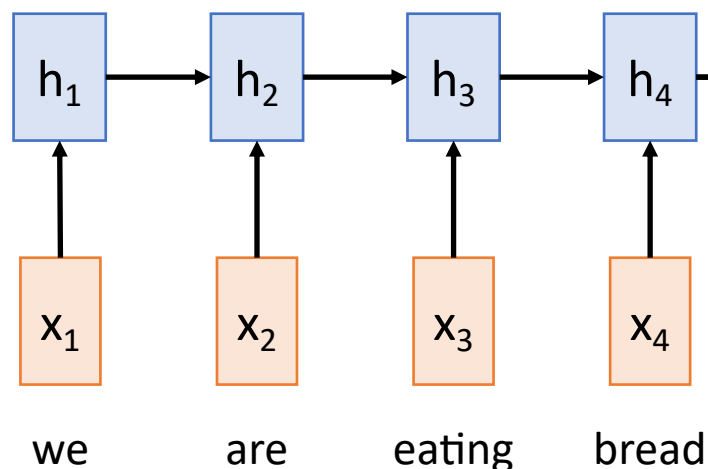


Sequence-to-Sequence with RNNs

Input: Sequence x_1, \dots, x_T

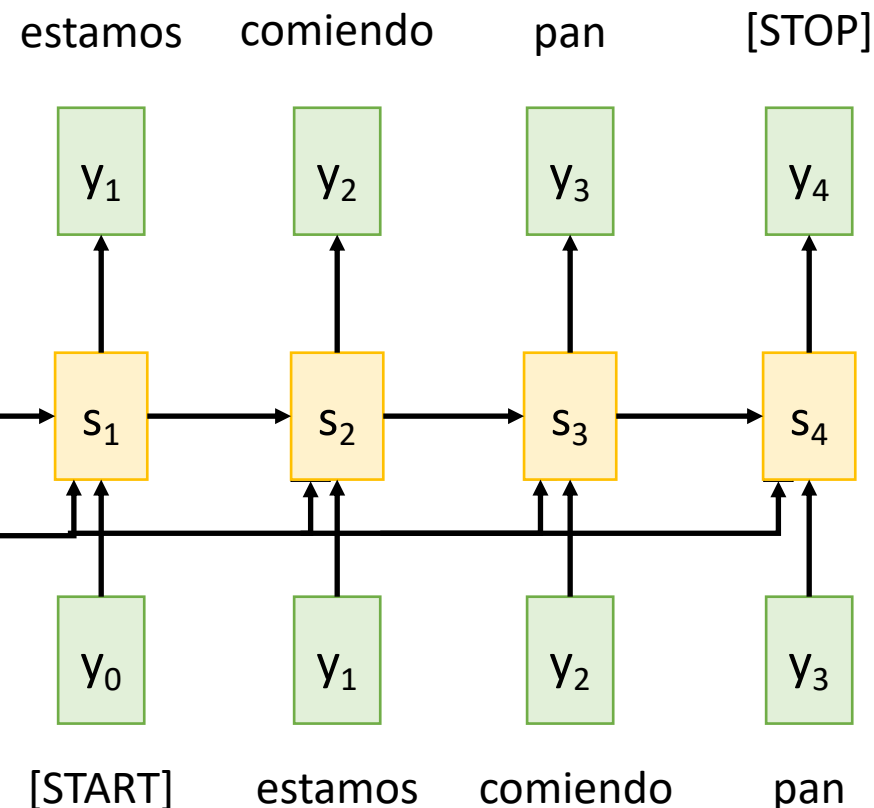
Output: Sequence y_1, \dots, y_T'

Encoder: $h_t = f_W(x_t, h_{t-1})$



From final hidden state predict:
Initial decoder state s_0
Context vector c (often $c=h_T$)

Decoder: $s_t = g_U(y_{t-1}, s_{t-1}, c)$



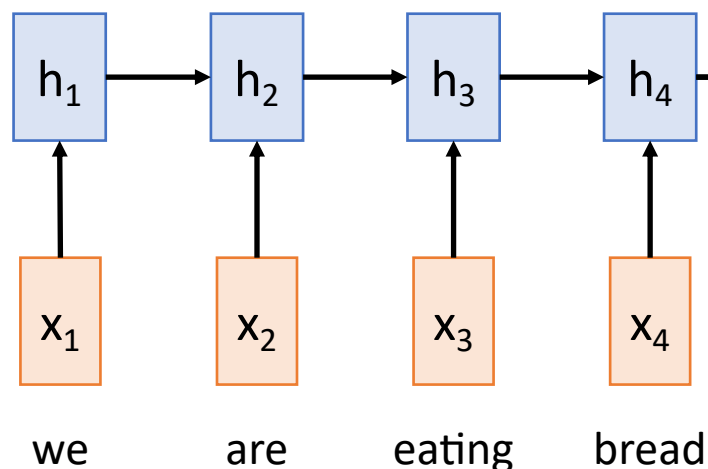
Problem: Input sequence bottlenecked through fixed-sized vector. What if $T=1000$?

Sequence-to-Sequence with RNNs

Input: Sequence x_1, \dots, x_T

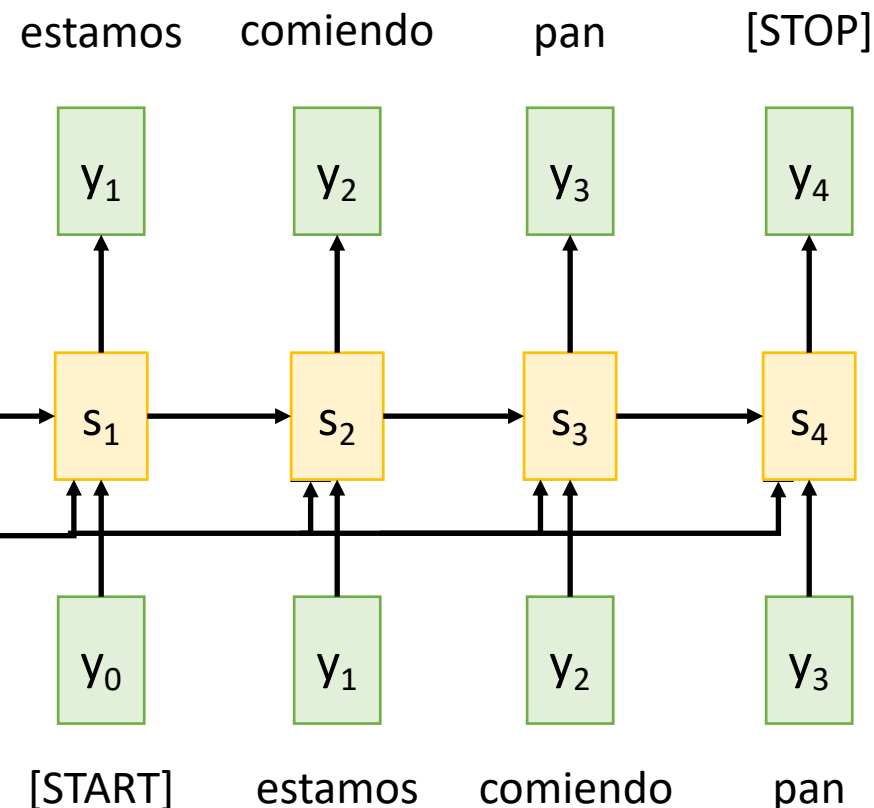
Output: Sequence y_1, \dots, y_T'

Encoder: $h_t = f_W(x_t, h_{t-1})$



From final hidden state predict:
Initial decoder state s_0
Context vector c (often $c=h_T$)

Decoder: $s_t = g_U(y_{t-1}, s_{t-1}, c)$



Problem: Input sequence bottlenecked through fixed-sized vector. What if $T=1000$?

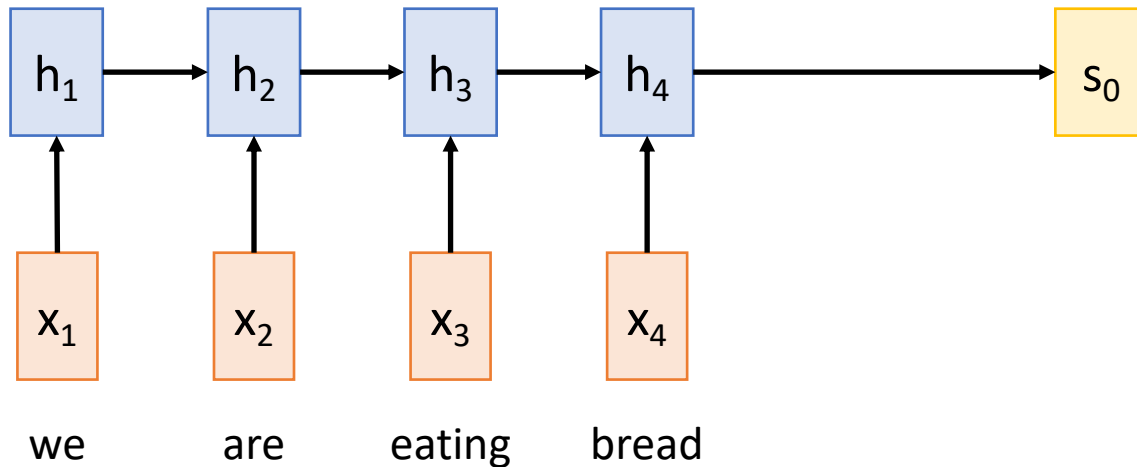
Idea: use new context vector at each step of decoder!

Sequence-to-Sequence with RNNs and Attention

Input: Sequence x_1, \dots, x_T

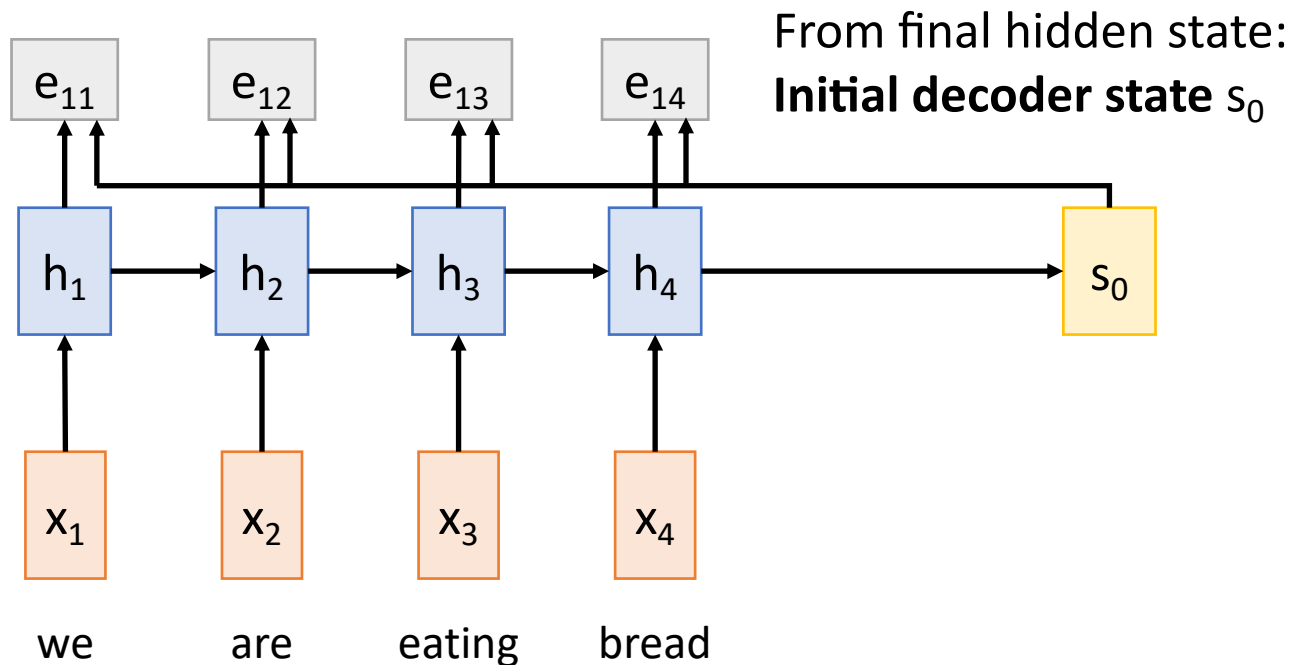
Output: Sequence y_1, \dots, y_T

Encoder: $h_t = f_W(x_t, h_{t-1})$ From final hidden state:
Initial decoder state s_0

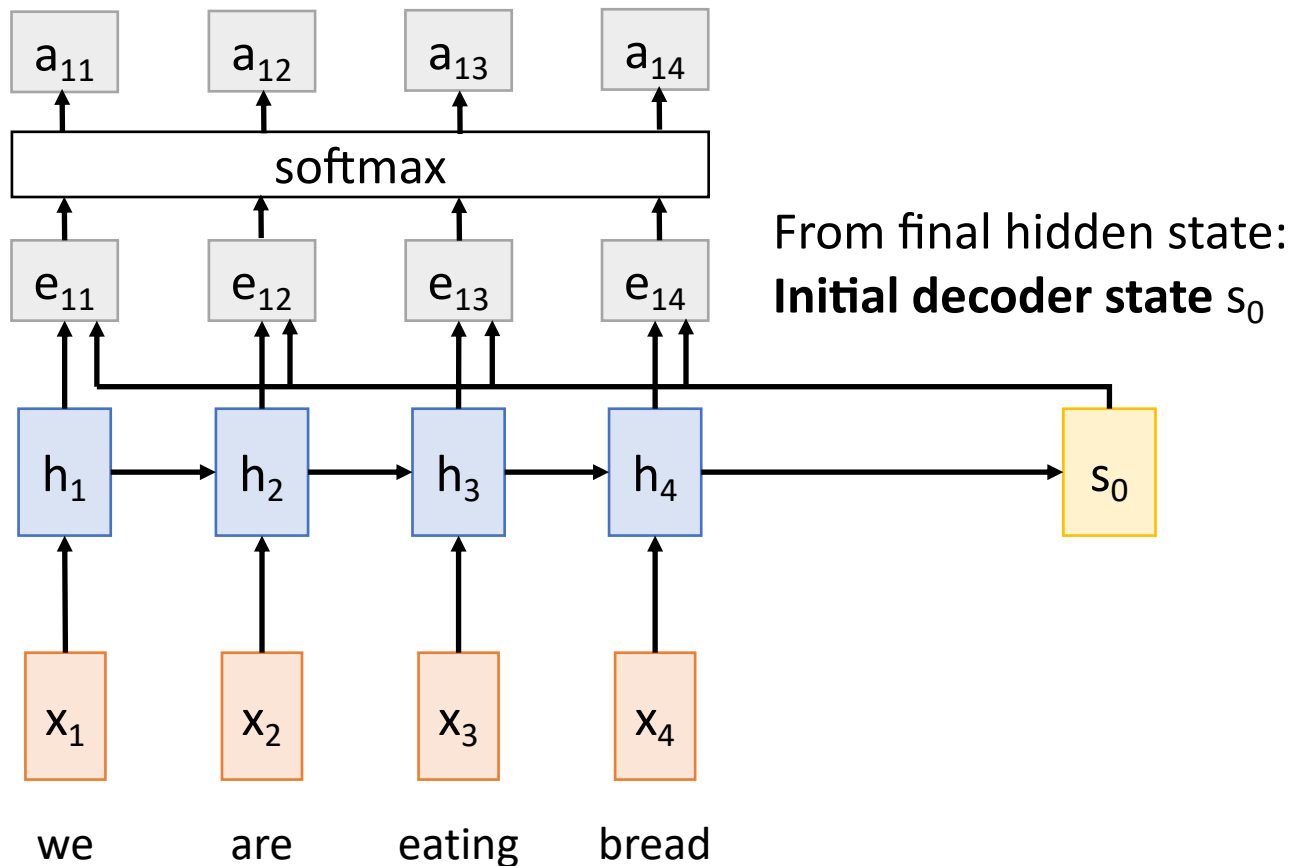


Sequence-to-Sequence with RNNs and Attention

Compute (scalar) **alignment scores**
 $e_{t,i} = f_{\text{att}}(s_{t-1}, h_i)$ (f_{att} is an MLP)



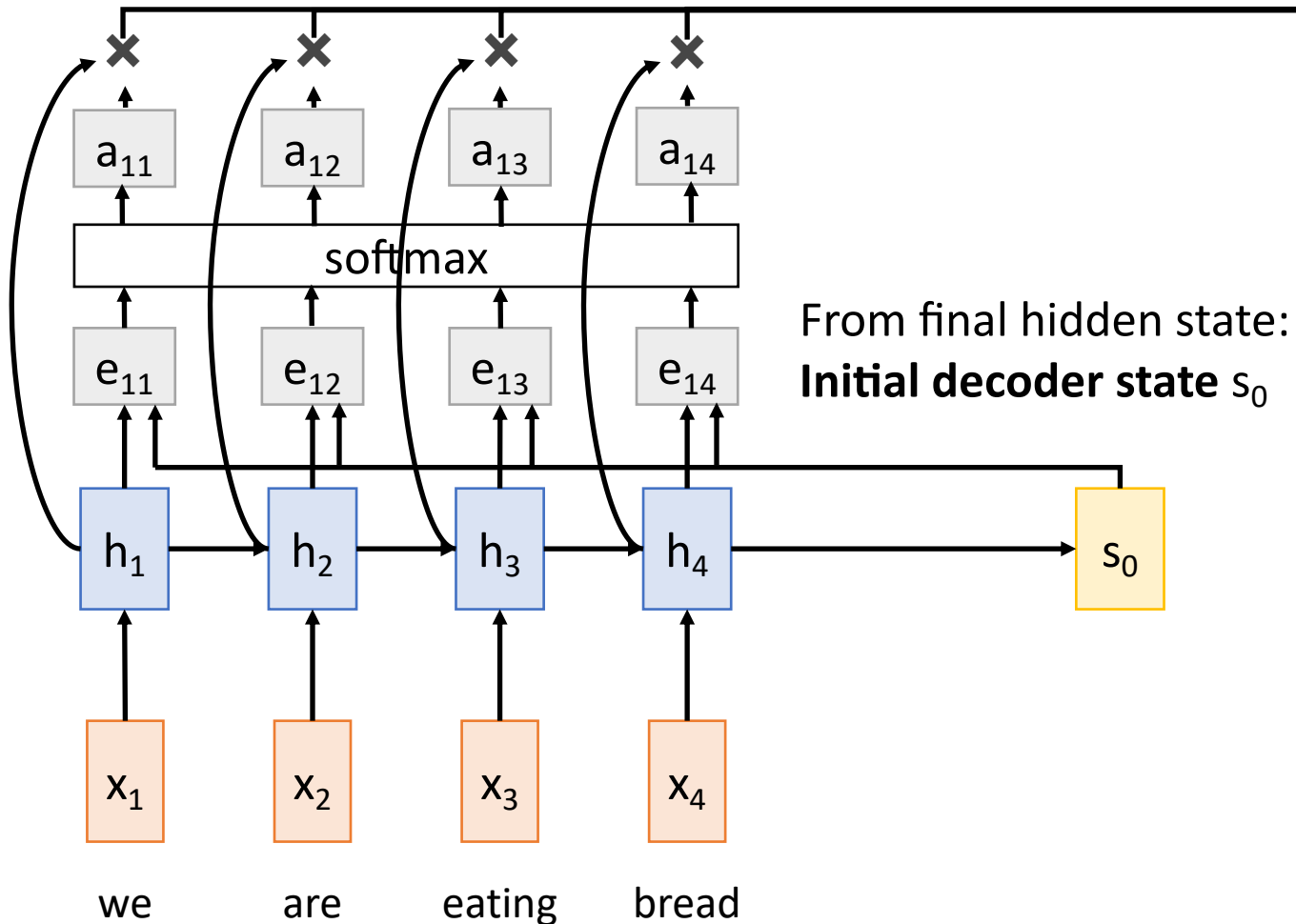
Sequence-to-Sequence with RNNs and Attention



Compute (scalar) **alignment scores**
 $e_{t,i} = f_{\text{att}}(s_{t-1}, h_i)$ (f_{att} is an MLP)

Normalize alignment scores
to get **attention weights**
 $0 < a_{t,i} < 1 \quad \sum_i a_{t,i} = 1$

Sequence-to-Sequence with RNNs and Attention



Compute (scalar) **alignment scores**
 $e_{t,i} = f_{\text{att}}(s_{t-1}, h_i)$ (f_{att} is an MLP)

Normalize alignment scores
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 $0 < a_{t,i} < 1 \quad \sum_i a_{t,i} = 1$

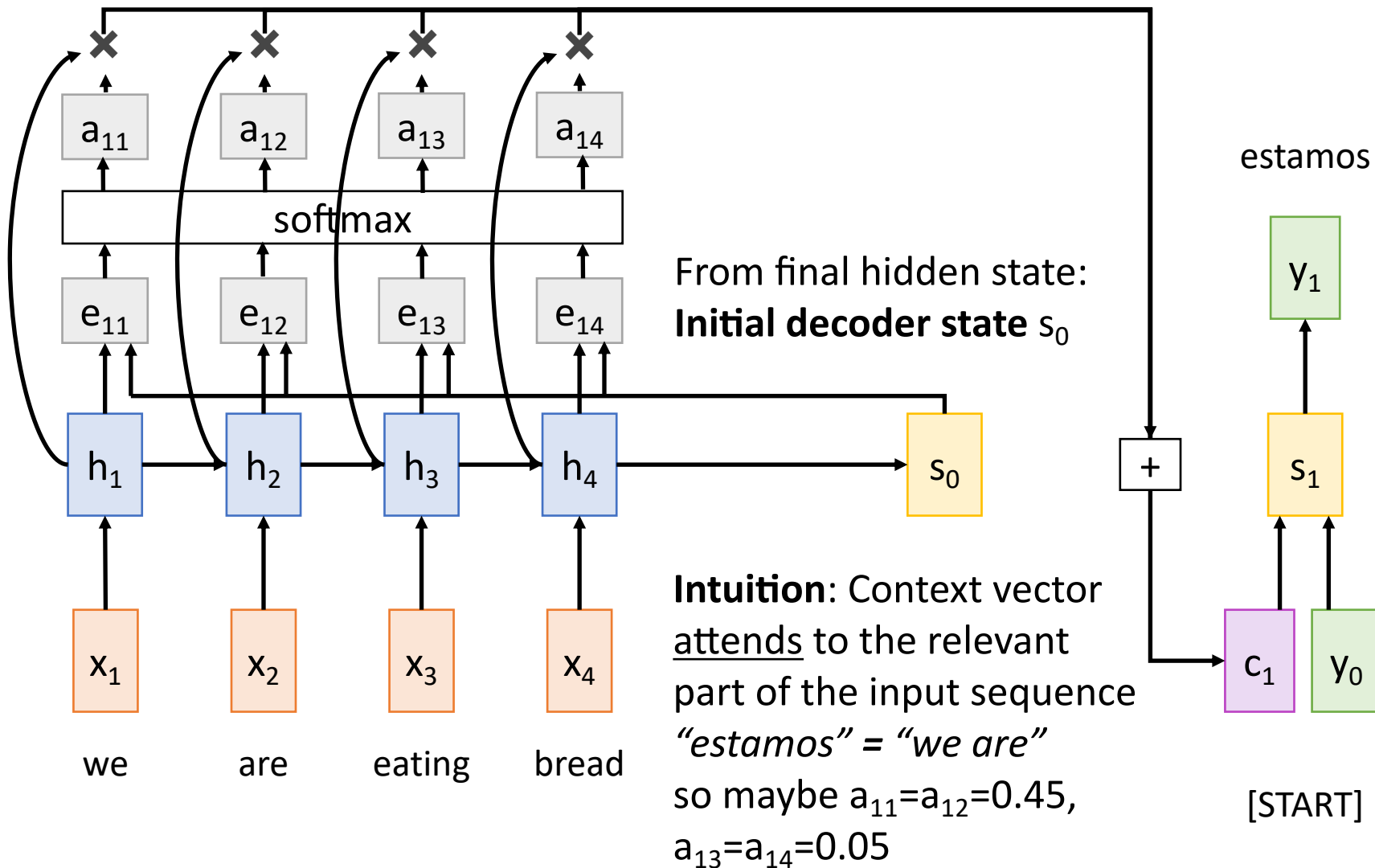
Compute context vector as linear
combination of hidden states
 $c_t = \sum_i a_{t,i} h_i$

Use context vector in
decoder: $s_t = g_U(y_{t-1}, s_{t-1}, c_t)$

[START]

This is all differentiable! Do not supervise attention weights – backprop through everything

Sequence-to-Sequence with RNNs and Attention



Compute (scalar) **alignment scores**
 $e_{t,i} = f_{\text{att}}(s_{t-1}, h_i)$ (f_{att} is an MLP)

Normalize alignment scores to get **attention weights**
 $0 < a_{t,i} < 1 \quad \sum_i a_{t,i} = 1$

Compute context vector as linear combination of hidden states
 $c_t = \sum_i a_{t,i} h_i$

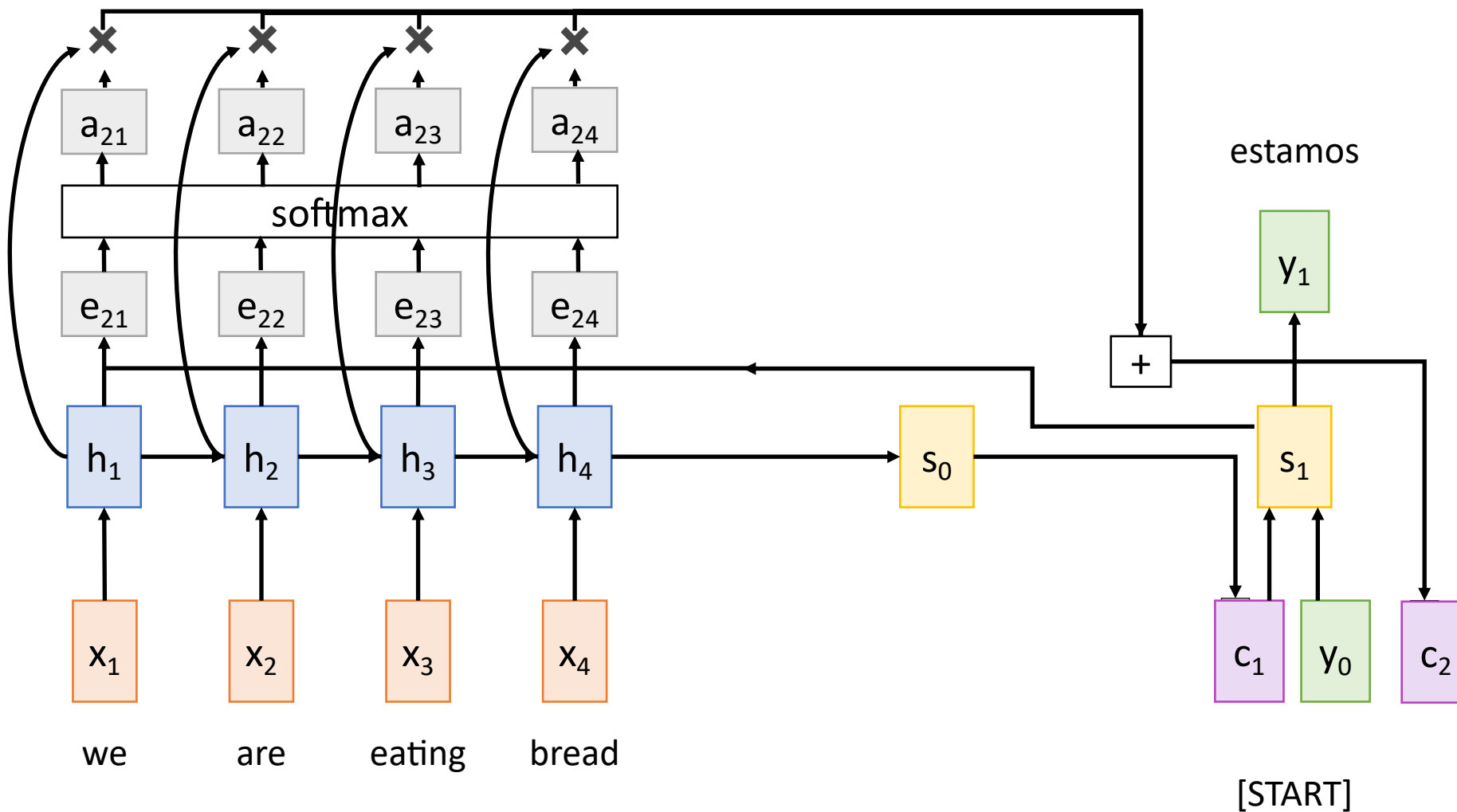
Use context vector in decoder: $s_t = g_U(y_{t-1}, s_{t-1}, c_t)$

This is all differentiable! Do not supervise attention weights – backprop through everything

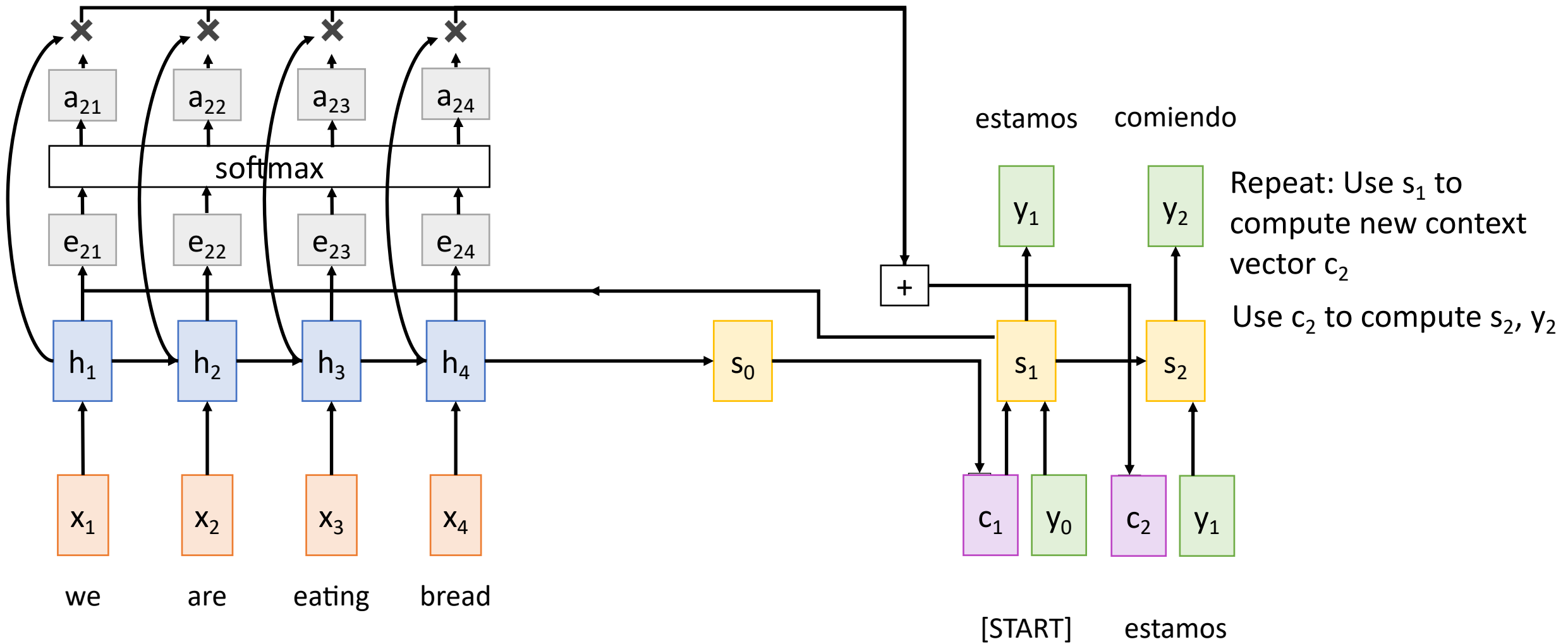
Bahdanau et al, “Neural machine translation by jointly learning to align and translate”, ICLR 2015

Sequence-to-Sequence with RNNs

Repeat: Use s_1 to compute new context vector c_2

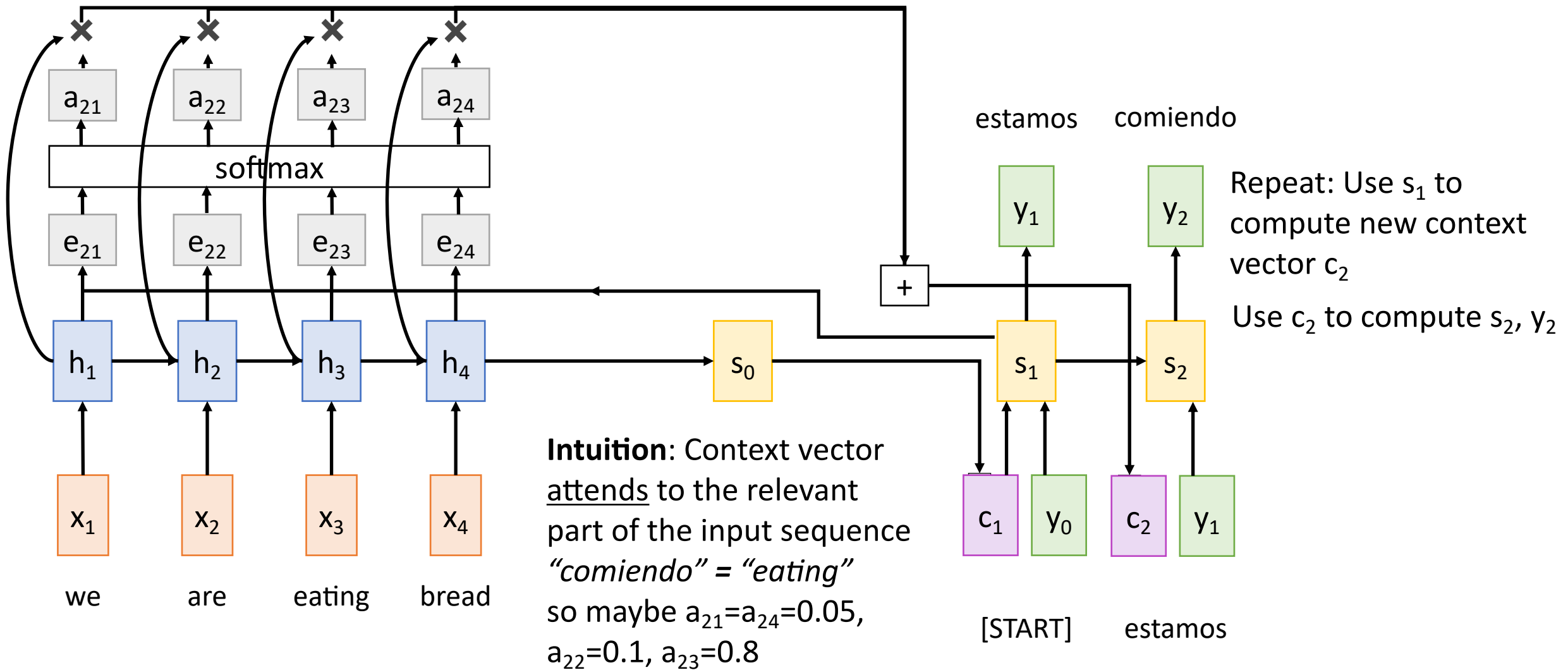


Sequence-to-Sequence with RNNs and Attention



Bahdanau et al, "Neural machine translation by jointly learning to align and translate", ICLR 2015

Sequence-to-Sequence with RNNs and Attention

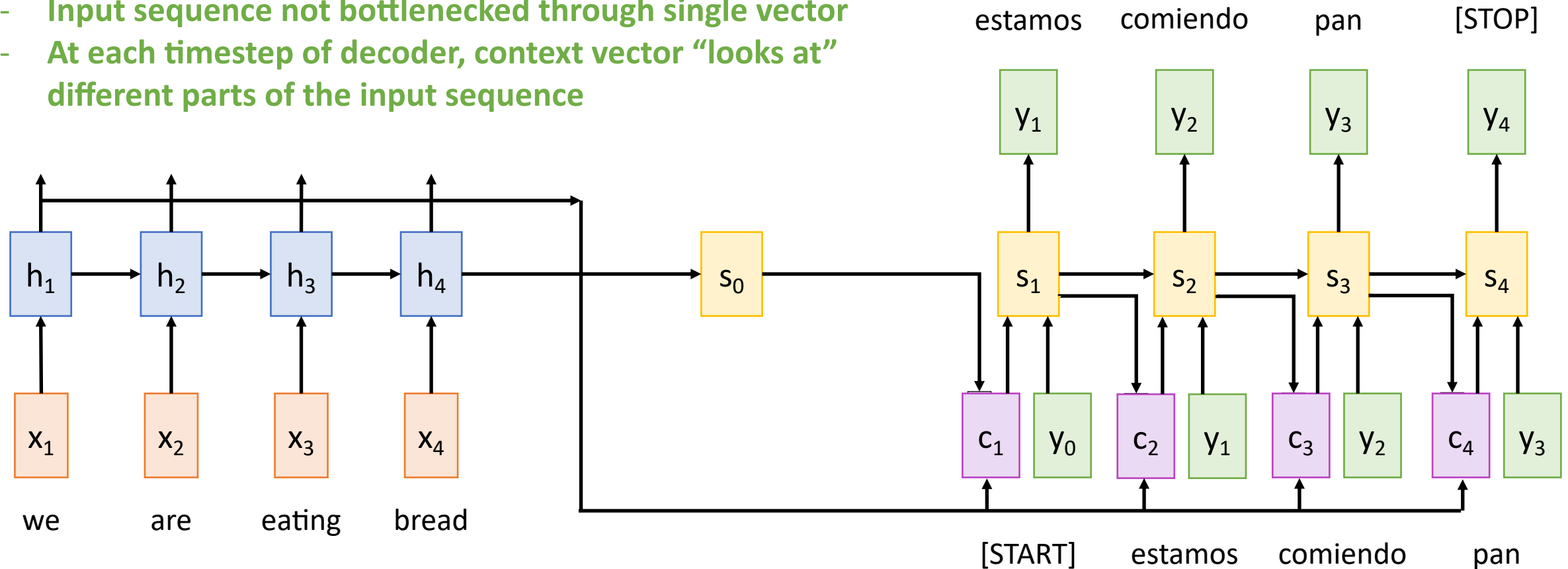


Bahdanau et al, "Neural machine translation by jointly learning to align and translate", ICLR 2015

Sequence-to-Sequence with RNNs and Attention

Use a different context vector in each timestep of decoder

- Input sequence not bottlenecked through single vector
- At each timestep of decoder, context vector “looks at” different parts of the input sequence



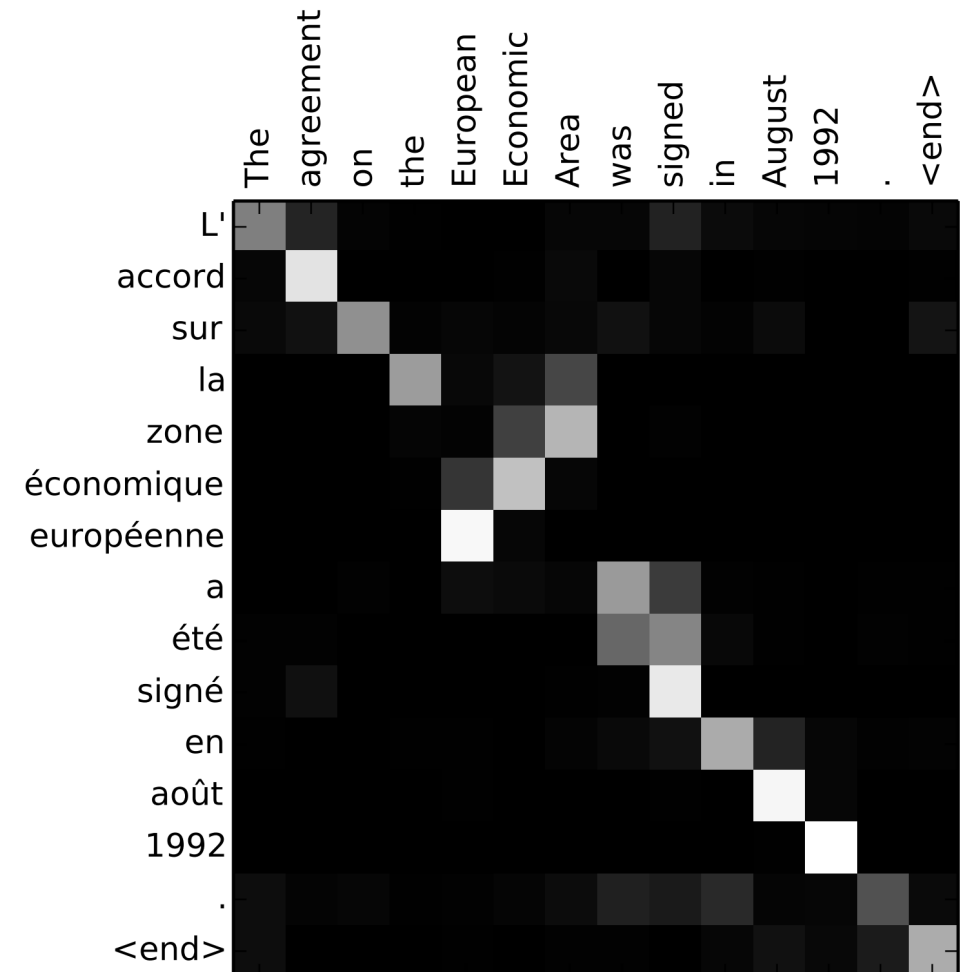
Sequence-to-Sequence with RNNs and Attention

Example: English to French translation

Input: “The agreement on the European Economic Area was signed in August 1992.”

Output: “L’accord sur la zone économique européenne a été signé en août 1992.”

Visualize attention weights $a_{t,i}$



Sequence-to-Sequence with RNNs and Attention

Example: English to French translation

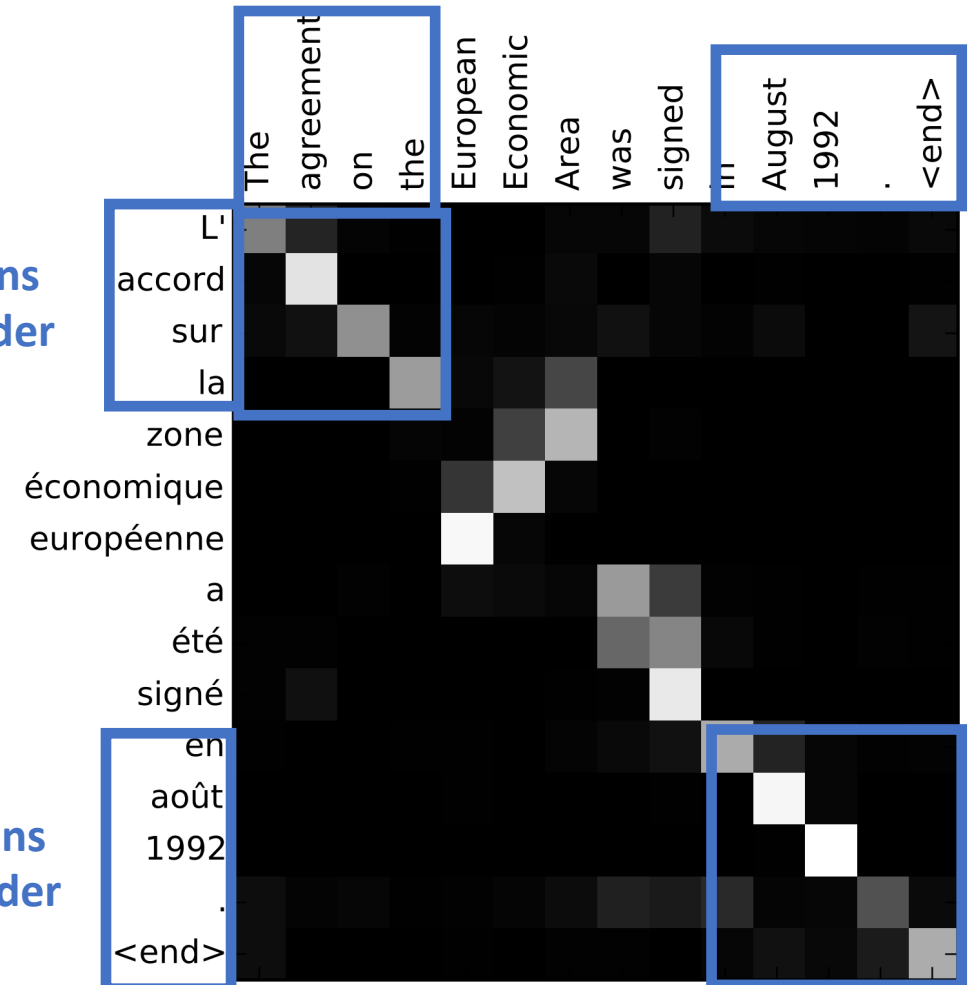
Input: “**The agreement on the** European Economic Area was signed **in August 1992.**”

Output: “**L'accord sur la** zone économique européenne a été signé **en août 1992.**”

Diagonal attention means words correspond in order

Diagonal attention means words correspond in order

Visualize attention weights $a_{t,i}$



Sequence-to-Sequence with RNNs and Attention

Example: English to French translation

Input: “The agreement on the European Economic Area was signed in August 1992.”

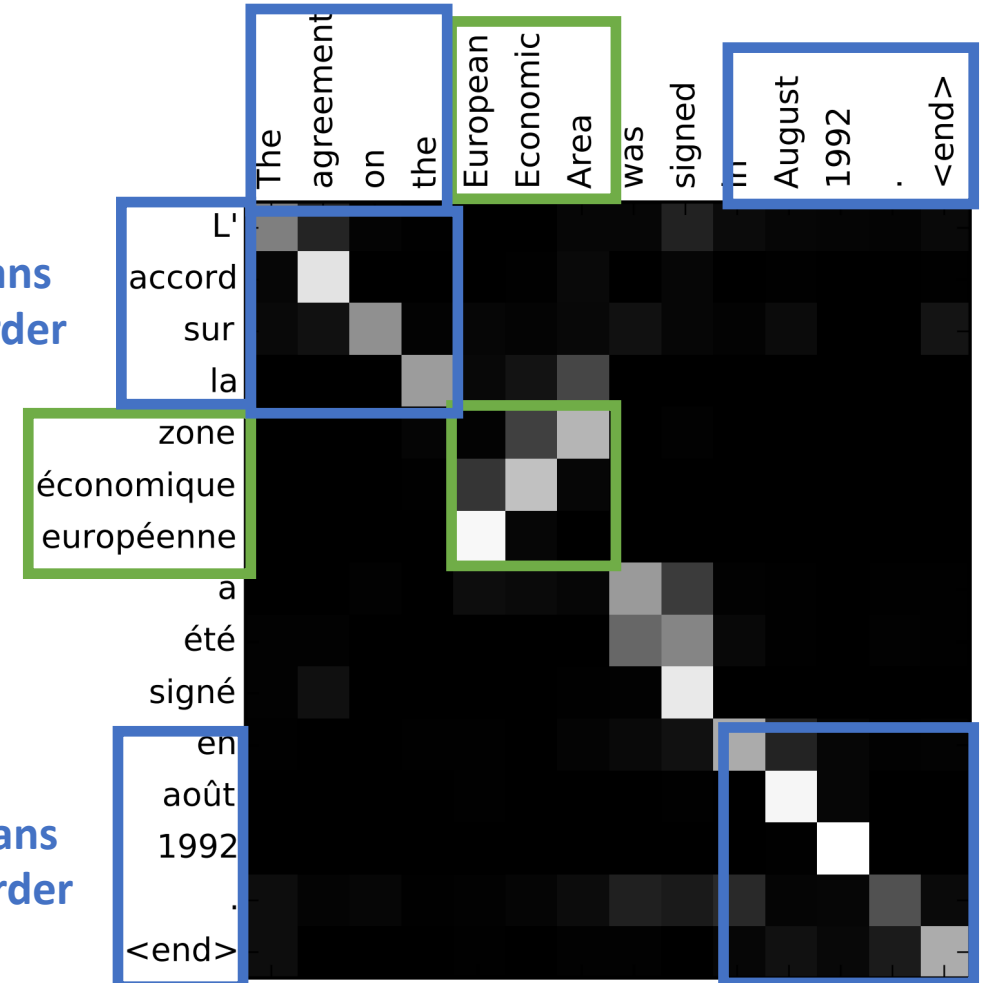
Output: “L'accord sur la zone économique européenne a été signé en août 1992.”

Visualize attention weights $a_{t,i}$

Diagonal attention means words correspond in order

Attention figures out different word orders

Diagonal attention means words correspond in order



Sequence-to-Sequence with RNNs and Attention

Example: English to French translation

Input: “The agreement on the European Economic Area was signed in August 1992.”

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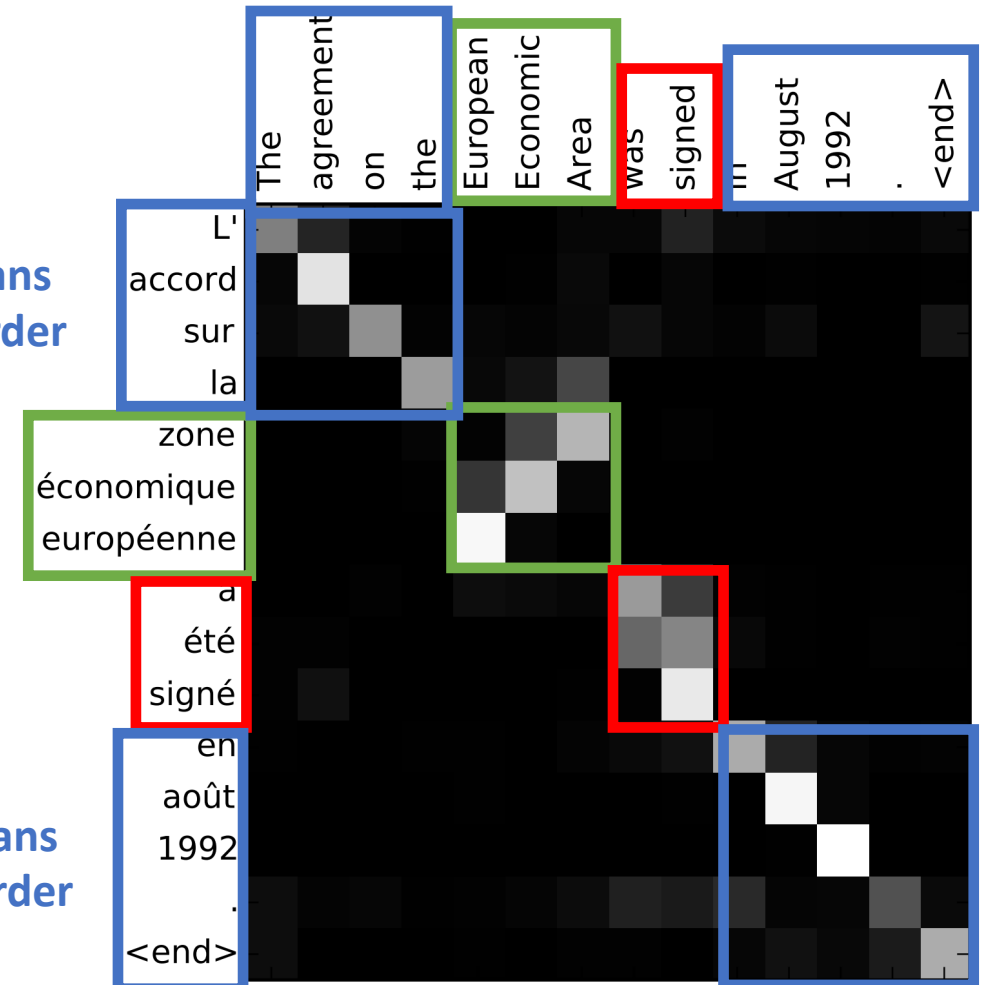
Visualize attention weights $a_{t,i}$

Diagonal attention means words correspond in order

Attention figures out different word orders

Verb conjugation

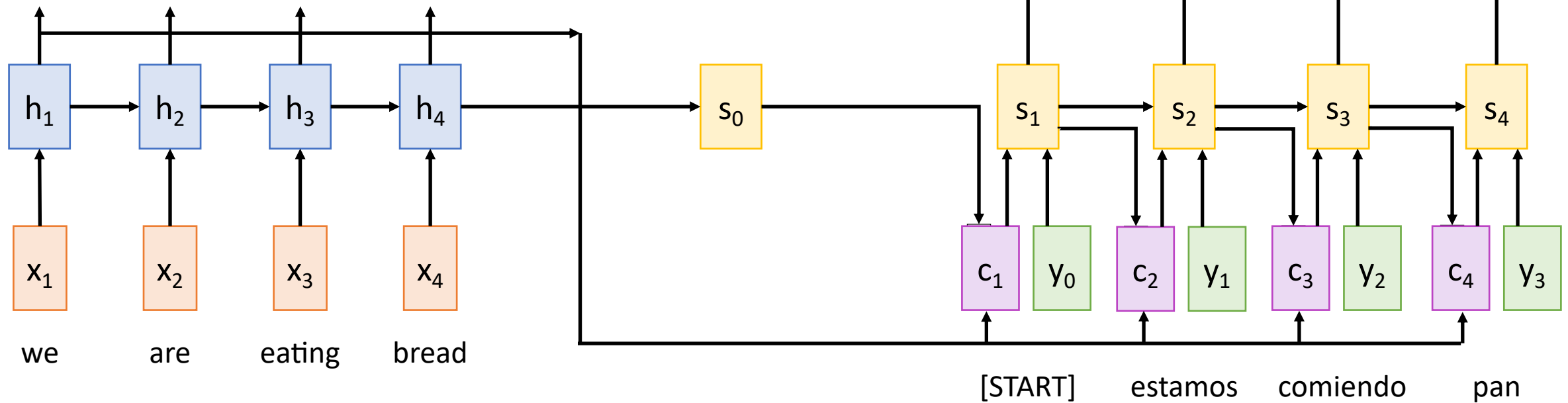
Diagonal attention means words correspond in order



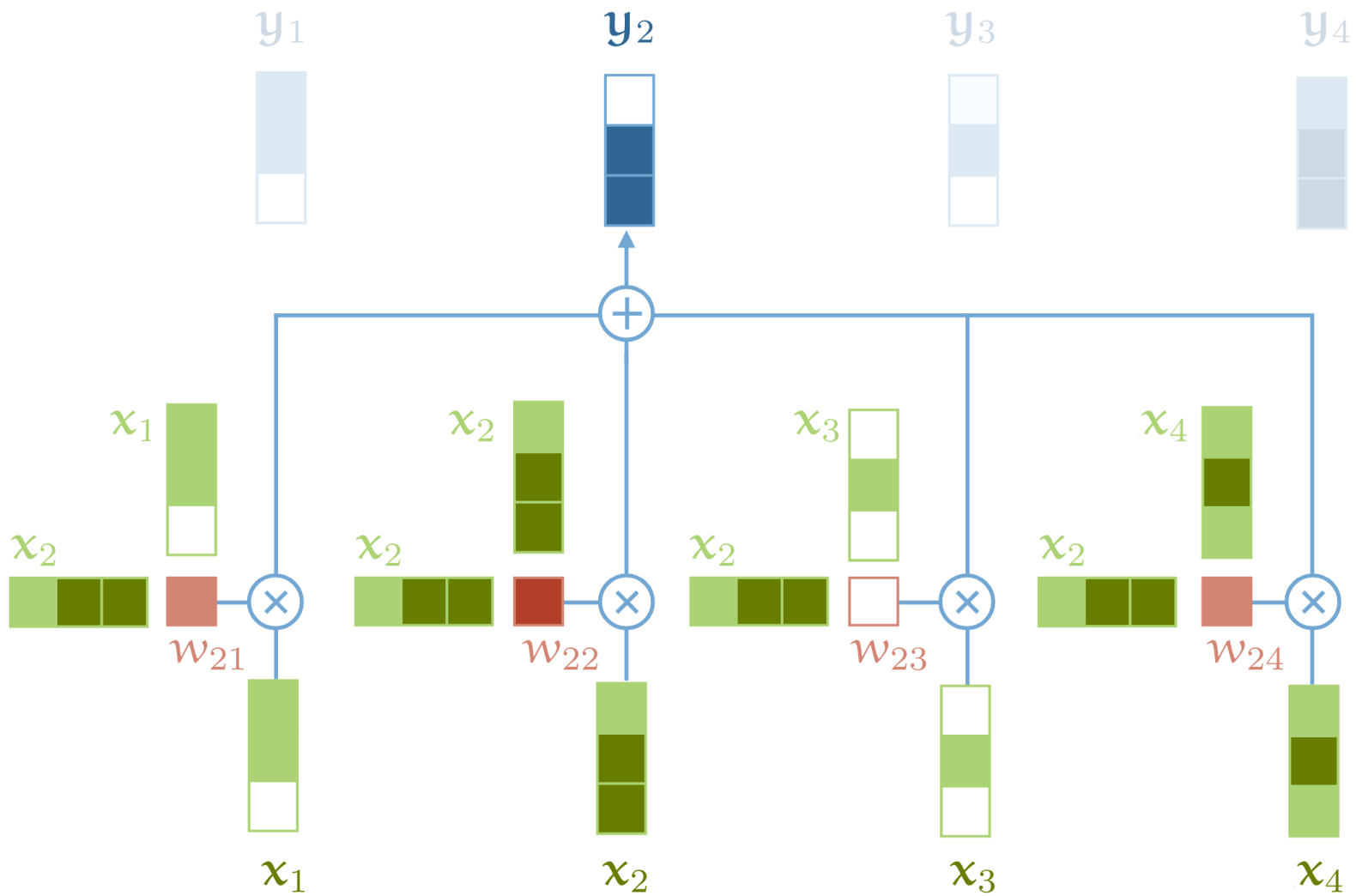
Sequence-to-Sequence with RNNs and Attention

The decoder doesn't use the fact that h_i form an ordered sequence – it just treats them as an unordered set $\{h_i\}$

Can use similar architecture given any set of input hidden vectors $\{h_i\}$!



Attention

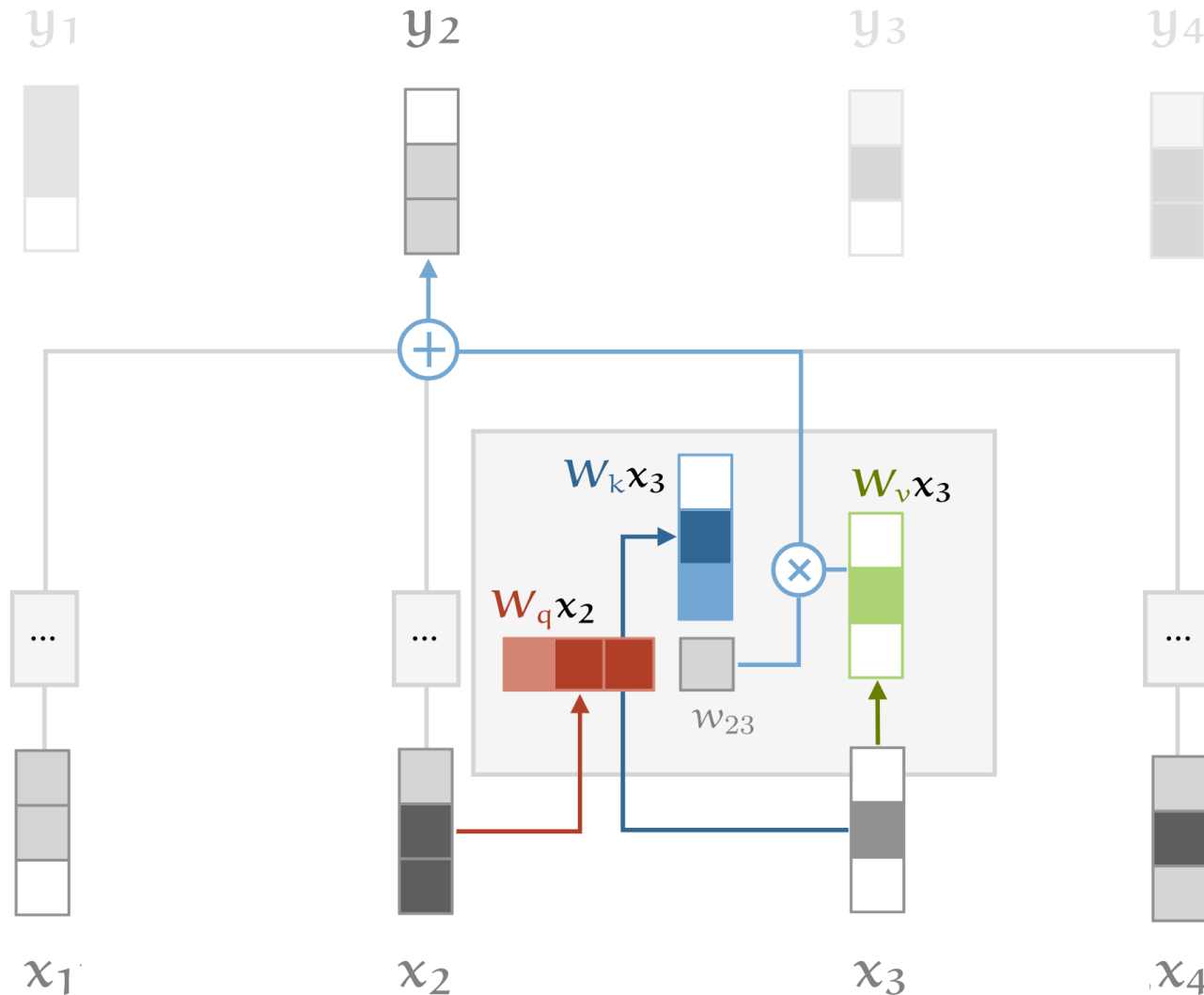


$$y_i = \sum_j w_{ij} x_{ij}$$

$$w_{ij} = \text{softmax}_j(x_i^T x_j / \sqrt{d_k})$$

$$w_{ij} = \frac{e^{x_i^T x_j}}{\sum_j e^{x_i^T x_j}}$$

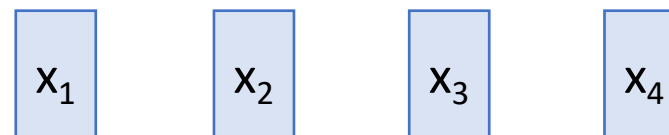
Attention (with key, query and value)



$$y_i = \sum_j w_{ij} W_v x_{ij}$$

$$w_{ij} = \text{softmax}_j((W_q x_i)^T W_k x_j / \sqrt{d_k})$$

The Transformer

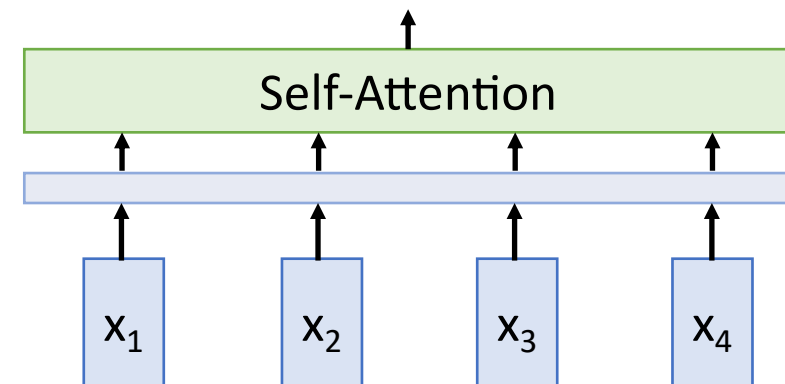


Vaswani et al, "Attention is all you need", NeurIPS 2017

Slide from Justin Johnson

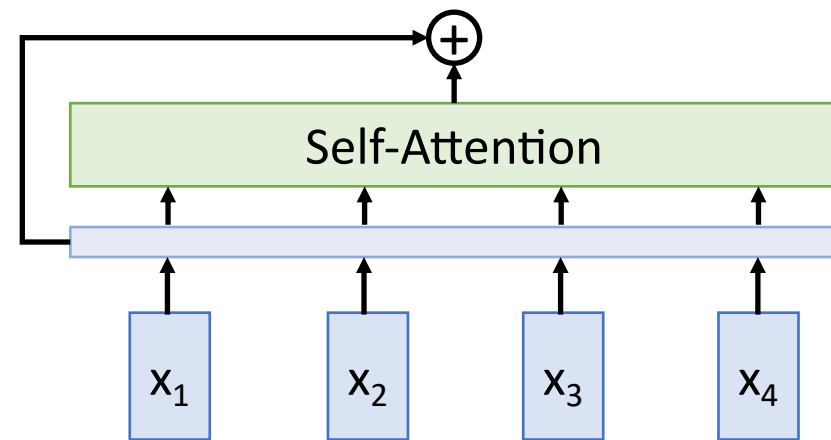
The Transformer

All vectors interact
with each other



The Transformer

Residual connection
All vectors interact
with each other



The Transformer

Recall **Layer Normalization**:

Given h_1, \dots, h_N (Shape: D)

scale: γ (Shape: D)

shift: β (Shape: D)

$\mu_i = (\sum_j h_{i,j})/D$ (scalar)

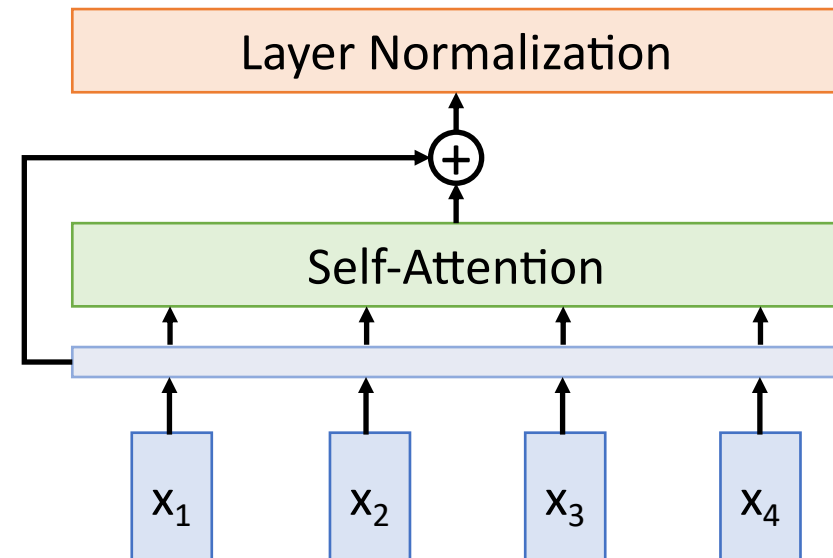
$\sigma_i = (\sum_j (h_{i,j} - \mu_i)^2/D)^{1/2}$ (scalar)

$z_i = (h_i - \mu_i) / \sigma_i$

$y_i = \gamma * z_i + \beta$

Ba et al, 2016

Residual connection
All vectors interact
with each other



The Transformer

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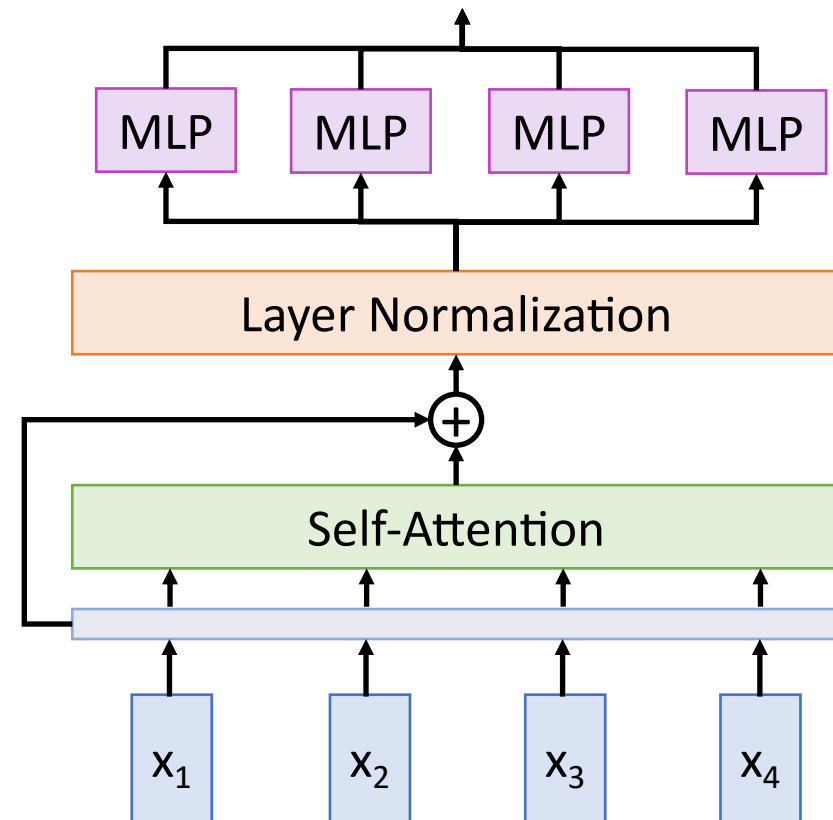
$z_i = (h_i - \mu_i) / \sigma_i$

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Ba et al, 2016

MLP independently
on each vector

Residual connection
All vectors interact
with each other



The Transformer

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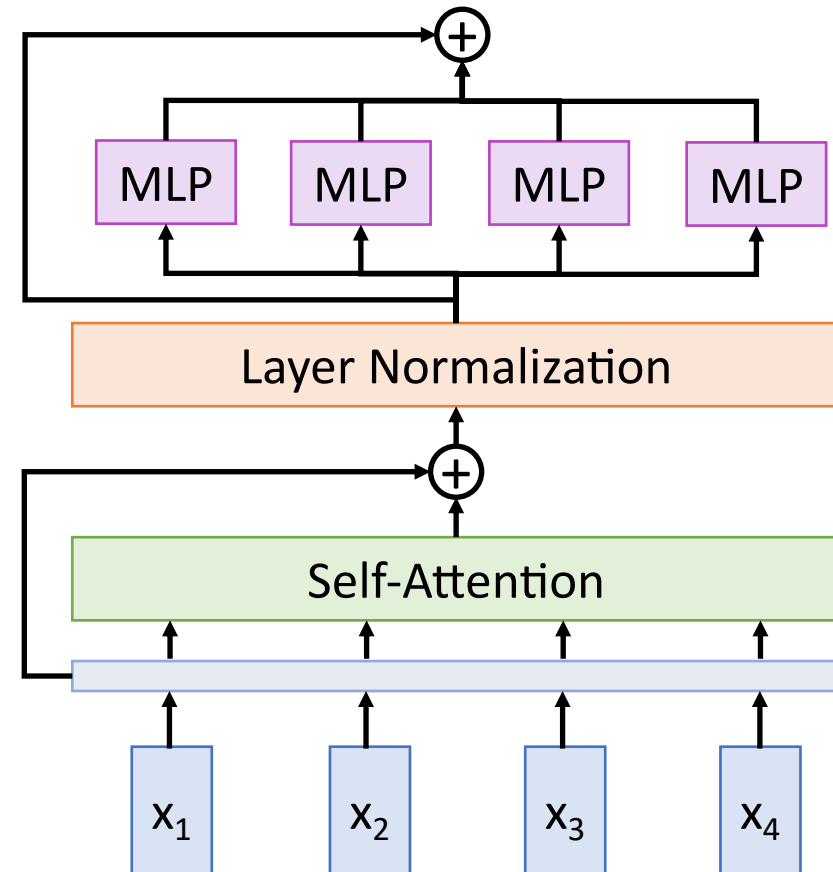
Ba et al, 2016

Residual connection

MLP independently
on each vector

Residual connection

All vectors interact
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The Transformer

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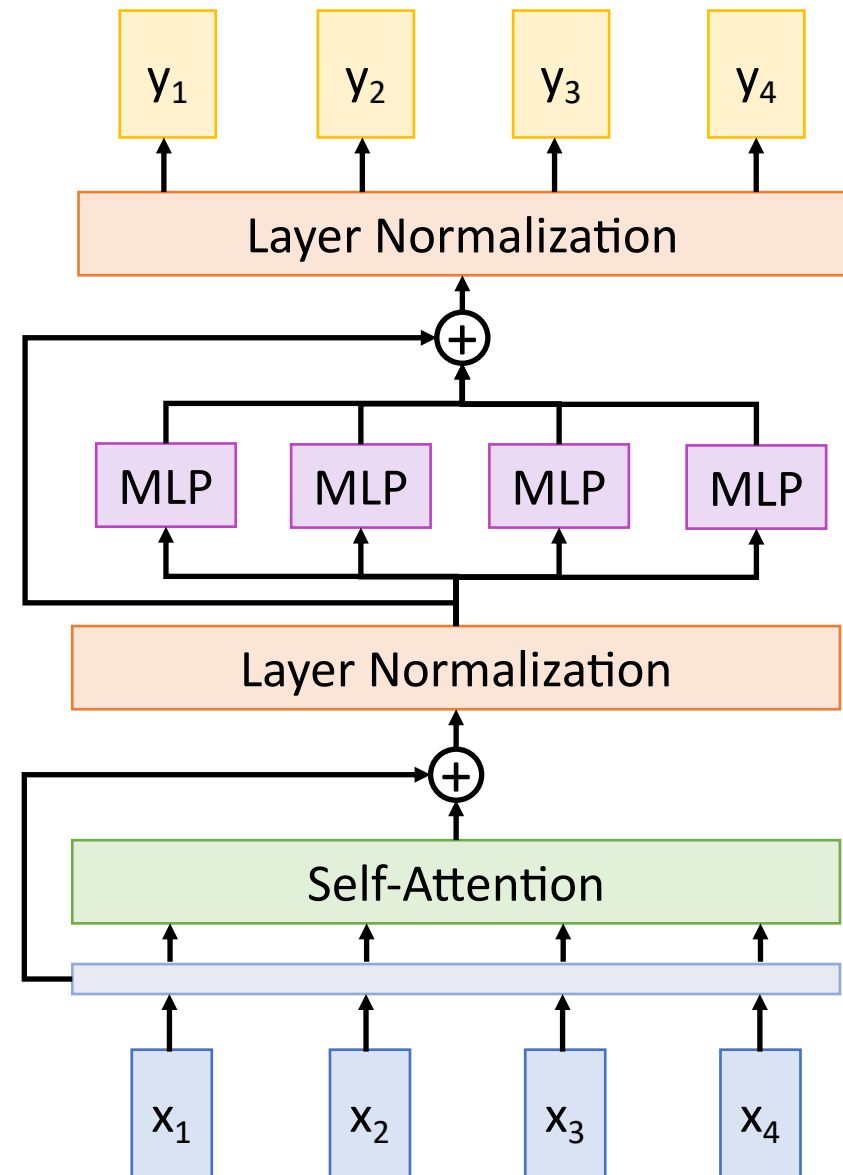
Ba et al, 2016

Residual connection

MLP independently
on each vector

Residual connection

All vectors interact
with each other



The Transformer

Transformer Block:

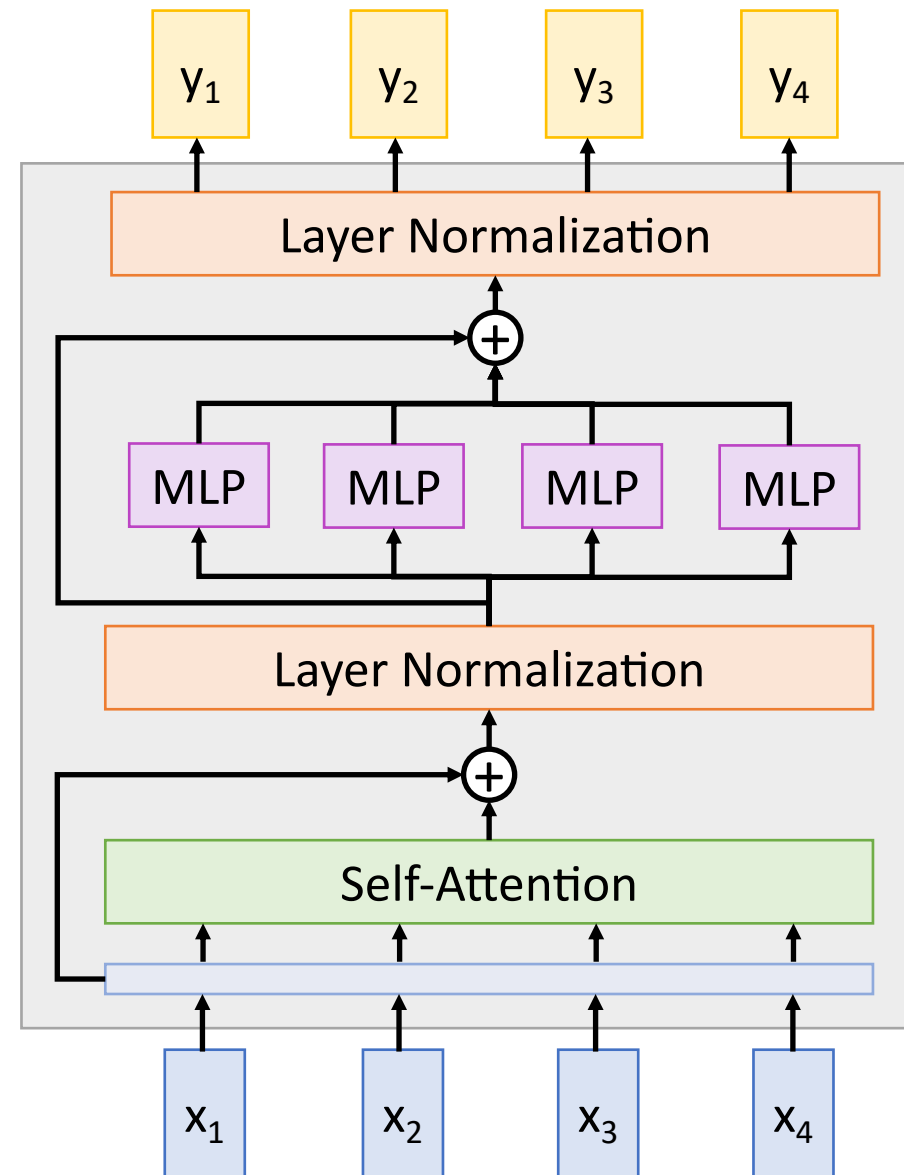
Input: Set of vectors x

Output: Set of vectors y

Self-attention is the only interaction between vectors!

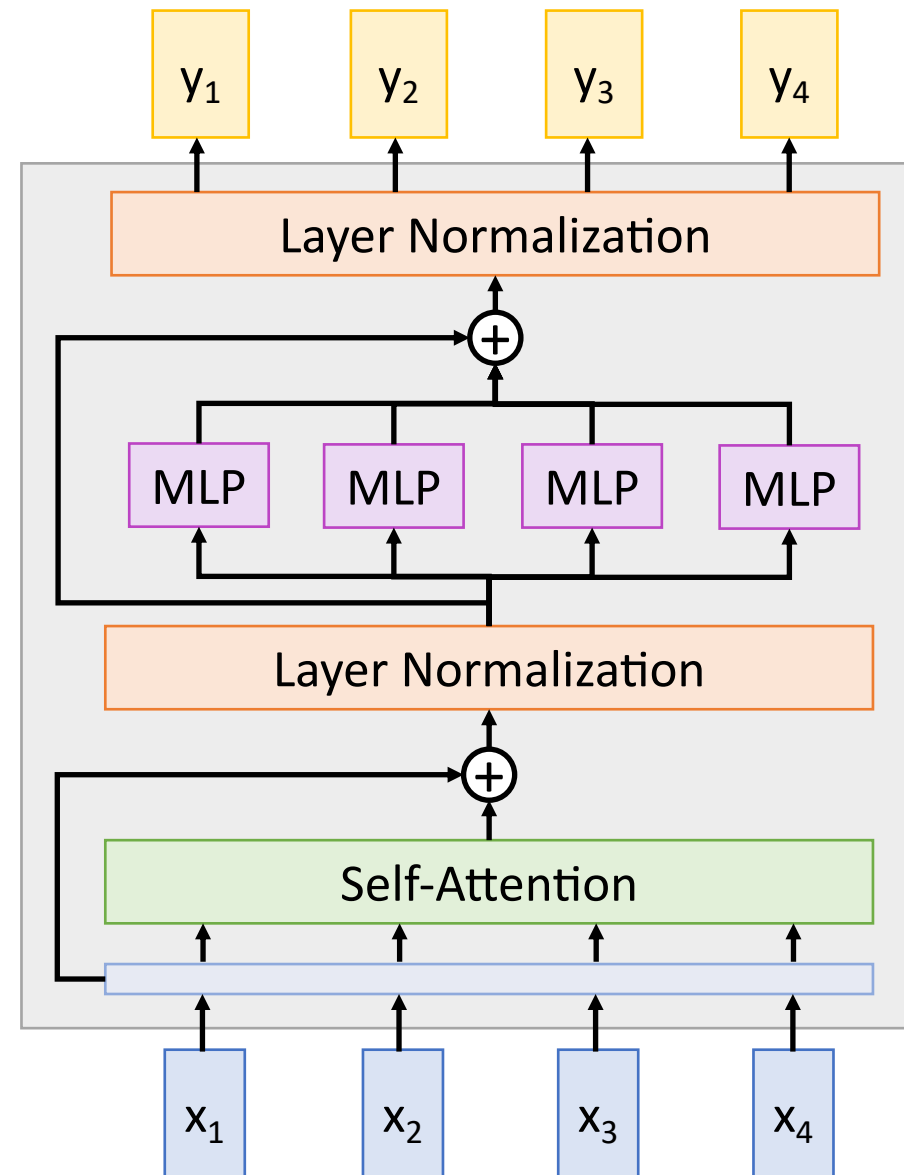
Layer norm and MLP work independently per vector

Highly scalable, highly parallelizable



Post-Norm Transformer

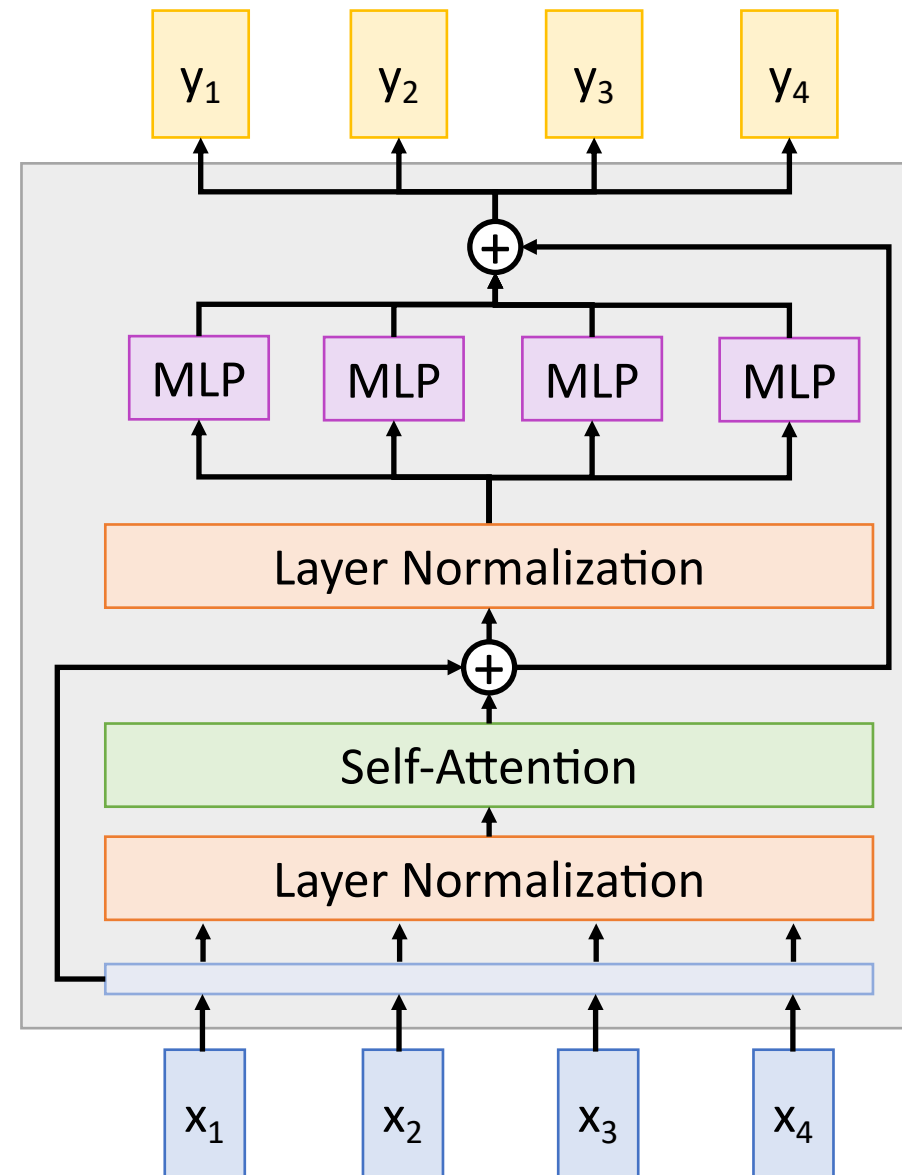
Layer normalization is
after residual connections



Pre-Norm Transformer

Layer normalization is
inside residual connections

Gives more stable training,
commonly used in practice



The Transformer

Transformer Block:

Input: Set of vectors x

Output: Set of vectors y

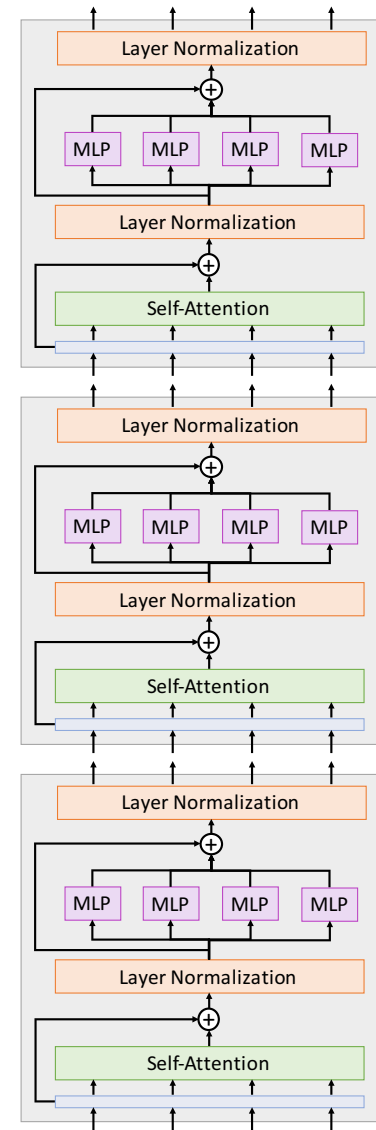
Self-attention is the only interaction between vectors!

Layer norm and MLP work independently per vector

Highly scalable, highly parallelizable

A **Transformer** is a sequence of transformer blocks

Vaswani et al:
12 blocks, $D_Q=512$, 6 heads



The Transformer

Table 2: The Transformer achieves better BLEU scores than previous state-of-the-art models on the English-to-German and English-to-French newstest2014 tests at a fraction of the training cost.

Model	BLEU		Training Cost (FLOPs)	
	EN-DE	EN-FR	EN-DE	EN-FR
ByteNet [18]	23.75			
Deep-Att + PosUnk [39]		39.2		$1.0 \cdot 10^{20}$
GNMT + RL [38]	24.6	39.92	$2.3 \cdot 10^{19}$	$1.4 \cdot 10^{20}$
ConvS2S [9]	25.16	40.46	$9.6 \cdot 10^{18}$	$1.5 \cdot 10^{20}$
MoE [32]	26.03	40.56	$2.0 \cdot 10^{19}$	$1.2 \cdot 10^{20}$
Deep-Att + PosUnk Ensemble [39]		40.4		$8.0 \cdot 10^{20}$
GNMT + RL Ensemble [38]	26.30	41.16	$1.8 \cdot 10^{20}$	$1.1 \cdot 10^{21}$
ConvS2S Ensemble [9]	26.36	41.29	$7.7 \cdot 10^{19}$	$1.2 \cdot 10^{21}$
Transformer (base model)	27.3	38.1	$3.3 \cdot 10^{18}$	
Transformer (big)	28.4	41.8	$2.3 \cdot 10^{19}$	

The Transformer: Transfer Learning

“ImageNet Moment for Natural Language Processing”

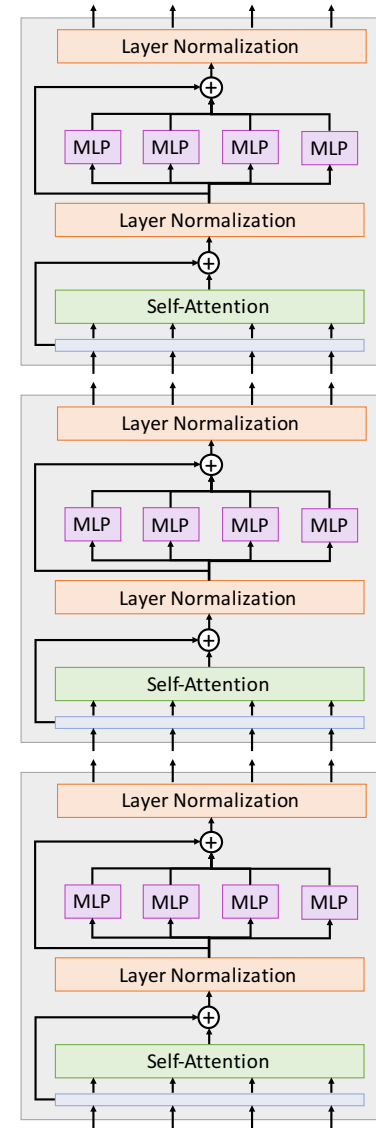
Pretraining:

Download a lot of text from the internet

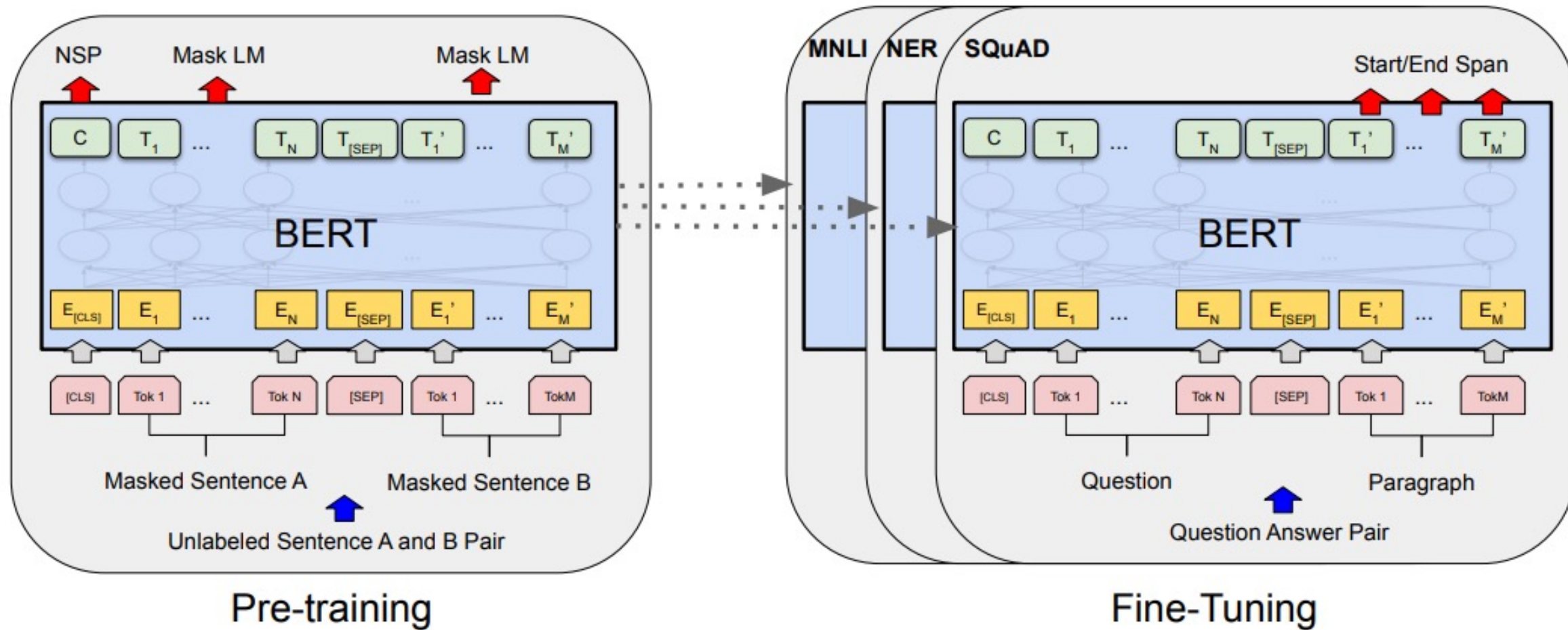
Train a giant Transformer model for language modeling

Finetuning:

Fine-tune the Transformer on your own NLP task



The Transformer: Transfer Learning



The Transformer: Transfer Learning

System	MNLI-(m/mm) 392k	QQP 363k	QNLI 108k	SST-2 67k	CoLA 8.5k	STS-B 5.7k	MRPC 3.5k	RTE 2.5k	Average
Pre-OpenAI SOTA	80.6/80.1	66.1	82.3	93.2	35.0	81.0	86.0	61.7	74.0
BiLSTM+ELMo+Attn	76.4/76.1	64.8	79.8	90.4	36.0	73.3	84.9	56.8	71.0
OpenAI GPT	82.1/81.4	70.3	87.4	91.3	45.4	80.0	82.3	56.0	75.1
BERT _{BASE}	84.6/83.4	71.2	90.5	93.5	52.1	85.8	88.9	66.4	79.6
BERT _{LARGE}	86.7/85.9	72.1	92.7	94.9	60.5	86.5	89.3	70.1	82.1

Table 1: GLUE Test results, scored by the evaluation server (<https://gluebenchmark.com/leaderboard>). The number below each task denotes the number of training examples. The “Average” column is slightly different than the official GLUE score, since we exclude the problematic WNLI set.⁸ BERT and OpenAI GPT are single-model, single task. F1 scores are reported for QQP and MRPC, Spearman correlations are reported for STS-B, and accuracy scores are reported for the other tasks. We exclude entries that use BERT as one of their components.